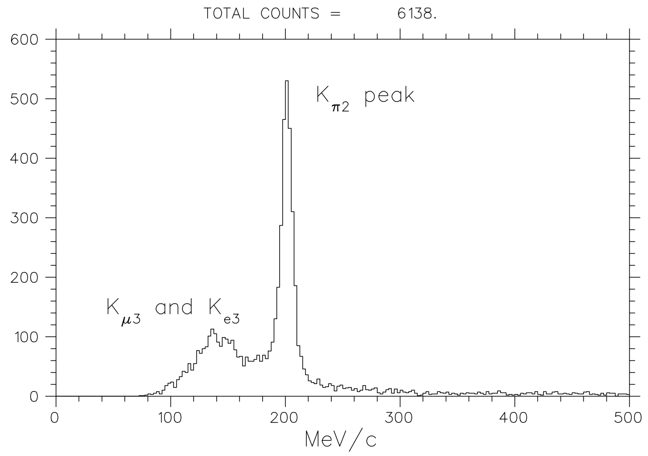


FIG. 4.9: Momentum distribution of reconstructed events in $\pi\gamma\gamma(2)$ triggers (after correction for energy loss in the Target).

The $K_{\mu 3}$ decays were extracted from the $\pi\gamma\gamma(2)$ trigger [4] after the following cuts:

1. One track consistent through Target, Drift Chamber, and Range Stack (RS-DC-TG track).
2. $P_{\text{track}} < 185$ MeV/c in order to reject the $K_{\pi 2}$ pions. (see Figure 4.9).
3. Two photon clusters whose invariant mass is near $m_{\pi^0} = 135.0$ MeV.
4. A reconstructed missing mass near $m_\nu = 0$ MeV. The reconstruction is based on the TG-corrected momentum of the track at the vertex with a muon hypothesis and using the reconstructed $\pi^0$ four-momentum.

This yielded a clean sample of low-momentum muons in the range stack.

### Protons

In order to have a consistent picture for any necessary empirical corrections to the RS-energy based on the measured masses, a sample of protons was also used. These come from the following chain [12]:

$$K^+ \rightarrow \pi^+\pi^0$$

$$\pi^+ + d \rightarrow p + p.$$

The quasi-deuterium represents the interacting nucleons within the Carbon nucleus. The 205.1 MeV/c $\pi^+$ has a 10 mbarn cross section in deuterium for the above reaction.

[4] $\pi\gamma\gamma(2) \equiv (T \cdot A) \cdot K_T \cdot \text{DelCo} \cdot \text{BV} \cdot \overline{(C + 11 + \cdots + 20 + 21 + \text{ECM} + \text{ECP})}$.

The resulting protons have momenta between 384 and 589 MeV/c. Such protons also come to rest within the A or B layers in the range stack thus providing another excellent source for testing and calibrating the measured energy for low-$KE$ particles in the RS.

### Saturation

Only a fraction of the kinetic energy lost by a charged particle in a scintillator is converted into fluorescent energy [13]. The remainder is dissipated non-radiatively, primarily in the form of lattice vibrations or heat. The fraction of the energy which is converted to light (the scintillation efficiency) depends on both the particle type and its energy. The scintillator's light yield for heavy charged particles is usually smaller as compared to electrons and is also non-linear at high initial energies. This is called the *saturation effect*. The response of organic scintillators to charged particles can best be described by the *Birks' equation*. It assumes that a high amount of ionization along the path of a charged particle leads to quenching from the damaged molecules and a consequent lowering of the scintillation efficiency. Furthermore, it is assumed that in the absence of quenching the light yield is proportional to energy loss,

$$\frac{dL}{dx} = S\frac{dE}{dx}$$

where $\frac{dL}{dx}$ is the fluorescent energy emitted per unit path length, $\frac{dE}{dx}$ is the actual energy loss for the charged particle, and $S$ is the normal scintillation efficiency. Then in the presence of quenching,

$$\frac{dL}{dx} = \frac{S\,\frac{dE}{dx}}{1 + K_B\,\frac{dE}{dx}}.$$

$K_B$ is called the *Birks' constant* and is experimentally treated as an adjustable parameter which gives the best fit to the data. In the scintillators used in the RS, the manufacturer's specified value of the Birks' constant is 0.013. However, a reasonable value of 0.010 has been widely used in the experiment's analyses and calibrations.

In measuring the true kinetic energy $E$ of the particle in the RS, it is necessary to "unsaturate" the measured energy $L$. The saturation effect is mostly important as a charged particle is coming to rest and hence is heavily ionizing. For $\pi\mu\mu$ triggers, the low-momentum particles come to a stop soon after they enter the RS making the saturation effects very noticeable.

The saturation effect was studied extensively for all three heavy particles, $p$, $\pi$ and $\mu$. The goal was to verify that a correct measurement of the masses in a known sample of each particle can be made. This was done by calculating the mass from $P$ and $K$, determining the peak position of the gaussian distribution, and comparing it

| Particle | Expected mass (MeV) | Peak Position (MeV) | sigma (MeV) |
|---|---|---|---|
| $\pi^+$ | 139.6 | 141.9 | 21.0 |
| $\mu^+$ | 105.7 | 105.0 | 12.0 |
| $p^+$ | 938.3 | 949.7 | 137.5 |

Table 4.6: Expected and measured masses for the $\pi^+$, $\mu^+$, and $p^+$ samples.

to the expected value.

With a Birks' constant of 0.01, it was found to be necessary to add 2.5 MeV to the saturated energy of each track in order to obtain the correct peak position for all three masses. The consistency among the three samples by using this method was grounds for adopting addition of 2.5 MeV as an empirical correction to the true kinetic energy of each track.

#### Spectra

Figure 4.10 shows the results of the mass measurements before and after the 2.5 MeV correction. Unsaturation based on the corresponding particle-id hypothesis was included in all measurements. The results are summarized in Table 4.6.

#### Cuts

The electron cut required the measured mass of any track (negative or positive) to be greater than 60 MeV/c$^2$. The mass was calculated twice based on both a pion and a muon hypothesis, and the same cut was applied for each value. For a muon, the mass is over-estimated with a pion hypothesis. Therefore muons with $\mu$-hypothesis masses slightly above 60 MeV/c$^2$ are not eliminated by either cut. On the high tail of the mass distributions, another cut was placed on positive tracks at 190 MeV/c$^2$, again, based on both hypotheses. The negative tracks were required to be below 135 MeV/c$^2$ to be consistent with a $\mu^-$. These cuts confined the positive and negative tracks roughly within $[m_\mu - 3\sigma_\mu, m_\pi + 3\sigma_\pi]$ and $[m_\mu - 3\sigma_\mu, m_\mu + 3\sigma_\mu]$, respectively.

No reasonable measurement of the mass can be made if two DC tracks end up in the same RS cluster. This scenario almost always occurs with a pair of positive and negative tracks. Events of this type were rejected before calculating the mass.

### 4.4.2 Combination of Charges

The mass cuts reduced the total sample by a factor of $\sim 2$. In particular, the number of Dalitz decays were greatly lowered since at least one of the $e^+e^-$ pair was subject to a mass measurement. In addition, the positive-track mass cut has a large rejection