CALCULATING THE ORBIT
OF COMET HALE-BOPP

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Abstract

In this project I have photographically measured the position of Comet Hale-Bopp in the sky, for a period of 4-5 months, and reduced the data to astronometric positions and times from a known place on Earth. From this data at three times, combined with a knowledge of the position of the Sun, I have used Gauss's method to determine the actual position of the Comet in the Solar System. I programmed Gauss's method into MathCad to do the numerical computations. Then I have converted the positions into orbital elements for the Comet, and compared them to actual Jet Propulsion Lab solutions based on the best data in the world. Although my methods could be improved, they yielded a very satisfactory orbit which puts my "calculated comet" very close to the actual position of Hale-Bopp for the last few, and next few years.

Purpose

I want to learn how to measure and calculate the orbit of a comet. Since Comet Hale-Bopp could be a very bright comet, I'll try to measure it, and calculate its orbital path around the sun. I also want to learn interesting applications of mathematics, and more about cameras, telescopes, and astronomy.

Hypothesis

By observing the position of the comet for a number of nights, I will be able to calculate the orbit of the comet using algebra, geometry, and trigonometry, just like professional astronomers and mathematicians.

Figure 1: My camera is mounted piggy-back on the large telescope, which I used for guiding on a star while taking pictures for 1 minute to 10 minute exposure times, depending on the film speed I was using, or whether clouds or the neighbor's lights were bad.

Procedure

I took three kinds of data. The first kind was aligning the telescope with the comet and reading off the dials for Right Ascension (RA) and Declination (Dec). However, this was really tricky and not very accurate, because you could read it backwards for each of the things, subtract instead of add, or misread it in the dark. The second way I took data was by sketching the comet through binoculars on a tripod. Then

I compared the position of the stars in my sketch to the star map on my computer program Redshift, and got the relative position of the comet. The third and most accurate way I got data was using a 300mm camera attached to the telescope.

I used the telescope to accurately guide to the comet or a nearby star for the long 5 or 10 minute exposures. Then I compared my picture to the computer program "Redshift", which has most star positions down to visual magnitude 10. It is more accurate than the binoculars because there is less human error in pictures as opposed to sketching. I always used the same lens to take my data. I have a slide scanner on my Dad's computer at home, and used it to scan the picture into the computer, at about 3000 dots per inch resolution. Later, to get the data even more accurate, I adjusted the Redshift magnification and overlayed the printed picture on the computer screen, and readout the comet coordinates using the computer cross-hairs. That way there is less human error.

Once I had my comet data, I also need to get the position of the sun for the same times as I have pictures of the comet. To simplify my calculations, I used "the Astronomical Almanac for the year 1996" from the Naval Research Lab to get the positions of the Sun in geocentric coordinates. Then, I used a method first invented by Karl Friedrich Gauss, a famous German astronomer, mathematician, and physicist (see the picture on the 10 Mark German banknote!), in the 1830's, to determine the orbit of any object in the solar system from "three complete observations". I followed the explanation of these orbit equations, and copied them out of a book ("Introduction to Celestial Mechanics", by S. W. McCuskey, Addison-Wesley, 1963, Sec 4-3 Gauss's Method, pg. 86-91). In order to actually calculate with them, I typed them into a computer program for solving equations, called "Mathcad". Mathcad is really neat because if you lay things out in the right order (defining all your terms as you go), it can actually do the calculations for you! I had lots of problems with vectors, cross products, dot products, typos in the book, and finding multiple variables, but with my dad's help I got all the bugs out. I entered my data into the equations and got orbital parameters. Then I put my orbital parameters into an astronomy program called First Light, which displayed the orbit for me.

Gauss's technique has some problems with very eccentric orbits (which is what Hale-Bopp has), and also only works with three observations. I really wanted to use more data points, to get more accuracy in my orbit parameters. Next, I contacted (using the Internet) the scientist at the Jet Propulsion Lab in Pasadena who calculates orbits of everything, Dr. Donald Yeomans in the Solar Systems Dynamics group of NASA! He was very helpful, and suggested a book that would be useful, and he offered to use my data in his fancy orbit code, which even takes into account the gravity of other planets as perturbations on the orbit, as well as fitting all my data points at the same time. He found two data points which had large errors, and I was able to go back and check them, and saw that they were my two worst pictures (biggest tracking errors). The fit from all of my data turned out very well, and you can see it compared to the real JPL latest fit.

Figure 2: Sept 26, 1996 photo of Comet Hale-Bopp in Ophiuchus. The Comet spent a lot of time in the "box" as seen from Earth.

My Data

Local Date	Local Time	Univ. Time	Julian Date	Sun X	Sun Y	Sun Z	Comet RA	Comet Dec.	Decimal RA	Decimal Dec.
4-Sep	10:00 PM	4:00 AM	331.6667	-0.963664	0.271679	0.117785	17 36 15	-6 20 25	264.0625000	-6.3402777
16-Sep	10:05 PM	4:05 AM	343.6701	-1.00053858	0.086789	0.037641	17 31 11	-5 39 11	262.7958200	-5.6530555
22-Sep	9:50 PM	3:50 AM	349.6597	-1.00346602	-0.007243	-0.00314	17 30 57	-5 38 01	262.7374600	-5.6336111
26-Sep	8:35 PM	2:35 AM	353.6076	-0.99933612	-0.069236	-0.03002	17 29 14	-5 07 39	262.3083332	-5.1275
27-Sep	10:17 PM	4:17 AM	354.6785	-0.99747388	-0.024482	-0.031224	17 29 39	-5 07 02	262.4125000	-5.117222222
28-Sep	7:54 PM	1:54 AM	355.5792	-0.99566541	-0.100094	-0.0434	17 29 38	-5 04 28	262.4083333	-5.0744444
29-Sep	8:45 PM	2:45 AM	356.6146	-0.99327759	-0.116256	-0.050408	17 29 25	-5 02 42	262.3541667	-5.045
2-Oct	7:55 PM	1:55 AM	359.5799	-0.98472633	-0.162331	-0.070386	17 29 38	-4 52 17	262.4125000	-4.871388888
22-Oct	8:00 PM	2:00 AM	379.5833	-0.86156452	-0.456282	-0.197827	17 35 04	-3 51 31	263.7666666	-3.858611111
Daylight	Savings	Time	Switch							
29-Oct	7:50 PM	2:50 AM	386.6181	-0.79285057	-0.548364	-0.237755	17 38 34	-3 25 13	264.6416664	-3.420277777
5-Nov	7:02 PM	2:02 AM	393.5847	-0.71314668	-0.631545	-0.273817	17 42 48	-2 59 19	265.6416667	-2.9886111
7-Nov	6:50 PM	1:50 AM	395.5764	-0.68836575	-0.653675	-0.28341	17 44 20	-2 47 49	266.0833333	-2.7969444
9-Nov	6:50 PM	1:50 AM	397.5764	-0.66263854	-0.675102	-0.292699	17 45 33	-2 39 37	266.3875000	-2.6602777
27-Nov	6:00 PM	1:00 AM	415.5417	-0.39929822	-0.8276	-0.35882	18 01 10	-0 59 28	270.2916667	-0.9911111
1-Dec	6:00 PM	1:00 AM	419.5417	-0.33433726	-0.850871	-0.368908	18 05 01	-0 28 55	271.2541667	-0.4819444
22-Jan-97	6:13 AM	1:13 PM	471.0507				19 19 46	11 24 13	289.76662	11.4036111

I estimate the readout error in my data is uncertain to 30 arc seconds, in the declination, and similarly in the right ascension. Sources of data input error, are the graininess of the film, precision of Redshift star coordinates, the computer monitor display distortions, and some eyeball overlay inaccuracies.

The Equations

My solution uses Gauss's method to determine the orbital elements. Karl Friedrich Gauss was a famous 19th century German mathematician and scientist (b. 1777, d. 1855). His method depends on the gravitational force law between two bodies, and Kepler's Laws for planetary motion. There are six quantities that need to be determined. You need the position (three quantities), and the velocity (three quantities), at one point in time. However, what I measure are the positions at three different times but I am missing the distances, so instead of having nine quantities, I only have six (two angles, at three times). This is enough to calculate with, though.

Joahannes Kepler was an imperial astronomer and mathematician in Poland (b. 1571, d. 1630). Kepler's First Law says that the orbit of a planet is an ellipse with one focus at the sun. Kepler's Second Law says that the radius vector joining the sun to an orbiting object sweeps over equal areas in equal intervals of time. This is because when the comet is far away, the vectors are longer but the comet is going slower. Kepler's Third Law says that the period of an orbit is proportional to the distance to the Sun raised to the 3/2 power. Sir Isaac Newton (b. 1642, d. 1727) was a very famous mathematician and scientist, who used Kepler's laws to help derive his law of gravitation. Newton's gravitational force law says that with two masses, m_1 and m_2, and distance r between them, the force along a line between their centers would be $F = -Gm_1m_2/r^2$, where G is the gravitational constant. Gravity obeys an inverse square force law. Another main equation I used was how to find the third vector of a triangle if I know the first two: $\vec{r} = \vec{\rho} - \vec{R}$. Another important equation I used was that of an ellipse, where $r=a(1-e^2)/(1+e\ cos\ f)$. The ellipse is described by a, the semi-major axis, f, the mean anomaly or angle to where the object is now, and e or eccentricity.

My six orbital elements are very important. Eccentricity is how curved the orbit is. An eccentricity of zero is a perfect circle, anything less than one is an ellipse, an eccentricity equaling one is a parabola and never returns, and anything over one is a hyperbola and which also doesn't return. Perihelion passage distance is how far away the comet is from the sun when it is as close as it ever gets to the sun, measured in Astronomical Units, or the average distance from the Earth to the sun. Longitude of the ascending node is the angle between where the Earth goes above the Plane of the Ecliptic, and where the object does, measured counter-clockwise. Argument of the perihelion is the angle from the line of the object's node to the line of the object's perihelion. Inclination is how tilted the orbit of the object is compared to the Ecliptic Plane. Perihelion passage time is when the perihelion occurs, in Julian date. Julian date is the number of days since the first day of January at 12 pm in 4713 BC. I drop the prefix and just use the last three numbers, so that Julian Date (JD) 2450398.1234 becomes 398.1234.

Calculation using MathCad

See the Appendix of my report for the actual calculations. Here is one set of my calculation results, using data from Sept. 4, Oct. 22, and Dec. 1, 1996; compared with the Jet Propulsion Laboratory data, as you can see below:

Orbital Parameter	Gauss's Method, My Data, 3 points	My Data, JPL's Equation, 14 points	JPL's Solution, 1772 points
Eccentricity	.97927548	.97615365	.995107808
Perihelion Passage Distance (AU)	.94719896	.898994201	.914103842
Longitude of Ascending Node (°)	282.84	282.714352	282.470692
Argument of Perihelion(°)	129.61	131.698036	130.590916
Inclination(°)	90.26	89.682385	89.429449
Perihelion Passage Time (JD)	541.690	539.374	539.635

The more I spread my data out, the better my calculations. I found that if I used my data too close together, say for example from Nov. 5, 7, and 9, then it was difficult to find a meaningful solution.. This data gave me the either the orbit of the Earth, or even no solution. I used both a graphical solution, and a computer root finder to solve some of my equation pairs. The graphical method was good to figure out where to pick a value for the root finder to start with. Since my technique only used three points at a time, it isn't as good as if I could fit them all at once. The fit with all 16 of my data points (with the two worst ones thrown out), was done by Dr. Don Yeomans at JPL, using a much fancier computer code.

Conclusions

I was surprised that my answers could come so close to the real thing, and I think I fulfilled my hypothesis. My data that I took is within about 1 minute (sometimes better), compared to the best fits of JPL. The reason that my calculated solution is off has to do with both my input data errors, and the fact that I use a simple equation, without corrections for many things. The hardest part in this experiment was understanding as much of Gauss' methods as I can. Debugging the equation in Mathcad was also really hard and time consuming. I even had one problem at first, which was due to a typo in the book I was using! I also had at first some problems forgetting to convert from hours/minutes/seconds into radians. This really messes up sines and cosines! After my code was all done, and working, I was able to compare some test data from Boulet's book, and his answers, with my code output. My code works great, with about 4 digits of accuracy in most orbital parameters, with model input data. With real input data it is slightly worse, but still close. Bad input data of course can really cause wild results.

Sources of Errors and Possible Improvements
• Find a method that fits all my data points together, not just three at a time.
• Guide telescope better while taking pictures, use higher magnification, and use better films.
• Figure how much the atmosphere bends light, especially near the horizon.
• I'm observing from the surface, not the center of the Earth, so my sun coordinates and Redshift are slightly wrong
• Keep more terms in a power series expansion equation that comes from the force equation.
• My cursor readout accuracy is limited in alt-azimuth mode in Redshift to degrees and minutes. This means about 30 arcseconds when I convert back into RA and Declination.
• Try to reduce parallax due to the glass of my computer screen getting in the way when I overlay for coordinates.
• Correct for the right Epoch on my star charts and times, and precession (wobble) of the Earth's axis.
• Do more iterations on the approximate part of the computer solution, using the previous position estimates.
• Try to track the head of the comet better, when reading out coordinates (the fuzzy ball is difficult).
• Record my times of observation always, as I take the photo. (One time, this cost me ½ hour uncertainty).

Bibliography:

1). *Introduction to Celestial Mechanics*, S. W. McCuskey, Addison-Wesley, 1963.
    My main reference for Gauss's method and orbit formulas

2). *Norton's Star Atlas*, Sky Publishing Corp, Cambridge, 1973.
    My main star map

3). *The Astronomical Almanac for the year 1996*, US Naval Observatory, US Government Printing Office.
    Used to get the Sun coordinates at my observation times.

4). *Methods of Orbit Determination for the Micro Computer*, Dan Boulet, William-Bell, Inc., 1991.
    A book which I got very late in my project. Useful for clarity, and numerical examples

5). *Unitron Telescope Instructions*, Unitron Scientific, 66 Needham Street, Newton Highlands, MA 02161.
    Used to learn to align and read telescope settings.

6). *Sky and Telescope magazine*, August 1996 through March 1997.
    Used for observing hints, comet charts, and latest sky happenings

7). *Nightwatch: An Equinox Guide to Viewing the Universe*, Terence Dickinson, Firefly Press, 1996.
    Used for general reading on astronomy, and good finders charts.

8). *Mechanics*, Keith R. Symon, Chapt. 3 on Vector Algebra, Addison-Wesley, 1971.
    Used for learning about vectors, dot and cross products

Electronic Materials:

1). REDSHIFT Astronomy program for Windows, Maris Multimedia Ltd, 1994
    Used for obtaining actual coordinates from my photos, by overlaying the photos on the screen Star Catalog is good down to magnitude 10.

2). FIRST LIGHT 1.1 Astronomy program for Windows, Virtual Reality Laboratories, 1990

Used to plot orbital elements in 3-dimensions, and you can create your own orbits. Good for testing how close my solutions are, graphically. But the star catalog is not as good as REDSHIFT.

3). MATHCAD Ver 6.0, general math solver program for Windows, MathSoft Inc., 1995
Used to for all my math calculations, including interpolating from tables, programming Gauss's solutions, and finding the orbital elements.

4). EXCEL and WORD for Windows, Ver. 7.0, Microsoft 1996.
Spreadsheet and wordprocessor used for making charts of data and documents.

5). Adobe PHOTOSHOP Ver 3.05 for Windows, Adobe Systems, 1996.
My image processing program. Used with slide scanner to input my pictures from their negatives, and then adjust colors, or print to large 8x10 format on an Epson InkJet Printer.

6). Finally, various sites on the Web, for the latest Jet Propulsion Lab orbit information, or for Comet light curves, or information about comets.