Solitary waves stability in Maxwell-Duffing equation

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Dielectric mediums with large concentration of metallic nanoparticles have attracted much attention recently due to to the increasing interest in the left-handed materials [1]. The propagation of electromagnetic pulse in such media containing metallic nanoparticles is described by the Maxwell-Duffing equations (MDE). These equations are non-integrable however they have solitary wave solutions which possess many properties of the solitons which are observed in the integrable equations such as Nonlinear Schroedinger equation (NLSE) or Korteveg-De-Vries (KdV) equation. The stability properties are the most striking difference between the solitary waves in MDE and the well-studied solitons in NLSE and KdV. It was observed numerically that the solitary waves in the MDE show semistable behaviour. The dynamics of the solitary wave crucially depends on the sign of the initial perturbation. The solitary wave survives when the perturbation has one sign and it totally breaks down for the other sign. Such behaviour is property of non-integrable equations and was not observed in any known integrable models. Analytical description of such semistability is the main aim of this work.

The Maxwell-Duffing equations can be written in the form

$$iE_x = Q, \tag{1}$$
$$iQ_t + \delta Q + |Q|^2 Q = E, \tag{2}$$

where E and Q are the slow varying envelopes of the electromagnetic field and plasmonic oscillator displacement. One can find the explicit expression for the moving solitary wave $Q(t,x) = Q_{sol}(x - vt)$, however it is too bulky for this text. In order to study the stability properties of this solution one should analyze the equations which describe the dynamics of the small perturbation over the solitary wave. Parametrizing the perturbation by the two fields u, v, such that $Q = Q_{sol} + u + iv$, one finds that in linear approximation the spinor $\psi = \begin{pmatrix} u \\ v \end{pmatrix}$ obeys the following equation:

$$i\partial_t \psi = \hat{L}\psi, \tag{3}$$

where $\hat{L}$ is some non-self-adjoint operator which depends on the solitary wave solutions. The similar operators in the other non-integrable equations possess some non-trivial eigenfunctions such as localized internal modes [2] or even linearly unstable modes [3]. Simple analysis shows that if any mode of this type exists in our problem the second order interaction between the solitary wave and the localized mode can lead to the semistable behaviour where the long-time solitary wave dynamics crucially depends on the sign of the initial perturbation. Unfortunately the spectrum of the $\hat{L}$ operator can be hardly found analytically and thus it is a problem a future work to study it numerically.

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