The World's Appetite for Light: A Simple Empirical Expression Spanning Three Centuries and Six Continents

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A simple empirical expression, in which consumption of artificial light depends linearly on the ratio between gross domestic product and cost of light, is found to be consistent with historical and contemporary data spanning three centuries, six continents, six lighting technologies, and over five orders of magnitude. The implication is that the income and price elasticities of demand for artificial light over the most recent three centuries have been unity (or near unity). From a practical perspective, this result represents the historically consistent baseline assumption for constructing future scenarios of consumption of light and associated energy. Given that lighting accounts for about 6.5% of world energy consumption and is poised at the brink of a technology revolution, these scenarios may be useful for forecasting future energy consumption and informing public policy. From a theoretical perspective, this result has implications on the "rebound effect," of current interest in energy economics.

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1 INTRODUCTION

Artificial light has long been a significant factor contributing to the quality and productivity of human life. It expands the productive day into the non-sunlit hours of the evening and night, and during the day it expands productive spaces into the non-sunlit areas of enclosed dwellings, offices and buildings (Bowers 1998; Boyce 2003; Schivelbusch 1988).

Because we value artificial light so highly, we consume huge amounts of energy to produce it. The production of artificial light consumed an estimated 6.5% of total global primary energy in 2005. This percentage is large and, coupled with increasing concern over energy consumption, has inspired a number of projections of light and associated-energy consumption into the future (Kendall and Scholand 2001; Tsao 2002; Navigant 2006). Such projections are of special interest at this point in history when lighting technologies are evolving rapidly. Filament-based incandescent technology is giving way to gas-plasma-based fluorescent and high-intensity-discharge (HID) technology; and over the coming 10-30 years both may give way to solid-state lighting (SSL) technology (Tsao 2004; Krames et al. 2007; Schubert et al. 2006; Shur and Zukauskas 2005; Krames et al. 2007).

Projections of the consumption of light and associated power are difficult, however, because there is no consensus regarding the factors that underlie the demand for light. Hence, relatively arbitrary assumptions must be made, the most common of which is that demand for light is independent of the efficiency (and hence cost) with which it is produced and delivered. If true, then technology evolution leading to efficiency improvement would not lead to an increase in light consumption, but rather to a decrease in energy consumption. If not true, however, there might instead be an increase in light consumption, a type of "rebound" effect (Khazzoom 1980; Brookes 1990) that would lessen the decrease in energy consumption.

Indeed, the possibility of rebound effects are of intense current interest (UKERC 2007) not just for lighting, but for all the energy services (e.g., transport of people and goods, heating and cooling of spaces, and process machinery and appliances). These services are the dominant consumers of energy in our modern economy, and whether (and by how much) improvements in their energy efficiencies increase or decrease energy consumption has important ramifications on public policies aimed at reducing energy consumption and risk of human-induced climate change.

Because of the importance of possible rebound effects, much work has been expended trying to understand and quantify them, both theoretically (Saunders 1992) and empirically (Greening et al. 2000). For any particular energy service, however, its magnitude has been difficult to quantify, especially over longer time periods for which its magnitude can be anticipated to be largest. Nearly all empirical studies of which we are aware focus on relatively short (months to years) time periods during which societal-use paradigms for an energy service are relatively static. It is only over longer (decades to centuries) time periods that radically new societal-use paradigms may be expected to emerge, with associated radical changes in consumption of that service. It is in fact these radically new societal-use paradigms that were envisaged in the first formulation of the rebound effect (Jevons 1906; Alcott 2005).

Recently, a number of careful estimates have been made of the consumption of light in various nations over diverse geographic, economic and temporal circumstances. In this work, we have built on these estimates -- filling in gaps in the datasets, estimating demand factors auxiliary to the datasets, and self-consistently integrating across the datasets -- to create a quantitative picture of the consumption of light and associated energy. These estimates span a wide enough (over five orders of magnitude) dynamic range to enable accurate correlations between the consumption of light and its underlying demand factors. They also span a long enough (decades to centuries) time period to enable quantitative conclusions to be drawn about the rebound effect in this important energy service over historically significant time scales.

Indeed, lighting appears to be uniquely well suited amongst the various energy services for such a quantitative study. Its output (light), is more easily defined and estimated than the outputs (e.g., weight times distance traveled, or change in temperature times volume of space) of other energy services. Though it has had a long history of technology innovation, each major lighting technology has had a reasonably well-defined historical period of maturity or dominance, without the accounting difficulties associated with a proliferation of subtechnology variants, each with a different energy efficiency, market penetration and cost structure.

The remainder of this paper is organized as follows. In Section 2, we discuss how the estimates, taken from a number of sources, were self-consistently integrated and interpreted. In Section 3, we describe what we have found to be the primary factors underlying the consumption of light: gross domestic product and cost of light. In Section 4, we describe what some of the secondary (non-income and non-price) factors are that might underlie the consumption of light at a higher level of detail. Some of these factors might also become more important in the future, and hence may cause a deviation from the simple dependence on gross domestic product and cost of light found in the past. In Section 5, we discuss the implications of this work on future scenarios of world consumption of light and associated energy. Finally, in Section 6, we discuss our results in the context of a simple Cobb-Douglas analytical model of the rebound effect.

2 DATA, ESTIMATES AND ASSUMPTIONS

In this Section, we discuss estimates of the consumption of light, along with how we have built on these estimates -- filling in gaps in the datasets, estimating demand factors auxiliary to the datasets, and self-consistently integrating across the datasets -- to create a quantitative picture of the consumption of light and associated power. We organize our discussion according to the quantity being estimated: consumption of light, luminous efficacy, cost of energy and light, consumption of associated energy, and finally gross domestic product and population. Before we begin, though, we make a few comments regarding scope, nomenclature and units.

First, wherever monetary units are used, we use year 2005 US$, using exchange rate conversions across nations from the XE Interactive Currency Table (XE 2007) and deflation conversions across years from Measuring Worth (MW 2007).

Second, we choose as our units for light and associated energy: petalumen-hours (Plmh) and petawatt-hours (PWh). These units are large, but appropriate for nation-scale quantitites. As the usual unit for time scale of consumption is the year, we then choose as our units for the rates of consumption of light and associated energy: petalumen-hours per year (Plmh/yr), denoted by the symbol Φ, and petawatt-hours per year (PWh/yr), denoted by the symbol $\dot{E}_{\Phi}$. We will often refer to these simply as consumption of light or energy, though technically speaking they are *rates* of consumption of light or energy. Also, we choose as our unit of population billions of persons (Gper), so that our units for per capita rates of consumption of light and associated energy become: megalumen-hours per person-year (Mlmh/(per-yr)), denoted by the symbol φ, and megawatt-hours per person-year (MWh/(per-yr)), denoted by the symbol $\dot{e}_{\varphi}$. Analogously, we denote gross domestic product *GDP*, with units of billions of dollars per year G$/yr, and we denote per capita gross domestic product *gdp*, with units of $/(per-yr).

Third, our focus throughout is on consumption of light in those applications in which light is used to illuminate (and hence is viewed indirectly, after it scatters from an object or scene) but not those in which light is used to signal or display information (and hence is viewed directly). We note here that the energy economics of these two broad classes of applications for light are quite different. For illumination, the cost of light is mostly the cost of the energy that is converted into light;[1] while for signaling or information display, the cost of light is mostly the cost of the capital equipment used to convert energy into light.[2] Hence, by including illumination but not signaling or information display, we are focusing on those applications for light which are most energy-intensive.

Fourth, within this broad class of illumination application, our intent is to be comprehensive, and hence to include consumption of light produced from all types of energy sources: from electricity in those populations with access to the electrical grid, from chemical fuel in those populations without access to grid electricity, and from electricity produced in situ from chemical fuel in vehicles. We think of these as defining three energy-source sectors and, for simplicity, refer to them as the vehicle, grid electricity, and fuel-based energy-source sectors. We note that, even in modern times, the fuel-based sector is not insubstantial. It has been estimated that, as recently as 1999, 2 billion persons did not have access to grid electricity, and were largely dependent on kerosene lamps for their lighting (Mills 2005).

Fifth, because we want to integrate data across these different energy-source sectors, we distinguish their energy units by using subscripts: "e" for electricity and "c" for chemical. Then, we convert between units by assuming efficiencies for the conversion of chemical fuel to electricity followed by transport of the electricity to point-of-use. For grid electricity, we use an efficiency of $\sigma_{grid} = 0.316\ W_e/W_c$ (DOE 2007, Chap. 6). For vehicle electricity, we use an efficiency of $\sigma_{veh} = 0.15\ W_e/W_c$, which is basically the product of engine (assuming a mix of gas and diesel) and alternator efficiencies (Navigant 2003). Thus, luminous efficacies (denoted by the

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[1] A typical 30W compact fluorescent light bulb (equivalent to a 100W incandescent light bulb) had, in early 2008, a retail capital cost of about $3, but, powered by electricity at $0.08/kWh, will use about $19 worth of electricity over a typical 8,000 hour operating life (http://www.bulbs.com/eSpec.aspx?ID=13178&Ref=Compact+Fluorescent+Screw-in&RefId=20&Ref2=Light+Bulbs).

[2] A typical 48W 22" liquid-crystal display television had, in early 2008, a retail capital cost of about $400, but, powered by electricity at $0.08/kWh, will only use about $200 worth of electricity if used over its product life of 50,000 hours (6 hours per day for 23 years), less if used for less than its product life, as is typical for advanced consumer electronics (http://www.viewsonic.com/products/lcdtv/NX2232w/).

symbol[3] η_φ) in units of lm/W_c are equivalent to those in units of lm/W_e multiplied by one of these efficiency factors; energy consumption in units of PW_ch/yr is equivalent to that in units of PW_eh/yr divided by one of these efficiency factors; and costs of energy (*CoE*) in units of \$/MW_ch are equivalent to those in units of \$/MW_eh multiplied by one of these efficiency factors.

2.1 Consumption of Light

The starting point for our estimates of the consumption of light is the five datasets summarized in Table 1. The first dataset (in brown) we refer to as the "Fouquet-Pearson" dataset: it represents estimates from the monumental work by Fouquet and Pearson on consumption of light in the United Kingdom over a 300-year time span (Fouquet and Pearson 2006). The second dataset (in dark grey) we refer to as the "IEA" dataset: it represents estimates from the recent comprehensive study by the International Energy Agency on consumption of light in various nations or groups of nations for which grid electricity is available, mostly in the year 2005 (IEA 2006). The third dataset (in blue) we refer to as the "Navigant" dataset: it represents an estimate from the extremely thorough bottoms-up survey by Navigant of consumption of light in the United States in 2001 (Navigant 2002). The fourth dataset (in green) we refer to as the "Mills" dataset: it represents estimates by Mills and co-workers of the consumption of light in China in 1993 (Min et al. 1997) and in populations in 1999 for which grid electricity was not available (Mills 2005). The fifth dataset (in red) we refer to as the "Li" dataset: it represents an estimate of consumption of light in China in 2006 (Li 2007a).

Of the estimates in these datasets, we consider those of contemporary consumption of light to be much more accurate than those for historical consumption of light. Despite the care with which the historical estimates were made, such estimates are fraught with difficulties, not the least of which are assumptions on the mix of lighting technologies used during periods when the efficiencies (or luminous efficacies) of these technologies were evolving rapidly. And of the estimates of contemporary consumption of light, we consider that of the United States in 2001 to be the most accurate, and those for China in 1993 and 2006 to be the least accurate.

All five of the datasets provide estimates of consumption of light for two of the energy-source sectors (grid electricity and fuel-based). Although it is a small (of order 1%) contribution, for completeness we have added to the contemporary (post 1950) data estimates of consumption of light for the third energy-source sector (vehicles). To do this, in anticipation of the result for all energy-source sectors discussed in Section 3, we assume that per capita consumption of light associated with vehicles is simply proportional to the ratio of the *gdp* (\$/(per-yr)) of a nation (or group of nations) to the cost of light (*CoL*, in \$/Mlmh) in that nation (or group of nations):

$$\varphi_{veh} = \beta_{veh} \cdot \frac{gdp}{CoL_{veh}}. \qquad (2.1)$$

[3] This symbol is often used both, in illumination engineering, for luminous efficacy (Ohno 2004) and, in economics, for elasticities. To avoid confusion, in this paper we use it only for luminous efficacy.

			per capita Consumption of Light (φ)				per capita Consumption of Energy ($\dot{e}_\varphi$)				Luminous Efficacy (η_φ)				Cost of Energy (*CoE*)				Cost of Light (*CoL*)				Population, *GDP*, *gdp*			Predicted Consumption of Light and Energy			
Nation or Nations	Year	Predominant Energy Source or Lighting Technology	Vehicle	Grid	Fuel-Based	All	Vehicle	Grid	Fuel-Based	All	Vehicle	Grid	Fuel-Based	φ-Weighted Inverse Average	Vehicle	Grid	Fuel-Based	All	Vehicle	Grid	Fuel-Based	All	Population (N)	Gross Domestic Product (*GDP*)	per capita gross domestic product (*gdp*)	per capita Consumption of Light $\beta \cdot gdp/CoL$	per capita Consumption of Energy $\beta \cdot gdp/[(1+\kappa_\varphi) \cdot CoE]$	Consumption of Light $\beta \cdot GDP/CoL$	Consumption of Energy $\beta \cdot GDP/[(1+\kappa_\varphi) \cdot CoE]$
			Mlmh/(per-yr)				MW_eh/(per-yr)				lm/W_e				\$/MW_eh				\$/Mlmh				Gper	G\$/yr	\$/(per-yr)	Mlmh/(per-yr)	MW_eh/(per-yr)	Plmh/yr	PW_eh/yr
UK	1700	Can			0.00058	0.00058			0.0068	0.0068			0.085	0.085			2,188	2,188			29,253	29,253.1	0.0086	16	1,863	0.00046	0.0046	0.0000039	0.000039
UK	1750	C			0.00060	0.00060			0.0065	0.0065			0.092	0.092			2,360	2,360			29,386	29,385.8	0.0125	27	2,120	0.00052	0.0048	0.0000065	0.000060
UK	1800	C+O			0.00274	0.00274			0.0247	0.0247			0.111	0.111			1,439	1,439			14,846	14,846.5	0.0183	44	2,414	0.00116	0.0090	0.0000212	0.000164
UK	1850	Gas+Ker			0.01288	0.01288			0.0271	0.0271			0.475	0.475			576	576			1,386	1,385.7	0.0272	94	3,472	0.01792	0.0324	0.0004872	0.000879
UK	1900	Gas+Ker			0.26728	0.26728			0.3519	0.3519			0.759	0.759			345	345			520	519.6	0.0412	275	6,693	0.09212	0.1039	0.0037913	0.004278
UK	1950	Inc		4.99		4.98733		0.43		0.4299		12		11.600		182		182		20.9		20.9	0.0501	518	10,340	3.53664	0.3049	0.1772812	0.015283
UK	2000	Inc+Flu+HID	0.32	46.09		46.40813	0.0146	0.85		0.8681	22	54		53.462	748	73		84	45	1.8		2.1	0.0595	1,788	30,037	102.38166	1.9151	6.0940090	0.113989
US	2001	Inc+Flu+HID	1.22	134.89		136.10225	0.0552	2.75		2.8029	22	49		48.558	278	72		76	17	2.0		2.1	0.2850	12,039	42,237	144.32354	2.9722	41.1356576	0.847145
China	2006	Inc+Flu+HID	0.13	16.25		16.38264	0.0074	0.28		0.2876	18	58		56.964	441	79		88	33	1.8		2.1	1.3108	11,842	9,034	31.33592	0.5501	41.0758653	0.721085
FSU	2000	Inc+Flu+HID	0.18	38.52		38.70621	0.0103	0.90		0.9061	18	43		42.717	235	49		51	17	1.5		1.6	0.2891	1,918	6,636	29.83051	0.6983	8.6243805	0.201898
OECD Eur	2005	Inc+Flu+HID	0.24	45.68		45.92144	0.0109	0.85		0.8569	22	54		53.591	944	138		149	57	3.4		3.7	0.4859	13,800	28,404	54.89360	1.0243	26.6701446	0.497659
JP+KR	2005	Inc+Flu+HID	0.31	71.62		71.93434	0.0141	1.10		1.1160	22	65		64.457	799	142		150	48	2.9		3.1	0.1761	5,452	30,967	71.22009	1.1049	12.5388598	0.194531
China	2005	Inc+Flu+HID	0.14	13.22		13.36237	0.0080	0.23		0.2359	18	58		56.644	374	78		88	28	1.8		2.1	1.3032	10,717	8,224	28.39031	0.5012	36.9977484	0.653164
AU+NZ	2005	Inc+Flu+HID	0.49	62.96		63.45005	0.0222	1.28		1.3071	22	49		48.541	568	98		106	34	2.7		2.9	0.0241	836	34,671	84.86073	1.7482	2.0473412	0.042177
Wrld Grid	2005	Inc+Flu+HID	0.18	32.70		32.87727	0.0082	0.65		0.6584	22	50		49.933	600	110		116	36	2.9		3.1	4.0767	54,821	13,447	31.04136	0.6217	126.5469030	2.534354
China	1993	Inc	0.13	2.57		2.70515	0.0074	0.10		0.1108	18	25		24.415	168	104		109	12	5.6		5.9	1.1784	4,059	3,445	4.15132	0.1700	4.8920810	0.200372
Wrld Non-Grid	1999	Ker			0.04275	0.04275			0.1228	0.1228			0.348	0.348			183	183			600	600.2	2.0000	4,404	2,202	0.02624	0.0647	0.0524755	0.129386
Wrld	2005	Ker+Inc+Flu+HID												47.527				119				3.3	6.4234	60,670	9,445	20.19001	0.4248	129.6880619	2.728729
Wrld	2050	Ker+Inc+Flu+HID												47.527				119				3.3	9.4000	158,200	16,830	35.97555	0.7570	338.1702104	7.115341
Wrld	2050	Flu+HID												75.432				119				2.1	9.4000	158,200	16,830	57.09855	0.7570	536.7263589	7.115341
Wrld	2050	SSL												148.000				119				1.1	9.4000	158,200	16,830	112.02878	0.7570	1053.0704859	7.115341
Wrld + ΔCoE	2050	Ker+Inc+Flu+HID												47.527				311				8.7	9.4000	158,200	16,830	13.79660	0.2903	129.6880619	2.728729
Wrld + ΔCoE	2050	Flu+HID												75.432				311				5.5	9.4000	158,200	16,830	21.89726	0.2903	205.8342194	2.728729
Wrld + ΔCoE	2050	SSL												148.000				311				2.8	9.4000	158,200	16,830	42.96296	0.2903	403.8518657	2.728729

Table 1. Per capita consumption of light (φ) and associated energy ($\dot{e}_\varphi$), luminous efficacies (η_φ), costs of energy (*CoE*) and light (*CoL*), population (*N*), gross domestic product (*GDP*) and per capita gross domestic product (*gdp*), for the five datasets (in brown, blue, pink, grey and green) discussed in Section 2. Estimates are also given for aggregate luminous efficacy and costs of light and associated energy for the World 2005 and projected future Worlds 2050. Monetary units are all year 2005 US\$. The various nation abbreviations are: UK = United Kingdom; FSU = Former Soviet Union; OECD Eur = Organization for Economic Development Europe = Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, Switzerland, UK, Ireland, Greece, Portugal, Spain, Hungary, Poland, Czech Republic, Slovak Republic, Turkey, Iceland, Luxembourg; JP+KR = Japan + South Korea; AU+NZ = Australia + New Zealand; Wrld = World. The various lighting technology abbreviations are: Can = candle; Oil = oil; Gas = gas; Ker = kerosene; Inc = incandescence; Flu = fluorescence; HID = high-intensity discharge; SSL = solid-state lighting. The various energy-source sectors are vehicle, grid electricity and fuel-based.

For the proportionality constant we use β_{veh} = 0.000485, deduced from Navigant's study of consumption of light in vehicles (autos, buses and trucks) in the United States in 2002 (Navigant 2003), where we have summed over only those lamps[4] used for illumination (rather than signaling) purposes. For gdp we use the estimates discussed in Subsection 2.6. For cost of light we use the expression discussed in Subsection 2.4, but particularized for vehicles: $CoL_{veh} \approx (1+\kappa_\varphi)\cdot CoE_{veh}/\eta_{\varphi,veh}$.

Finally, for each nation or group of nations, we sum the estimates of consumption of light from the three energy-source sectors to get an aggregate consumption of light across those sectors.

2.2 Luminous Efficacy

Luminous efficacy represents the efficiency with which energy is used to produce visible light. As has been discussed recently, there is a limiting luminous efficacy for the production of high quality white light which renders well the colors of typical environments: 408 lm/W_e (Phillips et al. 2007). In practice, the luminous efficacies of various lighting technologies are far less than this limiting value, and have evolved considerably throughout history. Indeed, as discussed first by Nordhaus (Nordhaus 1997), they have evolved spectacularly -- a key insight in the development of "hedonic" indices based on the price of consumed services or features rather than of the inputs to those services or features.[5]

For most of the datasets, because of the relationship between luminous efficacy (η_φ, in lm/W), per capita consumption of light (φ, in Mlmh/(per-yr)) and associated energy ($\dot{e}_\varphi$, in MWh/(per-yr)),

$$\frac{\varphi}{\eta_\varphi} = \dot{e}_\varphi, \tag{2.2}$$

two of the quantities were estimated and the third inferred. For example, in the Fouquet-Pearson dataset consumption of energy and luminous efficacy were estimated and consumption of light was inferred. Or, for example, in the Navigant dataset consumption of light and luminous efficacy were estimated and consumption of energy was inferred.

For the most part, we have used "as is" the estimates of luminous efficacy in the original datasets. The only exception was in the Fouquet and Pearson dataset, for which luminous efficacies were based on an evolved weighting of the proportions of old and new lighting technologies, with the underlying luminous efficacies of the various technologies based on estimates from Nordhaus' classic study (Nordhaus 1997). In this dataset, the luminous efficacy for 2000 appeared to be biased towards incandescent technology rather than reflecting a more accurate modern mix of incandescent, fluorescent and high-intensity discharge (HID) technology. Hence, instead of Fouquet and Pearson's estimate of 25 lm/W_e (based on Nordhaus' original estimate), we substituted the 2005 OECD Europe aggregate average of 54 lm/W_e from the IEA dataset.

Note that luminous efficacy relies on an assumption regarding the source of energy that is used to produce light, and these in turn differ according to the energy-

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[4] High- and low-beam headlamps, parking lamps, license plate lamps and fog lamps.

[5] See, e.g., U.S. Department of Labor discussions of hedonic adjustments to the U.S. consumer price index (http://www.bls.gov/cpi/home.htm).

source sectors (vehicle, grid electricity, and fuel-based) discussed in the introduction to Section 2. To compare across these sectors, and because electricity is now and likely in the future the dominant source of energy for lighting, we list in Table 1 luminous efficacies in units of lm/W_e, calculated as if electricity were the initial energy source.

For the grid electricity and vehicle energy-source sectors, the most common units for luminous efficacy are lm/W_e, calculated as if electricity were the initial energy source, and so these are listed "as is" in Table 1. Note that for the vehicle sector the range of luminous efficacies is not very great, varying from the η_{veh} = 18 lm/W_e typical of tungsten incandescent bulbs to the η_{veh} = 24 lm/W_e of tungsten-halogen incandescent bulbs (Denton 2004, p. 292). In newer vehicles, the latter is more common, and so we have assumed luminous efficacies for the various nations and groups of nations closer to the latter for recent years in more developed nations, and closer to the former for less recent years in less developed nations.

For the fuel-based energy-source sector, the starting point is the luminous efficacy in units of lm/W_c calculated as if chemical fuel were the initial energy source. Then, we divide by the σ_{grid} = 0.316 W_e/W_c efficiency of conversion-and-transport-to-point-of-use factor to get the effective luminous efficacy in units of lm/W_e as if grid electricity were the initial energy source.

Finally, given the luminous efficacies and consumptions of light of the various sectors for a particular nation or group of nations, an aggregate luminous efficacy for all the sectors combined is calculated by averaging the inverse luminous efficacies of each sector weighted by the fraction of light consumed by that sector,

$$\frac{\varphi}{\eta_\varphi} = \frac{\varphi_{grid}}{\eta_{\varphi,grid}} + \frac{\varphi_{fuel}}{\eta_{\varphi,fuel}} + \frac{\varphi_{veh}}{\eta_{\varphi,veh}}, \qquad (2.3)$$

where $\varphi = \varphi_{grid} + \varphi_{fuel} + \varphi_{veh}$ is the consumption of light for all three sectors. This weighting allows Equation 2.2 to be valid for each sector individually as well as for the sum over all sectors.

2.3 Cost of Energy

By cost of energy (*CoE*), we mean the point-of-use cost to the consumer who is converting the energy into light. Just as for luminous efficacy, however, the initial energy source is important to keep in mind. And, just as for luminous efficacy, to compare across these sectors, and because electricity is now and likely in the future the dominant source of energy for lighting, we list in Table 1 cost of energy in units of $\$/MW_eh$ calculated as if electricity were the initial energy source.

For the Fouquet and Pearson historical UK dataset, we used their estimates of the cost of energy "as is," but assumed that for 1900 and earlier the dominant energy source was chemical fuel, while for 1950 and later it was grid electricity. For the IEA and Navigant datasets (except for China), we used international residential and industrial electricity prices compiled (EIA 2007b) by the U.S. Energy Information Administration.[6] For China, estimates were spliced together from a number of sources (Li 2007b).

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[6] Since the cost and use of energy for lighting varies across the residential, commercial and industrial sectors, the aggregate cost of energy for lighting across these sectors can be written as: $CoE = (CoE_{Res}\cdot\dot{E}_{Res}+CoE_{Com}\cdot\dot{E}_{Com}+CoE_{Ind}\cdot\dot{E}_{Ind})/\dot{E}$, where $\dot{E} = \dot{E}_{Res}+\dot{E}_{Com}+\dot{E}_{Ind}$ is the energy consumed for lighting. In the U.S., the cost of energy in the form of electricity for the

For the Mills non-grid world, we used his estimate of \$0.5/liter for kerosene (in year 1999 US\$), divided by the energy content of kerosene (36.5 MJ/liter), then multiplied by 60·60 s/h (number of seconds in an hour) and a year 1999 to year 2005 exchange rate conversion, to derive a *CoE* of 58 \$/MW$_c$h. Then, we divide by the σ_{grid} = 0.316 W$_e$/W$_c$ efficiency-of-conversion-and-transport-to-point-of-use factor to get an effective *CoE* of 183 \$/MW$_e$h as if grid electricity were the initial energy source.

For the vehicle sector, we use international gasoline costs per unit volume (\$/gallon) taken from a compilation by the German Federal Ministry for Economic Cooperation and Development (GTZ 2007), divided by the σ_{veh} = 0.15 W$_e$/W$_c$ efficiency factor, then divided by the energy content of gasoline (38.3 kW$_c$h/gallon), to get the cost of energy in \$/MW$_e$h as if electricity were the initial energy source.

In all cases, for groups of nations, we used *GDP*-weighted averages.

2.4 Cost of Light

By cost of light (*CoL*, in units of \$/Mlmh), we mean the ownership cost of light, which includes (Rea 2000): the cost of the energy that is converted into light, the purchase and maintenance cost of the lamp (or bulb) that converts the energy into light, and the purchase cost of the luminaire and lighting system that directs and controls the light. The first cost is an operating cost, the second and third costs are capital costs.

The operating cost is the dominant of these, and is just the cost of energy divided by luminous efficacy, CoE/η_φ, with luminous efficacy and cost of energy as discussed in Subsections 2.2 and 2.3.

The purchase and maintenance cost of the lamp is smaller, and can be thought of as a fraction of the operating cost. For modern incandescent, fluorescent and high-intensity-discharge (HID) lamps, the fraction is approximately 1/6 (Navigant 2002). For the replaceable parts of kerosene lamps (the wick and mantle) such as those used for fuel-based lighting, the fraction can be estimated to be very similar, approximately 1/7 (Mills 2005).

The purchase cost of the luminaire and lighting system is more difficult to estimate, though it has been characterized as being of the same order of magnitude as the purchase cost of the lamp (IEA 2006). In the absence of accurate historical and contemporary data across nations, we simply assume here that these costs are a similar fraction, 1/6 to 1/7, of the operating cost. This is an assumption, however, that would benefit from more detailed examination.

Taken together, then, we write the cost of light as:

$$CoL = \frac{CoE}{\eta_\varphi} \cdot (1 + \kappa_\varphi) , \tag{2.4}$$

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commercial sector is, very roughly (EIA 2007a), $CoE_{Com} \approx (2/3)\cdot CoE_{Res} + (1/3)\cdot CoE_{Ind}$, and the fractions of energy for lighting consumed by the various sectors are roughly (Navigant 2002) $\dot{E}_{Res}/\dot{E} \approx 4/9$, $\dot{E}_{Com}/\dot{E} \approx 3/9$ and $\dot{E}_{Ind}/\dot{E} \approx 2/9$. Hence, we can deduce, after some algebra, that $CoE \approx (2/3)\cdot CoE_{Res} + (1/3)\cdot CoE_{Ind}$. Though this formula is strictly valid only for the U.S., we use it, in the absence of similarly detailed inventories, for all other nations (except China) as well.

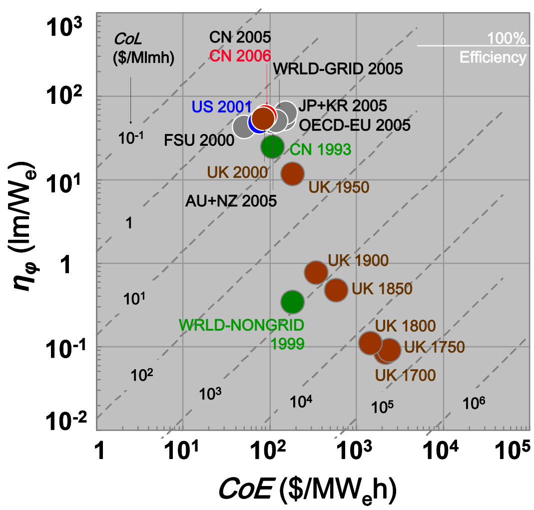

Figure 1. Scatterplot of the luminous efficacies (η_φ) and costs of energy (CoE) associated with the five datasets discussed in Section 2. Country abbreviations are given in the caption to Table 1. The dashed diaganol lines are contours of constant cost of light. The horizontal white line at the upper right indicates the luminous efficacy associated with 100% efficient conversion of energy into a high quality (color rendering index = 90) white light.

where $\kappa_\varphi = 1/3$ is the ratio of the capital to operating costs of light. The operating fraction of the cost of light is then $1/(1+\kappa_\varphi) \sim 3/4$ and the capital equipment fraction of the cost of light is $\kappa_\varphi/(1+\kappa_\varphi) \sim 1/4$.

To see the variation in cost of light over the various datasets, and how that variation is determined by variations in luminous efficacy and cost of energy, Figure 1 shows a scatterplot of the datasets on an η_φ versus CoE plot. The dashed diagonal lines are contours of constant CoL calculated according to Equation 2.4.

One sees that the cost of light varies across the datasets by ~4.3 orders of magnitude. The greater part of that variation is due to a ~2.8 order-of-magnitude variation in luminous efficacy; the lesser part is due to a ~1.5 order-of-magnitude variation in cost of energy. Note that in general the more recent data points have higher luminous efficacies and lower costs of energy. The glaring exception is the WRLD-NONGRID 1999 data point, which represents the world population in 1999 without access to grid electricity. Because of this population's reliance on relatively primitive kerosene lamp technology, its luminous efficacy is comparable to that of the United Kingdom in the 1850's, though its cost of energy is somewhat lower. Also note that even amongst the most contemporary (2000-2005) data points, there is a surprisingly large variation in cost of energy, with the FSU 2000 data point at the low end, and JP+KR 2005 at the high end. There is much less variation, however, in their luminous efficacies.

2.5 Consumption of Associated Energy

By consumption of energy associated with the consumption of light we should in principle include two contributions: consumption of energy associated with the operating cost of light, and consumption of energy "embodied" in the capital cost of light.

The first contribution is given by Equation 2.2: $\dot{e}_{\varphi,op} = \varphi/\eta_\varphi$. The second contribution can be written as: $\dot{e}_{\varphi,cap} = (\varphi \cdot \kappa_\varphi \cdot CoE/\eta_\varphi)/\eta_\mu$, where, using the capital cost-of-light part of Equation 2.4 multiplied by φ, $\varphi \cdot \kappa_\varphi \cdot CoE/\eta_\varphi$ can be deduced to be the cost of the capital equipment used to produce light, and $1/\eta_\mu$ is the energy intensity for manufacturing that capital equipment. The ratio between the two contributions is $\dot{e}_{\varphi,cap}/\dot{e}_{\varphi,op} = \kappa_\varphi \cdot CoE/\eta_\mu$.

The ratio contains three terms, each of which we can estimate. The capital equipment fraction of the cost of light we estimated in Subsection 2.4 to be $\kappa_\varphi \sim 1/3$. The cost of electricity in the U.S. in 1994, per unit of chemical fuel source energy, is estimated to be CoE ~ 28 $/MW$_c$h (EIA 2006, Table 8.10). The energy intensity for all manufacturing in the U.S. in 1994 is estimated to be $1/\eta_\mu$ ~ 5.91 kBtu/$ ~ 0.00019 MW_ch/$ (EIA 1998).

If we assume, conservatively, that manufacture of the capital equipment used to produce light is similarly energy intensive to the manufacture of all goods, then the ratio between the energy embodied in the capital cost of light to the energy associated with the operating cost of light is $\dot{e}_{\varphi,cap}/\dot{e}_{\varphi,op} \sim 1/85$. Note, though, that the energy intensity for manufacturing electronic and electrical equipment in the U.S. in 1994 was 0.91 kBTU/$, much lower than that for all manufacturing. Hence, if manufacturing capital equipment for lighting is more similar to that of electronic and electrical equipment than for the average over all manufacturing, then the ratio would be an even smaller $\dot{e}_{\varphi,cap}/\dot{e}_{\varphi,op} \sim 1/550$.

We conclude that the energy embodied in the capital cost of light is negligible, and for the remainder of this paper we assume that Equation 2.2 holds for the relationship between consumption of light and consumption of associated energy, both for the U.S. in 1994 as well as for all other nations in all other years.

2.6 Gross Domestic Product and Population

As we shall see, gross domestic product (*GDP*) and population (*N*) are key factors underlying consumption of light, so we have gathered together various estimates for these.

For individual nations our primary sources for historical and contemporary gross domestic products and populations were the comprehensive databases compiled by Angus Maddison (Maddison 2007) and the University of Groningen (GGDC 2007). Importantly, the GDPs in these databases were derived using purchase-power-parity, rather than exchange-rate, methods. Although we do not pursue this issue further in this paper, we did find that consumption of light had a significantly stronger correlation with such purchase-power-parity *GDP*s than with exchange-rate *GDP*s.

For most of the groups of nations, we simply summed the *GDP*s or populations of the individual nations. In the few cases where *GDP* or *N* for a particular year was not in the database, simple geometric interpolation between years was used.

To estimate GDPs and populations of those with (WRLD-GRID 2005) and those without (WRLD-NONGRID) access to grid electricity, we approximate the first to be those nations classified by the World Bank (WB 2007) as middle or high income, and the second to be those nations classified as low income. Doing so for the first in 2005 yields a population of 4.1 Gper and a GDP of 54.8 G$/yr, numbers we associate with the estimates in the IEA dataset of world consumption of light from grid electricity in 2005. Doing so for the second in 1999 yields a population of 2.1 Gper and a *GDP* of 4.6 G$/yr. We note that this population is very close to the estimate in the Mills dataset of 2.0 Gper without access to grid electricity in 1999. Since the deviation is small, and since we would like to use without modification Mills' associated estimates of the consumption of light, for our purpose we accept his estimate of 2.0 Gper and simply scale *GDP* proportionately down to (2.0/2.1) · 4.6 G$/yr = 4.4 G$/yr.

Finally, in our projections of the future, we used the "moderate assumption" scenario B1 from the Intergovernmental Panel on Climate Change report (Nakicenovic and Swart 2000), in which by 2050 world population has grown to 9.4 Gper and world *GDP* has grown to 158,200 G$/yr (in year 2005 US$).

3 DEPENDENCE OF CONSUMPTION OF LIGHT ON INCOME AND PRICE

In this Section, we describe what we have found to be the primary relationship underlying consumption of light. We start by describing how per capita consumption of light depends, to a good approximation, on the ratio between per capita gross domestic product and cost of light. We then discuss how this primary dependence can be improved slightly through higher-order non-linear dependences on per capita gross domestic product and cost of light, though the introduction of such dependences is not believed yet warranted by the accuracy of the underlying data.

3.1 Dependence of φ on *gdp/CoL*

The central result of this paper is that per capita consumption of light is, to a very good approximation, proportional to the ratio between per capita gross domestic product and cost of light, obeying the expression:

$$\varphi = \beta \cdot \frac{gdp}{CoL}. \qquad (3.1)$$

The surprising predictive power of this expression is illustrated[7] in Figure 2. The vertical axis of the Figure is per capita consumption of light, φ, in units of Mlmh/(per-yr). The horizontal axis of the Figure is β, a dimensionless proportionality constant, times per capita gross domestic product, *gdp*, in units of \$/(per-yr), divided by cost of light, *CoL*, in units of \$/Mlmh. Because the two axes have the same units, Mlmh/(per-yr), Figure 1 basically plots direct estimates of per capita consumption of light in a number of nations or groups of nations (vertical axis) against indirect predictions of per capita consumption of light based on independent estimates of *gdp* and *CoL* in those same nations or groups of nations (horizontal axis).

As illustrated in Figure 2, per capita consumption of light is predicted remarkably well by Equation 3.1, despite a span of data over: 3 centuries (1700-2006), 6 continents (Africa, Asia, Australia, Europe, North America, South America), 5 types of fuel (tallow, whale oil, gas, petroleum, electricity), 5 overall families of lighting technologies (candles, oil lamps, gas lamps, electric incandescent bulbs, electric gas-discharge bulbs or tubes), 1.4 orders of magnitude in per capita gross domestic product, 4.3 orders of magnitude in cost of light, and 5.4 orders of magnitude in per capita consumption of light.

That per capita consumption of light depends so simply on the ratio between *gdp* and *CoL* seems fortuitous, but allows for the following interpretation. People expend a fixed fraction (β) of their *gdp* on light, and per capita consumption of light is simply this expenditure ($\beta \cdot gdp$) divided by the cost of light (*CoL*). The fixed fraction can be determined, by a least squares fit of $log(\varphi)$ to $log(\beta \cdot gdp/CoL)$, to be $\beta = 0.0072$.[8]

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[7] Note that, since the axes of Figure 2 are logarithmic, we have effectively plotted the logarithmic form of Equation 3.1: $\log(\varphi) = \log(\beta) + \log(gdp) - \log(CoL)$.

[8] This procedure gives a β which is essentially the mean of the values for $\varphi \cdot CoL/gdp$ for all of the data points (see Table 1 in Section 5), weighted equally. We could instead have taken β to be the value of $\varphi \cdot CoL/GDP$ associated with the data point considered most accurate: the comprehensive Navigant study of the 2001 U.S. lighting market (Navigant 2002), which self-consistently aggregated bottom-up surveys, audits and inventories from a large number of independent sources. Doing so would give an β which is slightly lower, 0.0067 rather than 0.0072.

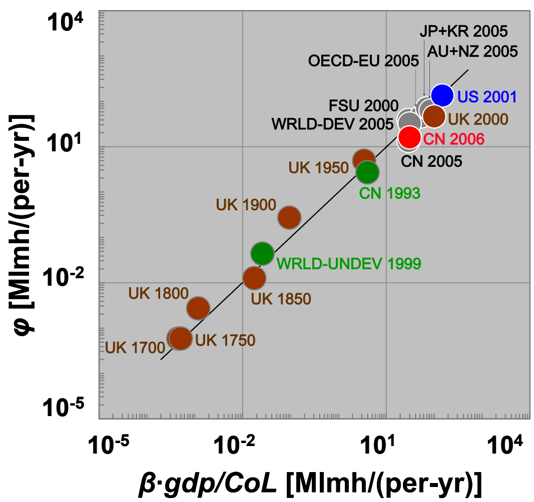

Figure 2. Data for per capita consumption of light (φ) plotted against the product of a constant factor (β) and per capita gross domestic product (*gdp*), divided by the cost of light (*CoL*). Country abbreviations are given in the caption to Table 1. The diagonal black line has slope unity and zero offset.

More precisely, logarithmic regression gives $log(\beta) = -2.15 \pm 0.26$ FWHM, with an adjusted coefficient of determination $R^2 = 0.986$.[9] Note that on an absolute scale the confidence interval for β is not small: its lower end is $\beta = 10^{-2.15-0.26} = 0.0039$ and its upper end is $\beta = 10^{-2.15+0.26} = 0.0130$. This range of $10^{2\cdot 0.26} = 3.3$ can be considered infinitesimal, however, compared to the dynamic range of $10^{5.36} = 230{,}000$ for per capita consumption of light itself.

We conclude that, to a very good approximation, people in nations over diverse temporal, geographic, technological and economic circumstances have expended 0.39% to 1.30% (with a best fit value of 0.72%) of their *gdp* on light.[10] We also conclude that the income elasticity (at constant price) and the price elasticity (at constant income) of the demand for light are both unity or nearly unity.

At first blush, such high elasticities are surprising, given the widely made assumption that demand for light is independent of efficiency (and hence cost), and the also widely made corollary assumption that energy consumption will decrease as technology evolution leads to improvement in lighting efficiency (Kendall and Scholand 2001; Tsao 2002; BES 2006; Navigant 2006).

At second blush, however, such high elasticities for lighting, over decades-to-centuries time periods, are perhaps not so surprising. The human visual system is among the most complex and developed of our sensory systems, and is key to how we experience the world around us. Humans are not indifferent to ways of enhancing this experience, including through use of artificial light. One can only speculate how altered the architecture of enclosed spaces and buildings would need to be if only natural sun- and moon-light were available to be exploited, and how expensive it would be to substitute enough capital, labor and materials to compensate.

Moreover, though an expenditure of 0.72% of *gdp* on any single good or service seems like a significant fraction, on an absolute scale it is relatively small. Hence, one can anticipate that it would be relatively painless in economic terms to maintain its magnitude under diverse temporal, geographic, technological, and economic

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[9] The adjusted and non-adjusted coefficients of determination are virtually the same, due to the large number (seventeen) of samples compared to the number (one) of fitting parameters.

[10] Note that while the confidence interval encompasses the percentage, 1.2%, found in the recent International Energy Agency study (IEA 2006), the best-fit value, 0.72%, is somewhat lower. The reasons are twofold: the IEA's use of exchange-rate based, but our use of purchase-power-parity based, *gdps*; and the IEA's estimates of $\varphi \cdot CoL/gdp$ being slightly high relative to those of the other datasets.

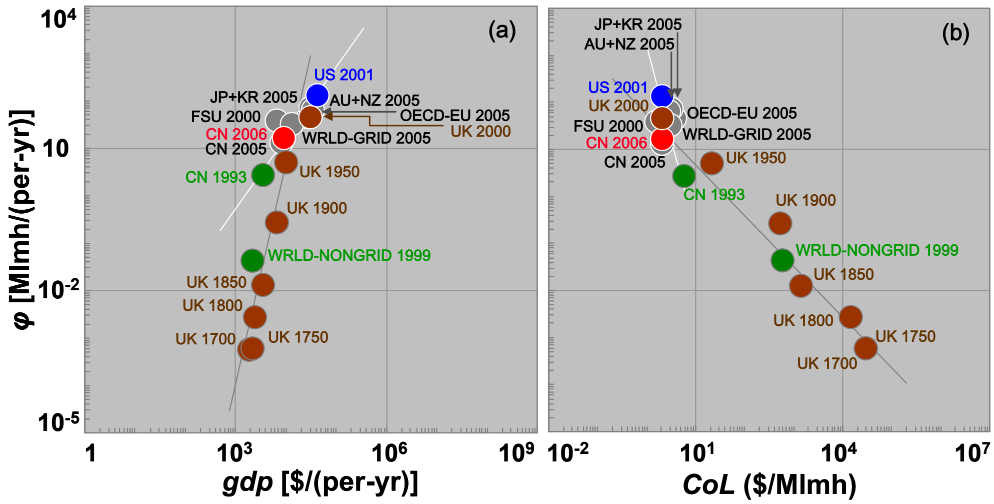

Figure 3. Data for per capita consumption of light (φ) versus (a) per capita gross domestic product (*gdp*) and (b) cost of light (*CoL*). Country abbreviations are given in the caption to Table 1. The black and white diagonal lines are independent power-law fits to the fuel-based lighting and grid-electricity-based lighting data points, respectively, and are intended to visually illustrate the different dependences on *gdp* and *CoL* of these data points.

circumstances, particularly if the consumption of light confers significant benefit to the productivity and quality of human life.

3.2 Other Possible Dependences of φ on *gdp* and *CoL*

Though the simple linear dependence of per capita consumption of light on the ratio between *gdp* and *CoL* is striking, it is interesting to explore other possible dependences on *gdp* and *CoL*.

φ depends solely on *gdp* or *CoL*

The simplest of these would be dependences of φ solely on either *gdp* or *CoL*. After all, over historical time, *gdp* has generally increased while *CoL* has generally decreased, and one might anticipate that consumption of light could be predicted using either variable alone. This, however, does not appear to be the case.

Consider Figure 3a, which plots per capita consumption of light against *gdp*. Per capita consumption of light has a larger apparent variation with *gdp* for the fuel-based data points than for the grid electricity data points. The reason is that, for the former (but not for the latter), the variation in *gdp* is augmented by a large (but hidden) variation in *CoL*.[11]

Likewise, consider Figure 3b, which plots the consumption of light against *CoL*. Here, the situation is reversed. Consumption of light has a larger apparent variation with *CoL* for the grid electricity data points than for the fuel-based data points. The reason is that, for the former, *CoL* varies hardly at all, hence most of its variation in consumption of light is due to the large (but hidden) variation in *gdp*.

[11] Note that if only the grid electricity datapoints are used, *gdp* is at least an *approximate* predictor for consumption of light. But *CoL* still plays a role, as can be seen from a compilation of data from 33 countries by Mills (Mills 2002) in which Norway is an outlier, most likely because of its low hydroelectricity cost and hence low *CoL*.

Of course, it is still *possible* that consumption of light depends either solely on either *gdp* or *CoL*, but that the dependences have power-law exponents that depend on type of energy source. For example, all that would be necessary for consistency with Figure 3a would be for the power-law exponent with respect to *gdp* to be relatively large (~4.7) for fuel-based lighting, then to become relatively small (~1.5) for grid-electricity-based lighting. Likewise, all that would be necessary for consistency with Figure 3b would be for the power-law exponent with respect to *CoL* to be relatively small (~-1.1) for fuel-based lighting, then to become relatively large (~-4) for grid-electricity-based lighting.

Although such changes in power-law exponents cannot be ruled out, we do not find any reason to invoke them. Instead, Occam's Razor suggests that it is much more likely that per capita consumption of light depends on both *gdp and CoL*, with a simple linear dependence that is the same across energy sources and across all the data sets.

Non-unit elasticities

Another possible dependence is one in which the dependences of φ on *gdp* and *CoL* are power law but not with unit elasticities. The dependence that is most consistent with the data is one in which consumption of light depends on *gdp* and *CoL* as

$$\varphi = \frac{0.0025 \cdot gdp^{1.08}}{CoL^{0.90}}, \tag{3.2}$$

with a (logarithmic) adjusted regression coefficient of determination that is increased (very slightly) to $R^2 = 0.989$. The implication is that the income elasticity (at constant price) of light consumption is slightly (8%) greater than unity, while the price elasticity (at constant income) of light consumption is slightly (10%) less than unity.

We note, however, that these deviations from non-unity elasticities of demand are small and, in our judgment, give insignificant improvement in consistency with the data compared to the likely errors in the data points themselves. As was discussed in in Section 2, each data point is associated with estimates of *three* quantities (φ, *gdp*, *CoL*). These estimates, made over diverse temporal, geographic, technological, and economic circumstances, are fraught with potential for error, particularly for the data points going back furthest in time, when the mixes of fuel and lamp technologies were undergoing radical changes.

Dependence of β on *gdp*

A third possible dependence might be one in which the proportionality factor β itself depends on per capita gross domestic product. If we assume an exponential form to that dependence, then we find that $\beta = 0.0056 + 0.0109 \cdot e^{gdp/gdpo}$, where *gdpo* = 6,300 \$/(per-yr). The "fit" to the data improves, but because there are more fitting parameters, the adjusted (logarithmic) regression coefficient of determination does not improve, but stays the same at $R^2 = 0.986$. Nevertheless, we cannot rule out the notion that β, the fraction of *gdp* spent on lighting, decreases slightly with *gdp*.[12]

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[12] We acknowledge Peter Dempster for prompting us to examine dependences of β on *gdp*.

4 POSSIBLE DEPENDENCES OF CONSUMPTION OF LIGHT ON NON-INCOME AND NON-PRICE FACTORS

From Figure 2 it seems apparent that per capita consumption of light has a *primary* dependence on per capita gross domestic product and cost of light. At a higher level of detail, however, we can anticipate that consumption of light might also have secondary dependences on other factors.

In this Section, we discuss some of the most important of these possible secondary dependences on other factors. We do so even though it does not appear currently possible to quantify them, due both to the uncertainties associated with the estimates of the consumption of light and to the incompleteness of the data associated with the other factors. Our reasons are twofold. First, some of these possible secondary dependences seem a priori more likely to have been primary dependences, and it is interesting to speculate on why they do not seem to have been in the past. Second, some of these secondary dependences may become more important (perhaps even primary) in the future, and it is of interest to speculate in what ways this may happen.

We start by discussing the demand for raw lumens – the aspect of light that enables us to see and that is presumably the principal motivation for its purchase. Then, we discuss the demand for other features beyond lumens – safety, reliability, quality, mood enhancement, convenience, etc. – that together comprise the lighting "experience."

4.1 Desire for Lumens

First and foremost, of course, we use light to illuminate our environment so that we can see. Because the dynamic range of the human visual system is large but ultimately still limited, one might anticipate a saturation in how brightly we would like our environment to be illuminated, and consequently a saturation in our appetite for light. If we let φ_{sat} represent a hypothetical saturation value for per capita consumption of light, then we might anticipate a dependence for per capita consumption of light along the lines of[13]:

$$\varphi = \left(1 - e^{-\frac{\beta \cdot gdp / CoL}{\varphi_{sat}}}\right) . \varphi_{sat} \, . \tag{4.1}$$

If so, consumption of light would increase linearly with *gdp/CoL* for small *gdp/CoL*, then saturate at φ_{sat} for large *gdp/CoL*.

At current levels of per capita consumption of light, a central result of this paper is that there is no evidence for such a saturation. It is nevertheless an open question whether we will approach such a saturation in the future, or whether per capita consumption of light will continue to scale linearly with *gdp/CoL*. To understand this question more quantitatively, we can decompose per capita consumption of light into three factors,

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[13] This particular function was chosen for simplicity and illustrative purposes only. Many other functions could be imagined with a similar dependence. We deliberately exclude, however, sigmoidal (e.g., Gompertz or logistic) functions which would increase nonlinearly with *gdp/CoL* for small *gdp/CoL*.

$$\varphi = I_N \cdot \left(\frac{\tau_{on}}{\tau_{on} + \tau_{off}} \right) \cdot \left(\frac{a_N}{1 + a_N \rho_N} \right). \tag{4.2}$$

each of whose potential for saturation can be discussed separately:

Illuminance: I_N

The first term in Equation 4.2 is I_N, the average illuminance[14] (or light per unit area, in units of lm/m^2) that a person is surrounded by during his or her waking hours.

Illuminances have gradually increased over the centuries and, for modern indoor office or living spaces, are now on the order of $I_N \approx 500$ lm/m^2. Such illuminances are, from a purely visual acuity point of view, clearly enough for most people for most tasks, and might be anticipated to be near a saturation level.

Furthermore, over the last decade many countries have introduced energy efficiency regulations that effectively constrain the degree to which interior light levels can be increased. These either limit the maximum permissible installed power demand of lighting per unit floor area or impose whole-building energy performance limits which include lighting. The scope of these requirements is increasingly being extended to apply to substantive interior refurbishments involving lighting systems, and not just to new construction.

Moreover, we do not always wish to be surrounded by illuminances suitable for tasks requiring high visual acuity. Ambient illuminances for enhancing particular moods or emotional states of mind can be much lower than 500 lm/m^2. And even when high visual acuity is desired, not all illuminance must be supplied artificially – artful use of sunlight can be an important supplement.

Nevertheless, arguments can be made that we have not yet approached saturation levels for illuminance. Considerable uncertainty exists regarding what constitutes optimal lighting -- despite over a century of research, recommended levels for comparable spaces still vary by a factor of up to 20. It is now recognized that optimal lighting conditions are contingent on numerous factors other than just average horizontal illuminance levels and include visual contrast and light distribution parameters.

And, even if one considers only horizontal illuminance, the evidence regarding the levels that humans would choose were affordability not a factor is far from complete. Humans might well choose higher illuminances than they do today, particularly to help mitigate losses in visual acuity in an aging world population, but perhaps also to function as neuropsychological modifiers (helping, e.g., to synchronize Circadian rhythms, to reduce seasonal affective disorder, and to enhance mood).

Indeed, the generally comfortable outdoor illuminance characteristic of an overcast or cloudy day is of the order 5,000 lm/m^2 – 10x higher than the 500 lm/m^2 mentioned above as typical of modern indoor office or living spaces. And the outdoor illuminance characteristic of a bright sunny day is of the order 30,000 lm/m^2, 60x higher than today's 500 lm/m^2. Though this latter illuminance is uncomfortable viewed from a close distance (requiring the use of sunglasses), it may well be

[14] We use the symbol *I* rather than the usual symbol for illuminance, *E*, as in this paper *E* refers to energy. The subscript "*N*" refers to local illuminance from the perspective of an average person, as opposed to global illuminance from the perspective of an average area of land.

desirable viewed from a farther distance, provided issues of glare associated with bright, localized point sources of light can be mitigated.

We conclude that it is possible that the developed countries are nearing a saturation point in average illuminance, but plausible arguments can be made that the saturation point may yet be factors of 10x or greater away.[15]

Illumination duty factor: $\tau_{on}/(\tau_{on}+\tau_{off})$

The second term in Equation 4.2 is $\tau_{on}/(\tau_{on}+\tau_{off})$, a dimensionless illumination duty factor that accounts for how many hours per year the area around a person is actually illuminated. The duty factor for a person who spends most of his or her time indoors, either at work or at home, is roughly the number of waking hours per year, or about $\tau_{on}/(\tau_{on}+\tau_{off}) \approx$ (16 h/day)·(365 days/yr) = 5,840 h/yr.

This is the term that is most clearly nearing saturation. Most people need on the order of 8 h of sleep each day. And most people need darkness to sleep, and even apart from sleep, to rest their human Circadian rhythms (IEA 2006, Chapter 2).

Unshared illuminated area: $a_N/(1+a_N\rho_N)$

The third term in Equation 4.2 is $a_N/(1+a_N\rho_N)$, the average unshared illuminated area (in units of m^2) that a person is surrounded by.[16] This area is a_N, the average illuminated area (in units of m^2) that a person is surrounded by (regardless of how many other persons share that area), divided by $1+a_N\rho_N$, the number of persons that share that area. Here, ρ_N (in units of per/m^2) is the density of people within the illuminated area that a person is surrounded by. When ρ_N is small, light is not shared, and $a_N/(1+a_N\rho_N)$ approaches a_N; when ρ_N is large, light *is* shared, and $a_N/(1+a_N\rho_N)$ approaches $1/\rho_N$.

The order of magnitude of a_N can be estimated as follows. As indicated in Table 1, per capita consumption of light in the U.S., representative of the high end in the world, was about $\varphi \approx$ 136 Mlmh/(per-yr) in 2001. As discussed above, the average illuminance in modern indoor office or living spaces is roughly $I_N \approx$ 500 lm/m^2, and the illumination duty factor is roughly $\tau_{on}/(\tau_{on}+\tau_{off}) \approx$ 5,840 h/yr. In the absence of light sharing ($\rho_N \approx 0$), the illuminated area that the average person is surrounded by is thus, using Equation 4.2, roughly $a_N \approx \varphi(\tau_{on}+\tau_{off})/(I_N\cdot\tau_{on}) \approx 46$ m^2. This area is plausible: larger than a typical one-person office area, but smaller than a typical one-person residential area.

Regarding how this term might evolve in the future, it is, just as for illuminance, possible that it be approaching a saturation level. Humans, often characterized as den

[15] We note in passing that, whatever its saturation value, average illuminance might be expected to vary with geography. In countries further from the equator, illuminance from the sun is lower, and illuminance from artificial sources might be expected to increase to compensate. The limited data which is available appears not to support this, however. For example, Japan consumes significantly less light per capita than Northern Europe despite being nearer the equator and despite a similar standard of living. A proximate explanation for this, via Equation 3.1, is the greater penetration of higher luminous efficacy fluorescence lighting technology, hence lower cost of light, in Japan than in Northern Europe. But an ultimate explanation for the greater penetration of fluorescence technology itself may be a desire for higher artificial illuminance levels so as not to provide too stark a contrast with outdoor illuminance levels.

[16] Just as for the symbol I, for the symbols a and ρ the subscript "N" refers to local illuminated area and local population density from the perspective of an average person, as opposed to from the perspective of an average area of land.

animals, find comfort in enclosed areas, and to be surrounded by a $(46 \text{ m}^2/\pi)^{1/2} = 3.8$ m ≈ 12.5 ft radius of illuminated area is surely sufficient for most people most of the time. Indeed, an increasing trend in modern buildings is the use of motion sensors to turn lights on and off when a person enters or exits a space, with typical coverage areas comparable to 46 m^2. With new technologies such as solid-state lighting, such opportunities for sensor-based intelligent control will only increase in the future.

Moreover, humans are not only den animals, they are social animals, and tend to cluster in groups. Indeed, local population density[17] can, in a typical office building or urban public space, easily be on the order of $\rho_N \approx 0.1/\text{m}^2$. Hence, for $a_N \approx 46 \text{ m}^2$ and $\rho_N \approx 0.1/\text{m}^2$, we have $1/(1+a_N\rho_N) \approx 1/5$, and for these environments the unshared illuminated area is reduced by a factor 5.

However, den and social animals though they may be, humans also like space. Environments in which local population density is so high, and space is shared so heavily, are not necessarily the desired norm. Even the most densely populated city in the U.S. (New York) only has an average population density of about $0.009/\text{m}^2$ (Gibson 1998), implying that its average resident has plenty of less-dense areas to "escape" to. Moreover, as nations develop, the densities of their cities tend to decrease, as transportation costs decrease relative to income (Tobler 1969; Stephan and Tedrow 1977). Clearly, humans do not *prefer* to share space to an extreme.

Indeed, if the average size of residences is an indication of the preferred size of spaces that humans prefer, it is clear that these can be rather large. The average area per person in new single-family homes in the U.S. increased from 27 m^2 in 1950 to 45 m^2 in 1970 to 78 m^2 in 2000, and can easily be 2-5x larger in "upper-end" homes. Hence, the saturation illuminated area surrounding each person could be more than 5-10x larger than the current 46 m^2 estimated above.

Moreover, even if the enclosed indoor areas in which we work and live might ultimately saturate, the unenclosed outdoor areas which we either occupy for short periods during the day or evening, or which are visible from enclosed indoor areas, may be less prone to saturation. Such unenclosed outdoor areas (e.g., streets, parks, and other recreation and public spaces) could all be rendered more useful if better illuminated in the evening hours (albeit at the cost of reducing the contrast of the night sky due to light pollution (Boyce 2003, pp. 504-512)). And there is a natural human tendency to gaze out (we value windows, not just because they are a portal for incoming light, but because of the view they afford (Boyce 2003, pp. 234, 256)) of faraway spaces, even if we do not directly occupy them.

We conclude that it is possible that the average unshared illuminated area is nearing a saturation point, but plausible arguments can be made that the saturation point may yet be factors of 10-100x or greater away.

4.2 Desire for Features Beyond Lumens

Although the primary demand for light is for raw lumens to illuminate our environment, there are many other features of light that are important to the consumer of light. Arguably, these features were just as important as cost in the historical transitions from one lighting technology to the next. It therefore comes somewhat as a surprise that these features do not seem to be reflected as significant

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[17] By local population density, we mean that seen from the perspective of a person, which includes the tendency towards clustering. As seen from the perspective of the land, median world population density is much lower, on the order of $4 \cdot 10^{-6}/\text{m}^2$ (Cohen and Small 1998).

breaks at various points in history, and it is interesting to speculate on whether such a break may occur at the current point in history, with the emergence of solid-state lighting technology.

On the one hand, the coming transition from incandescent, fluorescent and HID lighting to solid-state lighting will bring a significantly new set of performance attributes (Schubert and Kim 2005), including compactness, ruggedness, and the potential for real-time IP-addressable control of local illuminance, hue, saturation, color rendering, color temperature and perhaps even luminous efficacy itself. Their easy compatibility with video displays, either as back lights or as active pixels, even suggest the potential for integrated applications involving simultaneous illumination and information transfer. These new performance attributes at least have the *potential* to unleash new and unforeseen ways of consuming light, and to lead to greater-than-unity elasticities. They also, of course, have the potential to unleash new ways of consuming *less* light, through sensor-based control of light flux and directionality, and to lead to less-than-unity elasticities.

On the other hand, the transitions within chemical-fuel-based lighting (e.g., from candles to oil lamps to gas lamps), and the final transition from chemical-fuel-based to electricity-based lighting, also brimmed over with new performance attributes (Schivelbusch 1988), including increased cleanliness, faster turn-on and turn-off, greatly decreased concomitant room heating, and reduced fire hazard. These new performance attributes had similar tremendous potential, ultimately realized, to unleash new and unforeseen ways of consuming light. They also, of course, had a similar potential, ultimately not realized, to unleash new ways of consuming less light, through instant turn-on and turn-off, and through the increased ability to focus light sources with smaller spatial extent.

In other words, each transition from one technology to the next apparently brought with it similar potential for new ways to consume light, and these potentials are reflected in the historical constancy, at least back to 1700, of β, the fraction of *GDP* spent on lighting. It is difficult to guess whether the coming transition to solid-state lighting will be quantitatively similar, but plausible arguments can be made that it will be.

We mention here in particular one important feature of solid-state lighting: its potential to fill the visible spectrum with light of a precisely tailored mix of wavelengths and intensities. This potential would enable a tailoring of the rendering of the colors of natural objects in the environment, either to be as accurate as possible (as measured, e.g., by the color rendering index, or CRI), or to deliberately create subjective emotional responses (by mimicking, e.g., daylight, moonlight, candelight, etc.).

People might easily consume more of such high quality light, preferring it even at a higher price to light of lesser quality. And different use-sectors (residential, commercial, industrial, outdoor stationary, and vehicle) might have different preferences for light qualities, with the residential sector emphasizing subjective emotional response, and the commercial sector emphasizing raw human productivity. To some extent, such sector preferences are evident even with current lighting technology: the residential sector prefers higher CRI but also higher *CoL* incandescent over lower CRI but also lower *CoL* fluorescent and high-intensity-discharge technology, while the commercial sector prefers the opposite.

Indeed, given such sector preferences, it could well be that our datasets, which combine consumption of light across all sectors, reflect the cancelling of a slightly lower sensitivity to cost of light in the residential sector by a slightly higher sensitivity in the other sectors. Each sector could separately obey Equation (3.1), but with different values for β, the fraction of *gdp* spent on lighting. In fact, Equation (3.1) does appear to be separately consistent with a dataset (Waide 2007) of year 2000 residential light consumption for eleven international energy agency (IEA) nations, but with a fixed fraction β, of 0.0016. In other words, 0.16% of GDP is expended by the residential use-sector for light, roughly 0.0016/0.0072 ≈ 22% of that expended by all use-sectors for light.

5 IMPLICATIONS ON WORLD CONSUMPTION OF LIGHT AND ASSOCIATED ENERGY

In Section 3 we discussed how per capita consumption of light depends on the ratio between per capita gross domestic product and cost of light. In this Section, we discuss the implications of this dependence on world consumption of associated energy: for the past and present, and projected into the future.

5.1 Relation between Consumption of Light and Associated Energy

To start, note that, as discussed in Section 2, luminous efficacy connects two pairs of quantities. The first pair is per capita consumption of light and per capita consumption of associated energy, through Equation 2.2. The second pair is cost of light (*CoL*, in units of \$/Mlmh) and cost of associated energy (*CoE*, in units of \$/MW$_e$h), through Equation 2.4. Thus, we can rewrite Equation 3.1 as

$$\dot{e}_\varphi = \beta \cdot \frac{gdp}{(1+\kappa_\varphi)\cdot CoE}. \tag{5.1}$$

Likewise, we can replot the data of Figure 2 using the modified axes in Figure 4. Because Equation 5.1 is essentially equivalent to Equation 3.1, the data points in Figure 4 fall on a (logarithmic) unit-slope line just as did those in Figure 2. However, because luminous efficacy varies between time periods, and between nations, the relative placements of the data points are not the same.

Also note that per capita consumption of associated energy does not span as wide a dynamic range (2.6 orders of magnitude) as per capita consumption of light (5.4 orders of magnitude). The reason is that, as discussed in Section 2, cost of energy does not span as wide a range as cost of light, due to the steady advancement, over the centuries, in luminous efficacy.[18]

5.2 World Consumption of Light and Associated Energy: Present

Up until now, we have dealt exclusively with per capita quantities for consumption of light and associated energy. It is also of interest to estimate *total* consumption of light and associated energy, by multiplying by world population:

$$\Phi = N\cdot\varphi = \frac{\beta\cdot GDP}{CoL} \tag{5.2a}$$

$$\dot{E}_\Phi = N\cdot\dot{e}_\varphi = \frac{\beta\cdot GDP}{(1+\kappa_\varphi)\cdot CoE}. \tag{5.2b}$$

[18] This steady advancement was first made quantitative in W.D. Nordhaus' classic study of the luminous efficacies of lighting technologies throughout history (Nordhaus 1997).

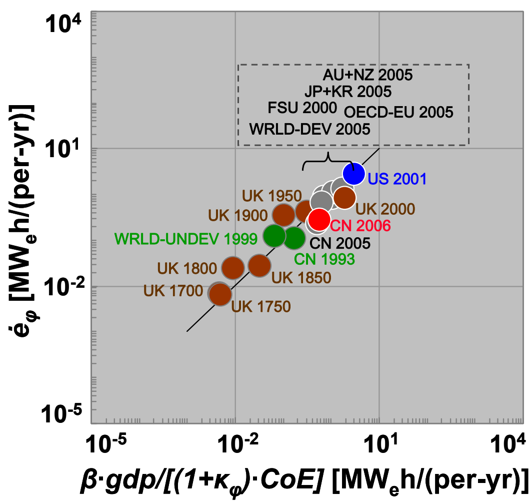

Figure 4. Data for per capita consumption of energy associated with consumption of light, plotted against the product of a constant factor (β) and per capita gross domestic product (*gdp*), divided by a factor that accounts for operating and capital cost of light ($1+\kappa$) and by cost of energy (*CoE*). Country abbreviations are given in the caption to Table 1. The diagonal black line has slope unity and zero offset.

In particular, we can estimate, using Equations 5.2, world consumption of light and associated energy in 2005. As for all other estimates, we use Maddison's estimates of *GDP*, EIA estimates for average price of energy, and light-consumption-weighted inverse luminous efficacies, all listed in Table 1. The result is an estimated world 2005 consumption of light and associated energy of 130 Plmh/yr and 2.7 $PW_e h$/yr, respectively. This represents about 16% of the world's total electrical energy generation of about 16.9 $PW_e h$/yr in 2005 (EIA 2007c). And, since 2.7 $PW_e h$/yr of electrical energy is equivalent to roughly 8.5 $PW_c h$/yr and 29.5 Quads/yr of primary chemical energy, this represents about 6.5% of the world's consumption of 457 Quads/yr of primary energy in 2005 (EIA 2007c).

Note that lighting therefore represents a much larger (6.5%) percentage of world energy consumption than of *GDP* (0.72%). This is an indication of the very high energy intensity of lighting relative to other goods and services, and hence the reasonableness of its classification, along with heat, power and transportation, as an "energy service."

5.3 World Consumption of Light and Associated Energy: Future

As discussed in Section 4, whether the historical trend represented by Equations 5.2 can be extrapolated into the future is highly uncertain. However, it is of interest to perform these extrapolations, even if only as baseline scenarios that spawn improved models which extrapolate differently.

Such scenarios, for the year 2050, are indicated in the bottom rows of Table 1. To enable the *GDP*, *CoL* and *CoE* contributions to these scenarios to be seen separately, they are also shown on the contour plots in Figure 5: on the left, plots of *GDP* vs *CoL* with contours of constant consumption of light, Φ; on the right, plots of *GDP* vs *CoE* with contours of constant consumption of associated energy, $\dot{E}$. The contours are all tilted at 45°, consistent with the dependence of consumption of light or associated energy on the simple ratios *GDP*/*CoL* or *GDP*/*CoE*, and all increase from the lower left to the upper right corners. As a point of comparison, the world in 2005 is shown as a grey point on each of the contour plots.

All scenarios assume an increase in world *GDP* from 61 G$/yr in 2005 to 158 G$/yr in 2050, consistent with "moderate" Scenario B2 from the Intergovernmental Panel on Climate Change (Nakicenovic and Swart 2000, pp. 48-55). The scenarios in the top two contour plots differ from those in the bottom two in assumed cost of energy: the top two contour plots assume a cost of energy in 2050 of 119 $/$MW_e h$, similar to that in 2005, and extrapolated from EIA projections to 2030 (EIA 2007a);

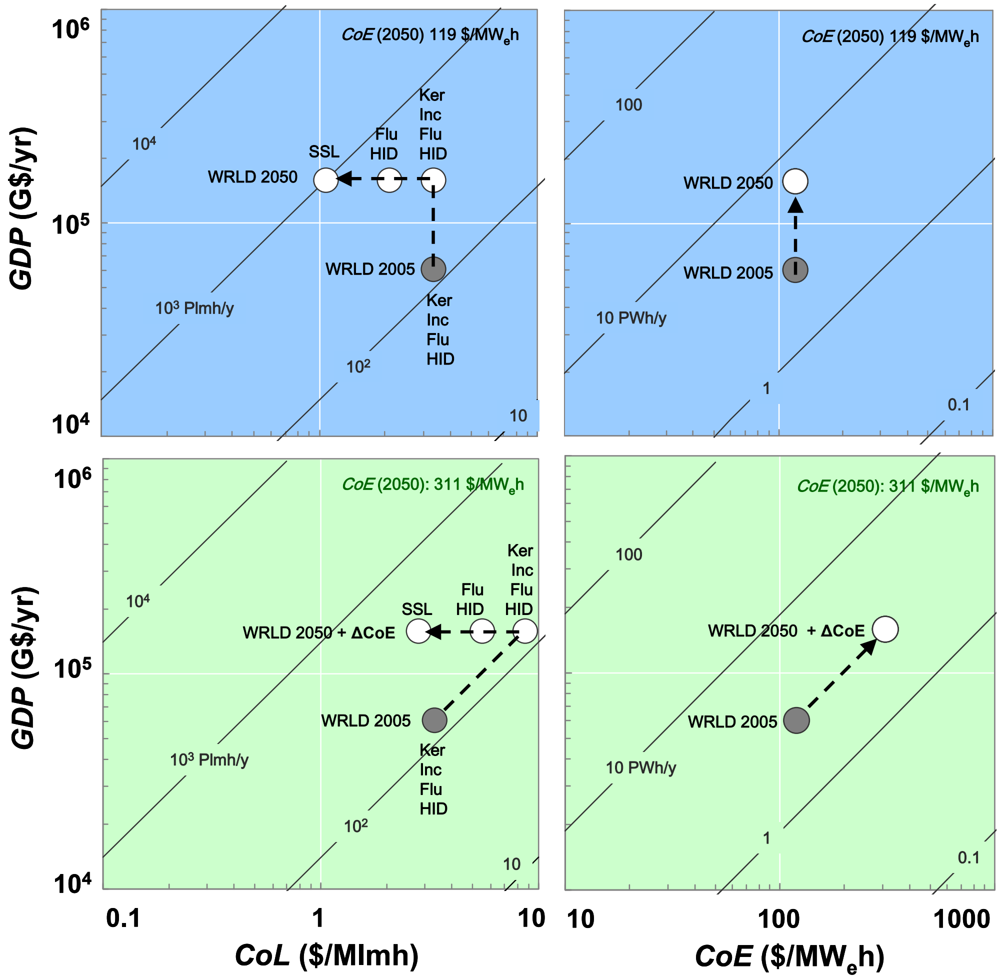

Figure 5. World 2050 consumption of light (left) and associated energy (right) scenarios plotted on *GDP* vs *CoL* (left) and *GDP* vs *CoE* (right) diagrams. In the blue (top) scenarios, cost of energy in 2050 is assumed to be 119 $/MW$_e$h, similar to that in 2005; in the green (bottom) scenarios, cost of energy is assumed to be a significantly higher 311 $/MW$_e$h. The world 2005 is shown as a (grey) point of reference. The dashed lines indicate a path by which the scenarios evolve from one in which lighting is provided by a mix of kerosene, incancescance, fluorescence and high-intensity discharge technologies in 2005 to one in which lighting is provided solely by solid-state lighting in 2050. Also shown are contours of constant consumption of light (left) and associated energy (right).

the bottom two contour plots assume a cost of energy, discussed later, increased to 311 $/MW$_e$h.

In each contour plot, three scenarios for the world in 2050 are shown as white points. The three scenarios differ in the assumptions on the technology mix used to produce light, and hence on aggregate luminous efficacy. In the "ker+inc+flu+HID" scenario, the current world mix of kerosene, incandescent, fluorescent and high intensity discharge (HID) technology is assumed, with an aggregate luminous efficacy of 48 lm/W$_e$ (roughly 12% efficiency, using the 408 lm/W$_e$ luminous efficacy discussed in Subsection 2.2). In the "flu+HID" scenario, a complete conversion from incandescent technology is assumed, with an aggregate luminous efficacy of 75 lm/W$_e$ (roughly 18% efficiency) associated with the current mix of fluorescent and HID technology in the U.S. (Navigant 2002). In the "SSL" scenario, a complete conversion from incandescent, fluorescent and HID to high-efficiency solid-state lighting technology is assumed, with an aggregate luminous efficacy of 148 lm/W_e (roughly 36% efficiency) as targeted by various roadmaps (Navigant 2007) and reports (BES 2006).

In the left two plots of Figure 5, the leftward progression of these three scenarios reflects the decrease in cost of light with increasing luminous efficacy. Accompanying this decrease in cost of light is an increase in consumption of light. In the top left plot, with a cost of energy of 119 \$/MW_eh, the increase is from 338 Plmh/yr to 537 Plmh/yr to 1,053 Plmh/yr; in the bottom left plot, with a cost of power of 311 \$/MW_eh, the increase is from 130 Plmh/yr to 206 Plmh/yr to 404 Plmh/yr.

Because these increases in consumption of light are driven entirely by increases in luminous efficacy (rather than decreases in cost of energy), however, consumption of associated energy does not increase similarly. The reason is that consumption of associated energy does not depend on either luminous efficacy or cost of light but, as described by Equation 5.2b, on cost of energy. Hence, in the right two plots of Figure 5, consumption of associated energy is constant across the three "World 2050" scenarios despite luminous efficacies that increase by a factor of 3.1. All three scenarios lie on top of each other, and all have the same consumption of associated energy.

Note that these scenarios are quite different from previous scenarios (Kendall and Scholand 2001; Tsao 2002; BES 2006; Navigant 2006). In those scenarios, consumption of light was assumed to increase with increasing *GDP*, but not with decreasing *CoL*. Therefore, for constant *GDP*, consumption of associated energy decreased with decreasing *CoL*. Instead, in the scenarios described here, consumption of light increases with decreasing *CoL* to exactly negate anticipated energy consumption decreases.

Note, though, that while in these scenarios consumption of associated energy does not decrease, light consumption increases, tremendously. This increase presumably leads to an increase in the quality and productivity of human life (Boyce 2003), and to its utility to consumers. Otherwise, consumption of light would be much less than unit elastic with respect to *GDP* and *CoL*. Viewed in this manner, the greater benefit of higher efficiency lighting technologies, both past and future, might not be as much to reduce consumption of associated energy, as *to enable consumption of associated energy to remain constant while consumption of light increases*.

Nevertheless, with heightened concern over climate change and a heightened desire to actually reduce energy consumption, it is of interest to ask under what conditions consumption of associated energy in 2050 *could* be reduced to that which was consumed in 2005. From Equation 5.2b, changes in consumption of energy are related to changes in GDP and CoE:

$$\frac{\Delta \dot{E}}{\dot{E}} = \frac{\Delta GDP}{GDP} - \frac{\Delta CoE}{CoE}. \tag{5.3}$$

Therefore, if we would like for $\Delta\dot{E}=0$ in going from 2005 to 2050, then we must have $\Delta GDP/GDP = \Delta CoE/CoE$, and the projected increase in *GDP* (160%) must be offset by an equivalent percentage increase in *CoE*. On a year 2005 *CoE* of \$119/MWh, such an increase would be \$192/MWh, resulting in a year 2050 *CoE* of \$311/MWh. As illustrated in the lower right plot of Figure 5, this increase in *CoE* causes the World 2050 scenario to lie on the same contour of constant consumption of associated energy as does the World 2005. Note, though, that despite having the same consumption of associated energy, the World 2050 scenario can be seen in the lower left plot of Figure 5 to have a consumption of light 3.1x higher than that of World 2005, because of the anticipated high luminous efficacy of solid-state lighting.

6 IMPLICATIONS ON THE REBOUND EFFECT

In Section 3, we discussed the central result of this paper: that per capita consumption of light is, *empirically*, very closely proportional to the ratio between per capita gross domestic product and cost of light. Then, in Section 5, we made the implicit assumption that gross domestic product is itself not influenced by the cost and consumption of light, but is determined exogenously. Finally, using projections of future gross domestic product, first order estimates of future consumption of light and associated energy were then made.

To second order, however, gross domestic product is bound to be influenced by the cost and consumption of light – light enables us to do useful work and enhances our productivity. And gross domestic product, in turn, is bound to influence the consumption of light. In other words, there is an *interplay* between consumption of light and economic productivity that self-consistently determines both.

In this Section, we model this interplay using a simple Cobb-Douglas function for production. First, we begin by tailoring the Cobb-Douglas production function to our purposes, and discuss various analytical relationships connecting the cost and consumption of light with gross domestic product – relationships that follow from the standard neoclassical economics optimization of profit. Second, we discuss the implications of these relationships on consumption of energy, paying particular attention to the various effects which collectively comprise the rebound effect mentioned in Section 1. Third, we make these rebound effects semi-quantitative by connecting to the empirical trend observed and discussed in Section 3. Finally, we discuss the implications on energy intensity – energy consumed per unit gross domestic product.

We emphasize that we do not intend for the semi-quantitative results of this Section to be taken too seriously. They undoubtedly depend on the exact form of the production function used. Though the Cobb-Douglas production function is compact, relatively easy to manipulate analytically, and is widely used in neoclassical economics, it is not, as discussed by Saunders (Saunders 2008a), “rebound-flexible,” and cannot represent the full range of possible rebound effects. Nevertheless, it enables us to illustrate semi-quantitatively the inter-relationships between gross domestic product and the cost and consumption of light and energy, and to connect these to the empirical results of this paper.

6.1 Cobb-Douglas Production Function

We begin with a nonlinear Cobb-Douglas per-capita production function

$$gdp(\chi,\varphi) = A \cdot \chi^{\alpha} \varphi^{\beta}, \tag{6.1}$$

and a linear per-capita cost function

$$cost(\chi,\varphi) = \chi \cdot CoX + \varphi \cdot CoL. \tag{6.2}$$

The per-capita production function, $gdp(\chi,\varphi)$, contains two production factors: φ (per-capita consumption of light), the production factor we wish to focus on here; and χ, all other production factors (including capital, materials, other energy services, etc) except labor. It also contains a proportionality constant, A, and two exponents, β and

α, representing the relative importance of the two production factors to *gdp*. Note that this per-capita production function is derived by normalizing the constant-returns-to-scale non-per-capita production function, $GDP(X,\Phi,N) = AX^{\alpha}\Phi^{\beta}N^{1-\alpha-\beta}$, by population, N. If we estimate the population (or labor) portion of production to be 0.7 (see, e.g., Jones 2002, p. 14), then we can also estimate $1\text{-}\alpha\text{-}\beta = 0.7$, or $\alpha+\beta = 0.3$.

The cost function, $cost(\chi,\varphi)$, is the sum of the same two production factors, φ and χ (which are of course also cost factors), weighted by their unit costs, *CoX* and *CoL*.

These two functions, $gdp(\chi,\varphi)$ and $cost(\chi,\varphi)$, can be thought of as surfaces above a two-dimensional (χ,φ) plane, with the shape of the *gdp* surface defined by the parameters A, α and β, and the shape of the *cost* surface defined by the parameters *CoX* and *CoL*. Profit is the difference between the two surfaces, $gdp(\chi,\varphi) - cost(\chi,\varphi)$, and is maximized at that point (χ,φ) where the planes tangent to the two surfaces are parallel. Since planes are defined by two slopes, this parallel tangent condition is equivalent to any two slopes being equal. Here, we choose these two slopes to be: those in the (χ,φ) plane at constant *gdp* or *cost*; and those in the (φ,gdp) or $(\varphi,cost)$ planes at constant χ.

The first parallel tangent condition is $\partial\chi/\partial\varphi|_{gdp} = \partial\chi/\partial\varphi|_{cost}$, which equates the marginal rates of technical substitution between χ and φ at constant *gdp* and *cost*. This condition defines a first relationship between φ and χ:

$$\frac{\alpha}{\chi \cdot CoX} = \frac{\beta}{\varphi \cdot CoL}. \tag{6.3}$$

The second parallel tangent condition is $\partial gdp/\partial\varphi|_{\chi} = \partial cost/\partial\varphi|_{\chi}$, which equates the marginal productivity and cost of φ at constant χ. This condition defines a second relationship between φ and χ:

$$(CoL)\cdot\varphi^{1-\beta} = (A\cdot\beta)\cdot\chi^{\alpha}. \tag{6.4}$$

Together, these two relationships enable that point (χ,φ) where profit is maximized to be defined exactly:

$$\chi = A^{\frac{1}{1-\alpha-\beta}}\left(\frac{\alpha}{CoX}\right)^{\frac{1-\beta}{1-\alpha-\beta}}\cdot\left(\frac{\beta}{CoL}\right)^{\frac{\beta}{1-\alpha-\beta}} \tag{6.5a}$$

$$\varphi = A^{\frac{1}{1-\alpha-\beta}}\cdot\left(\frac{\alpha}{CoX}\right)^{\frac{\alpha}{1-\alpha-\beta}}\cdot\left(\frac{\beta}{CoL}\right)^{\frac{1-\alpha}{1-\alpha-\beta}}. \tag{6.5b}$$

Substituting into Equations 6.1 and 6.2 then give the *gdp* and *cost* at the profit maximization point:

$$gdp = A^{\frac{1}{1-\alpha-\beta}}\cdot\left(\frac{\alpha}{CoX}\right)^{\frac{\alpha}{1-\alpha-\beta}}\cdot\left(\frac{\beta}{CoL}\right)^{\frac{\beta}{1-\alpha-\beta}} \tag{6.6a}$$

$$cost = A^{\frac{1}{1-\alpha-\beta}}\cdot\left(\frac{\alpha}{CoX}\right)^{\frac{\alpha}{1-\alpha-\beta}}\cdot\left(\frac{\beta}{CoL}\right)^{\frac{\beta}{1-\alpha-\beta}}(\alpha+\beta). \tag{6.6b}$$

Note that Equations 6.5 and 6.6 share many common factors, and can be rewritten more simply in terms of *gdp*:

$$\chi = \alpha\frac{gdp}{CoX} \tag{6.7a}$$

$$\varphi = \beta\frac{gdp}{CoL} \tag{6.7b}$$

$$cost = gdp\cdot(\alpha+\beta). \tag{6.7c}$$

The equivalence of Equations 6.7b and 3.1 allows us to equate the two β's, so that $\beta = 0.0072$. Using $\alpha+\beta = 0.3$, we then have $\alpha = 0.2928$. In other words, lighting, though important, is nonetheless a small fraction of a large world economy, with $\beta \ll \alpha$.

6.2 Rebound Effect for Lighting

In Subsection 6.1, we deduced, using the Cobb-Douglas production function, the dependences of φ and χ (from which could be inferred the dependences of *gdp* and *cost*) on *CoX* and *CoL*. Since, as described by Equation 2.4, $CoL = CoE \cdot (1+\kappa_\varphi)/\eta_\varphi$, *CoL* is related to *CoE* through luminous efficacy. Thus, these dependences on *CoL* can also be thought of as being dependences on η_φ (for constant *CoE*). In this Subsection, we derive expressions for the rebound effect in terms of the per capita consumption of energies associated with φ and χ, and ultimately in terms of how those per capita consumption of energies depend on η_φ.

Let us write economy-wide per capita consumption of energy as the sum of the per capita consumptions of energy associated with the two production factors χ and φ:

$$\dot{e} = \dot{e}_\chi + \dot{e}_\varphi . \tag{6.8}$$

The rebound, r, can be defined by

$$r - 1 = \frac{\eta_\varphi}{\dot{e}_\varphi} \frac{\partial \dot{e}}{\partial \eta_\varphi} , \tag{6.9}$$

where the left side of the equation is the rebound less unity, and the right side of the equation is the change in per capita consumption of energy for a change in lighting efficiency -- the first change normalized to per capita consumption of energy associated with lighting and the second change normalized to current lighting efficiency.

If the rebound is zero, then the fractional change in per capita consumption of energy is -1, exactly what one would anticipate if *gdp* and φ did not change, with any increase in η_φ going towards reducing consumption of energy associated with φ. If the rebound is unity, then the fractional change in energy consumption is zero. If the rebound is greater than unity, then the fractional change in energy consumption is greater than zero, and so-called "backfire" occurs.

The rebound itself is commonly broken out into a sum of two terms, so we rewrite the left side of Equation 6.8 as

$$r - 1 = r_\chi + r_\varphi - 1 . \tag{6.10}$$

The first term on the right side of Equation 6.10 represents "indirect rebound": increases in the consumption of χ (and in the energy associated with that consumption). The second term on the right side of Equation 6.10 represents "direct rebound": increases in the consumption of φ (and in the energy associated with that consumption). The third term on the right side of Equation 6.10 is negative unity, and represents the (negative) baseline percentage change in per capita consumption of energy associated with lighting for a percentage change in lighting efficiency: the so-called "engineering efficiency" reduction in per capita consumption of energy associated with an energy efficiency improvement (UKERC 2007).

The two per capita energy consumptions associated with the two production factors can be expressed as the two production factors themselves, χ and φ, weighted by the efficiencies η_χ and η_φ with which each of those production factors "uses" energy:

$$\dot{e}_\chi = \frac{\chi}{\eta_\chi} \tag{6.11a}$$

$$\dot{e}_\varphi = \frac{\varphi}{\eta_\varphi}. \tag{6.11b}$$

Note that the efficiency η_φ is simply luminous efficacy, as discussed in Subsection 2.2, where we have appended a subscript φ to distinguish it from the efficiency η_χ with which energy is used to produce χ. Also note that η_φ and η_χ include both the energy used to produce χ or φ, as well as energy input used to produce the capital equipment that in turn produces χ or φ. In the case of φ, we found in Subsection 2.5 that the energy input "embodied" in the lamps and luminaires that produce light is a very small (< 1/85) fraction of the direct energy input used to produce light, so for our purpose here we equate η_φ with simple luminous efficacy.

Using these expressions for per capita consumption of energy associated with the two production factors, we can write the right hand side of Equation 6.9 as:

$$\begin{aligned}\frac{\eta_\varphi}{\dot{e}_\varphi}\frac{\partial \dot{e}}{\partial \eta_\varphi} &= \frac{\eta_\varphi}{\dot{e}_\varphi}\cdot\left[\frac{\partial \dot{e}_\chi}{\partial \eta_\varphi}+\frac{\partial \dot{e}_\varphi}{\partial \eta_\varphi}\right] \\ &= \frac{\eta_\varphi}{\varphi/\eta_\varphi}\cdot\left[\frac{1}{\eta_\chi}\frac{\partial \chi}{\partial \eta_\varphi}+\frac{1}{\eta_\varphi}\frac{\partial \varphi}{\partial \eta_\varphi}-\frac{\varphi}{\eta_\varphi^2}\right]. \\ &= \left(\frac{\dot{e}_\chi}{\dot{e}_\varphi}\right)\cdot\frac{\eta_\varphi}{\chi}\frac{\partial \chi}{\partial \eta_\varphi}+\frac{\eta_\varphi}{\varphi}\frac{\partial \varphi}{\partial \eta_\varphi}-1\end{aligned} \tag{6.12}$$

Equating and comparing Equations 6.10 and 6.12 (so that Equation 6.9 holds) allows the indirect and direct rebounds to be expressed as:

$$r_\chi = \left(\frac{\dot{e}_\chi}{\dot{e}_\varphi}\right)\cdot\frac{\eta_\varphi}{\chi}\frac{\partial \chi}{\partial \eta_\varphi} \tag{6.13a}$$

$$r_\varphi = \frac{\eta_\varphi}{\varphi}\frac{\partial \varphi}{\partial \eta_\varphi}. \tag{6.13b}$$

Since $CoL = CoE\cdot(1+\kappa_\varphi)/\eta_\varphi$ (Equation 2.4), we have

$$\frac{\eta_\varphi}{CoL}\frac{dCoL}{d\eta_\varphi} = -1, \tag{6.14}$$

so the variations with respect to η_φ can be rewritten as variations with respect to CoL. Applying Equations 6.5 then gives:

$$r_\chi = \left(\frac{\dot{e}_\chi}{\dot{e}_\varphi}\right)\cdot\frac{\eta_\varphi}{\chi}\frac{\partial \chi}{\partial \eta_\varphi} = -\left(\frac{\dot{e}_\chi}{\dot{e}_\varphi}\right)\cdot\frac{CoL}{\chi}\left[\frac{\partial \chi}{\partial CoL}\right]_{CoX} = \left(\frac{\dot{e}_\chi}{\dot{e}_\varphi}\right)\cdot\frac{\beta}{1-\alpha-\beta} \tag{6.15a}$$

$$r_\varphi = \frac{\eta_\varphi}{\varphi}\frac{\partial \varphi}{\partial \eta_\varphi} = -\frac{CoL}{\varphi}\left[\frac{\partial \varphi}{\partial CoL}\right]_{CoX} = \frac{1-\alpha}{1-\alpha-\beta}. \tag{6.15b}$$

Note that the rebound, r_χ, is multiplied by a relatively large ratio between the per capita energy consumed to produce χ to the per capita energy consumed to produce φ. This ratio is due to our normalizing the right side of Equation 6.9 by per capita consumption of energy associated *only* with φ because that is the baseline per capita consumption of energy expected to first order to be affected by changes in lighting efficiency. Since, as discussed in Section 5, lighting accounts for about 6.5% of all energy consumption, we have $\dot{e}_\chi/\dot{e}_\varphi \sim (100\%\text{-}6.5\%)/6.5\% \sim 14$. The indirect rebound term, therefore, has the potential to be very large.

6.3 Magnitude and Uncertainty of the Rebound Effect for Lighting

As mentioned in Section 1, the magnitude of the rebound effect is of great current interest for its implications on the use of energy-efficiency technology to reduce overall energy consumption. It was discussed as early as 1865 by Jevons (Jevons 1906)[19] and the effect is sometimes known as Jevons' paradox (Alcott 2005). It has also been reviewed recently in a comprehensive set of reports from the United Kingdom Energy Research Centre (UKERC 2007).

As has been discussed by Brookes (Brookes 2004) and Saunders (Saunders 2000), it is relatively easy to find a theoretical basis for relatively large rebound effects (magnitudes of unity or greater). However, it has not been easy to find an empirical basis for such large rebound effects: most empirical studies have yielded rebound effects on the order of 0.05-0.15 (Greening et al. 2000) -- though one can infer an effect in the range 0.5-0.8 in one study of lighting in households without access to grid electricity (Roy 2000).

We suspect that the principal source of the discrepancy is time scale. The full response of an economy to a change in technology includes adjustments in lifestyle, and invention and diffusion of supporting complementary technologies (Rosenberg 1982), all of which requires time to develop. Hence, empirical studies over months-to-years time periods seem bound to reveal smaller effects than those over decades-to-centuries time periods.[20]

Indeed, by connecting the expressions derived in Subsections 6.1 and 6.2 with the empirical result of Section 3, we can estimate that the magnitude of the rebound effect associated with lighting could be unity or greater. To do this, we use for the coefficients in the Cobb-Douglas production function: $\beta = 0.0072$ and $\alpha = 0.2928$, as discussed in Subsection 6.1. Inserting these values into Equations 6.15 gives estimates for the various rebound effects: $r_\varphi \sim 1$ and $r_\chi \sim 0.2$, for a total rebound of $r \sim 1.2$.

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[19] From pp. 140-141:

"It is wholly a confusion of ideas to suppose that the economical use of fuel is equivalent to a diminished consumption. The very contrary is the truth. As a rule, new modes of economy will lead to an increase of consumption, according to a principle recognised in many parallel instances. The economy of labour effected by the introduction of new machinery, for the moment, throws labourers out of employment. But such is the increased demand for the cheapened products, that eventually the sphere of employment is greatly widened. Often the very labourers whose labour is saved find their more efficient labour more demanded than before. Seamstresses, for instance, have perhaps in no case been injured, but have often gained wages before unthought of, by the use of the sewing-machine, for which we are so much indebted to American inventors.

So it is a familiar rule of finance that the reduction of taxes and tolls leads to increased gross and sometimes even net revenue; and it is a maxim of trade, that a low rate of profits, with the multiplied business it begets, is more profitable than a small business at a high rate of profit.

Now the same principles apply, with even greater force and distinctness, to the use of such a general agent as coal. It is the very economy of its use which leads to its extensive consumption. It has been so in the past, and it will be so in the future."

[20] The lower-than-expected consumption of light for the data points associated with China (1993, 2005, 2006) seen in Figure 2 may be due to a lag time associated with consumption of light keeping pace with the extremely rapid rate at which *gdp* has grown in that nation.

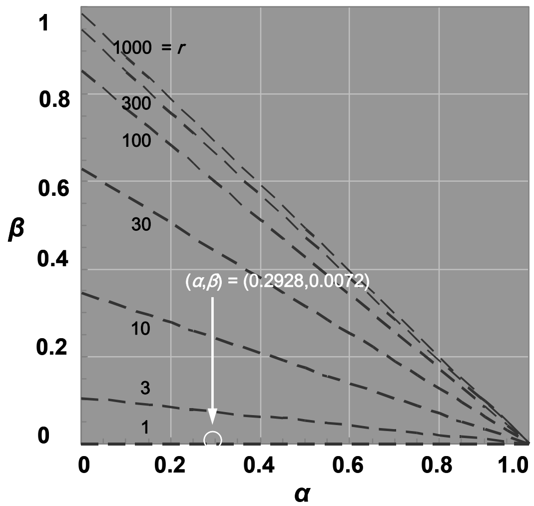

Figure 6: Contours of constant rebound as the two exponents β and α associated with the two production factors φ (light) and χ (everything else) vary.

Note that, for α and β in the range estimated here, the rebound effect is not particularly sensitive to their exact values. To illustrate this, we plot in Figure 6 contours of constant rebound on an (α,β) plot. Only for large (near-unity) α or β does the rebound effect become sensitive to their exact values. For α and β in the modest range estimated here, the rebound effect is near unity, and, as β approaches zero, the rebound effect becomes independent of β and approaches unity itself. It never decreases below unity, however, at least within the framework of the Cobb-Douglas production function.

6.4 Energy Intensity

Interestingly, even for unity or larger rebound effects, energy *intensities* within the framework of the Cobb-Douglas production function do not change (Saunders 2008b). This can be seen by writing per capita energy consumption, using Equations 6.8, 6.11, 6.5 and 2.4, as

$$\dot{e} = A^{\frac{1}{1-\alpha-\beta}} \cdot \left[\frac{\alpha}{CoX}\right]^{\frac{\alpha}{1-\alpha-\beta}} \cdot \left[\frac{\beta}{CoE_{\varphi}(1+\kappa_{\varphi})/\eta_{\varphi}}\right]^{\frac{\beta}{1-\alpha-\beta}} \cdot \left[\frac{\alpha}{\eta_{\chi}\cdot CoX} + \frac{\beta}{CoE_{\varphi}(1+\kappa_{\varphi})}\right], \quad (6.17)$$

and per capita *gdp*, using Equations 6.8 and 2.4, as

$$gdp = A^{\frac{1}{1-\alpha-\beta}} \cdot \left[\frac{\alpha}{CoX}\right]^{\frac{\alpha}{1-\alpha-\beta}} \cdot \left[\frac{\beta}{CoE_{\varphi}(1+\kappa_{\varphi})/\eta_{\varphi}}\right]^{\frac{\beta}{1-\alpha-\beta}}. \quad (6.18)$$

Here, note that we have appended a subscript "φ" to *CoE* to make it clear that we mean only the cost of energy used for lighting (φ), not the cost of energy used for the other production factors (χ).

On the one hand, per capita energy consumption and per capita gdp both increase with η_{φ} in exactly the same proportion, so energy intensity,

$$\frac{\dot{e}}{gdp} = \frac{\alpha}{\eta_{\chi}\cdot CoX} + \frac{\beta}{CoE_{\varphi}(1+\kappa_{\varphi})}, \quad (6.19)$$

is independent of luminous efficacy, η_{φ}.[21] On the other hand, per capita energy consumption and per capita gdp do not change with CoE_{φ} in the same proportion, so energy intensity is not independent of CoE_{φ}. If CoE_{φ} were to increase, due to market forces or policy, energy intensity *would* decrease. As can be seen from Equations

[21] Note that if Equation 2.4 did not hold, and instead *CoL* had a significant component independent of consumption of energy, then energy intensity would instead be $\dot{e}/gdp = [\alpha/(\eta_{\chi}\cdot CoX)] + [\beta/(\eta_{\varphi}\cdot CoL)]$, and would indeed decrease with increasing η_{φ}. From this, it would appear that historical decreases in energy intensity have not been due to primarily to increases in the energy efficiency of energy-intensive energy services, but rather to increases in the energy efficiency of energy-*un*intensive goods and services.

6.17 and 6.18, $\dot{e}$ decreases more rapidly than *gdp* with increases in CoE_φ. Because of this, increases in η_φ and CoE_φ which offset each other's effects on *gdp* will leave a residual decrease in $\dot{e}$, while those which offset each other's effects on $\dot{e}$ will leave a residual increase in *gdp*.

The beneficial nature of these residual increases and decreases is illustrated in Figure 7. That Figure shows a number of world 2050 scenarios similar to (but not exactly the same as) those shown in Table 1 and illustrated in Figure 5. The scenarios were calculated using values for the two factors $A^{1/(1-\alpha-\beta)}[\alpha/CoX]^{\alpha/(1-\alpha-\beta)}$ and $\alpha/[\eta_\chi \cdot CoX]$ determined by substituting projected 2050 values[22] of gdp = 16,830 \$/(per-yr) and $\dot{e}/gdp$ = 554 W_eh/\$ into Equations 6.17 and 6.19.

Scenario A shows a hypothetical world in 2050 identical to the WRLD 2050 Ker+Inc+Flu+HID scenario shown in Table 1. This world has a per capita gross domestic product of gdp = 16,830 \$/(per-yr), a per capita energy consumption of $\dot{e}$ = 9.3 MW_eh/(per-yr) (of which 6.5% is that associated with lighting), a cost of energy for lighting of CoE_φ = 119 \$/MW_eh, and a luminous efficacy of η_φ = 47.5 lm/W_e.

Scenario B shows a hypothetical world similar to that of Scenario A except with a luminous efficacy increased to η_φ = 148 lm/W_e. As anticipated from Equations 6.17 and 6.18, the roughly 103% = 2·(148-47.5)/(148+47.5) increase in η_φ manifests itself as roughly 103%·$[\beta/(1-\alpha-\beta)]$ ~ 1.03·0.01 ~ 1% increases in both *gdp* and $\dot{e}$, to gdp = 17,030 \$/(per-yr) and $\dot{e}$ = 9.4 MW_eh/(per-yr). But because, as discussed above, *gdp* and $\dot{e}$ both increase in the same proportion, energy intensity is constant at $\dot{e}/gdp$ = 554 W_eh/\$.

Scenario C shows a hypothetical world similar to that of Scenario B except with a cost of energy for lighting increased to CoE_φ = 137 \$/MW_eh. This increase in cost of energy for lighting causes $\dot{e}$ to decrease proportionately more than *gdp*, so that it is possible to return to the same $\dot{e}$ as that of Scenario A, while maintaining a large portion of the increase in *gdp* (now 17,000 \$/(per-yr)).

Scenario D shows a hypothetical world similar to that of Scenario C except with a cost of energy for lighting increased further to CoE_φ = 370 \$/MW_eh. This increase in cost of energy for lighting causes $\dot{e}$ to continue its proportionately greater decrease than *gdp*, now to the point that $\dot{e}$ decreases, to $\dot{e}$ = 8.8 MW_eh/(per-yr), even while *gdp* has returned to that of Scenario A. The strikingly larger percentage decrease in $\dot{e}$ than in *gdp* is due to the energy service nature of lighting and the larger percentage of $\dot{e}$ (6.5%) than of *gdp* (0.72%) that it consumes.

Scenario E shows a hypothetical world similar to that of Scenario C except with a luminous efficacy that has returned to the luminous efficacy η_φ = 47.5 lm/W_e of Scenario A. Scenario E is thus also the same as Scenario A except with a cost of energy for lighting that has increased to CoE_φ = 370 \$/MW_eh. Without the benefit of an increase in luminous efficacy, the effect of the increase in cost of energy for lighting is to decrease *gdp* by 1% and $\dot{e}$ by 6% to gdp = 16,630 \$/(per-yr) and $\dot{e}$ = 8.7 MW_eh/(per-yr).

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[22] As with Table 1 and the discussion in Section 5.3, the projected 2050 gdp is consistent with the "moderate" Scenario B2 from the Intergovernmental Panel on Climate Change (Nakicenovic and Swart 2000). The projected 2050 energy intensity is estimated to be 554 W_eh/\$ = 0.2 W_cy/\$ (Hoffert et al. 1998; Tsao et al. 2006). For simplicity and consistency with the rest of the paper, our unit for energy in this Section is W_eh of electricity rather than W_ch of chemical fuel, where we use the factor σ_{grid} = 0.316 W_e/W_c discussed in Section 2 for converting between these units.

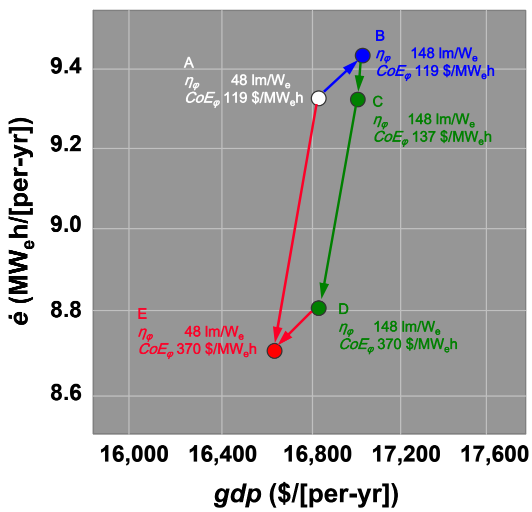

Figure 7: Projected world 2050 per capita energy and per capita gross domestic product scenarios for various assumptions on luminous efficacy (η_φ) and cost of energy for lighting (CoE_φ). Both axes are plotted logarithmically and gridded so that equal vertical or horizontal grid spacings represent equal percentage changes. All scenarios assume α = 0.2928 and β = 0.0072.

Again, the quantitative results illustrated in Figure 7 cannot be taken too seriously, as they depend on the uncertain applicability of the Cobb-Douglas production function. Nevertheless, the qualitative results illustrate the principal benefits to increased luminous efficacy.

On the one hand, for a constant cost of energy for lighting, increased luminous efficacy shifts the production frontier and enables per capita gdp to increase, albeit at the expense of an increased per capita energy consumption. In other words, increased luminous efficacy does not decrease per capita energy consumption, but it *does* increase human productivity and standard of living.

On the other hand, for a non-constant (increased) cost of energy for lighting, increased luminous efficacy can offset the decrease in per capita gdp that would otherwise occur. In other words, increased luminous efficacy allows human productivity and standard of living to be maintained even if policy or market forces cause increases in the cost of energy for lighting.

7 SUMMARY

We have found that a simple empirical expression, in which per capita consumption of artificial light depends linearly or nearly linearly on the ratio between per capita gross domestic product and cost of light, is consistent with data spanning a remarkable: 3 centuries (1700-2006), 6 continents (Africa, Asia, Australia, Europe, North America, South America), 5 types of fuel (tallow, whale oil, gas, petroleum, electricity), 5 overall families of lighting technologies (candles, oil lamps, gas lamps, electric incandescent bulbs, electric gas-discharge bulbs or tubes), 1.4 orders of magnitude in per capita gross domestic product, 4.3 orders of magnitude in cost of light, and 5.4 orders of magnitude in per capita consumption of light.

The implication is that over decades-to-centuries time periods the income and price elasticities of demand for artificial light have been unity or nearly unity.

From a practical perspective, this result represents the historically consistent baseline assumption for constructing future scenarios for consumption of light and associated energy. In other words, there is a massive *potential* for growth in the consumption of light if new lighting technologies are developed with higher luminous efficacies and lower cost of light.

Whether history is predictive of the future cannot be known, of course, and there are at least three ways in which this growth might be moderated. First, as discussed in Section 5, the cost of energy might increase, either through natural supply and demand considerations, or through energy and/or carbon taxes. Second, as discussed in Section 4, demand for light may be nearing saturation, both in terms of illuminances (lm/m^2) as well as in terms of the per capita illuminated areas surrounding people. Third, even if intrinsic demand for light is not nearing saturation, regulation and legislation aimed at efficient lighting system designs and light usage may nonetheless bring about a saturation.

Again, whether any of these ways are realized, or whether the historical trend does indeed play out into the future, cannot be known. Given, however, that lighting accounts for about 6.5% of world energy consumption and is poised at the brink of a technology revolution, this issue is of great current interest for forecasting future energy consumption and informing public policy.

From a theoretical perspective, this result also has implications on the "rebound effect," of great current interest in energy economics. From a simple analysis using a Cobb-Douglas production function, we can quantify crudely a rebound effect of ~1.2. Even with such a large rebound effect, however, increased luminous efficacy has two important benefits: an increase in economic productivity if the cost of energy for lighting is constant and hence energy consumption is free to increase; and a mitigation of the decrease in economic productivity were the cost of energy for lighting to increase in order to decrease energy consumption.

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