Implications of compressed supersymmetry with natural dark matter from annihilations to top quarks

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Executive Summary

Experimental results from LEP2, Tevatron, and WMAP put tension on the parameter space of minimal supersymmetry.

A possible resolution:

- "Compressed" supersymmetry: the spectrum of superpartner masses has a narrower range than is found in the most popular models.
- Dark matter with observed density explained by LSP† pair annihilation to top quark-antiquark pairs.

This scenario has sharp implications for collider physics and dark matter detection.

† LSP $=$ Lightest Supersymmetric Particle, assumed to be a neutralino $\tilde{N}_1$.

LEP2 did not find a Higgs boson

If the Higgs sector is Standard Model-like, then LEP2 implies $M_h \gtrsim 114$ GeV in most of SUSY parameter space.

A simplified form of the SUSY prediction is:

$$M_h^2 = m_Z^2 \cos^2(2\beta) + \frac{3y_t^2}{4\pi^2} \sin^2\beta \, m_t^2 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

Top squarks are spin-0 partners of top quark: $\tilde{t}_1$, $\tilde{t}_2$.

$\tan\beta = v_u/v_d$ = ratio of Higgs VEVs.

To evade discovery at LEP2, need $\sin\beta \approx 1$ and (naively)

$$\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \gtrsim 700 \text{ GeV}.$$

The logarithm apparently must be $\gtrsim 3$.

Meanwhile, the condition for Electroweak Symmetry Breaking is:

$$m_Z^2 \;=\; -2\left(|\mu|^2 + m_{H_u}^2\right) + \text{small loop corrections} \;+\; \mathcal{O}(1/\tan^2\beta).$$

Here $|\mu|^2$ is a SUSY-preserving Higgs squared mass,

$m_{H_u}^2$ is a SUSY-violating Higgs scalar squared mass.

The problem: assuming $m_{H_u}^2$ is comparable to $m_{\tilde{t}_1} m_{\tilde{t}_2} \;\gtrsim\; (700\ \mathrm{GeV})^2$ as found above, then the required cancellation here is of order 1%. This may seem like an amazing coincidence.

However, the fine-tuning required is actually not quite so severe as often portrayed, for at least two reasons:

- The previous expression for M_h^2 is too simplistic
- The relation between $m_{H_u}^2$ and stop masses is subtle

Include effects of a stop mixing angle with (cosine, sine) $= c_{\tilde{t}},\ s_{\tilde{t}}$:

$$M_h^2 = m_Z^2 + \frac{3y_t^2}{4\pi^2} m_t^2 \left[\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \frac{c_{\tilde{t}}^2 s_{\tilde{t}}^2}{m_t^2}(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2) \ln\left(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right) + \frac{c_{\tilde{t}}^4 s_{\tilde{t}}^4}{m_t^4} \left\{ (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2 - \frac{1}{2}(m_{\tilde{t}_2}^4 - m_{\tilde{t}_1}^4) \ln\left(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right) \right\} \right].$$

The Blue term is positive definite.

The Red term is negative definite.

Maximizing with respect to the stop mixing angle, one can show:

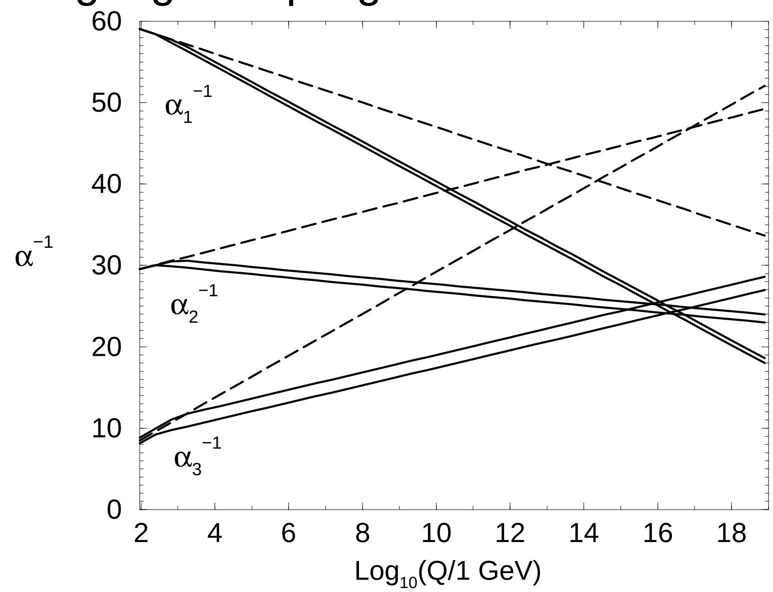
$$M_h^2 < m_Z^2 + \frac{3y_t^2}{4\pi^2} m_t^2 \left[\ln\left(m_{\tilde{t}_2}^2/m_t^2\right) + 3 \right]$$

within this approximation.

In cases below, the stop-mixing contribution is closer to 1 than to 3, but it helps.

Why should $m_{H_u}^2$ and the stop masses be related?

Apparent unification of gauge couplings is one clue:



This invites us to use renormalization group running to also evolve SUSY breaking masses down from the GUT scale. Schematically, scalar masses run like:

$$\frac{d}{d(\ln Q)} m_\phi^2 \sim \frac{1}{16\pi^2} \left[g_a^2 |M_a|^2 - y^2 \sum m_\Phi^2 \right]$$

where M_a are gaugino masses and m_Φ^2 are other scalar masses.

The so-called mSUGRA model says:

gluino, wino, bino masses unified: $\hat{M}_3 = \hat{M}_2 = \hat{M}_1 = m_{1/2}$

scalar masses unified: $\hat{m}^2_\phi = m^2_0$

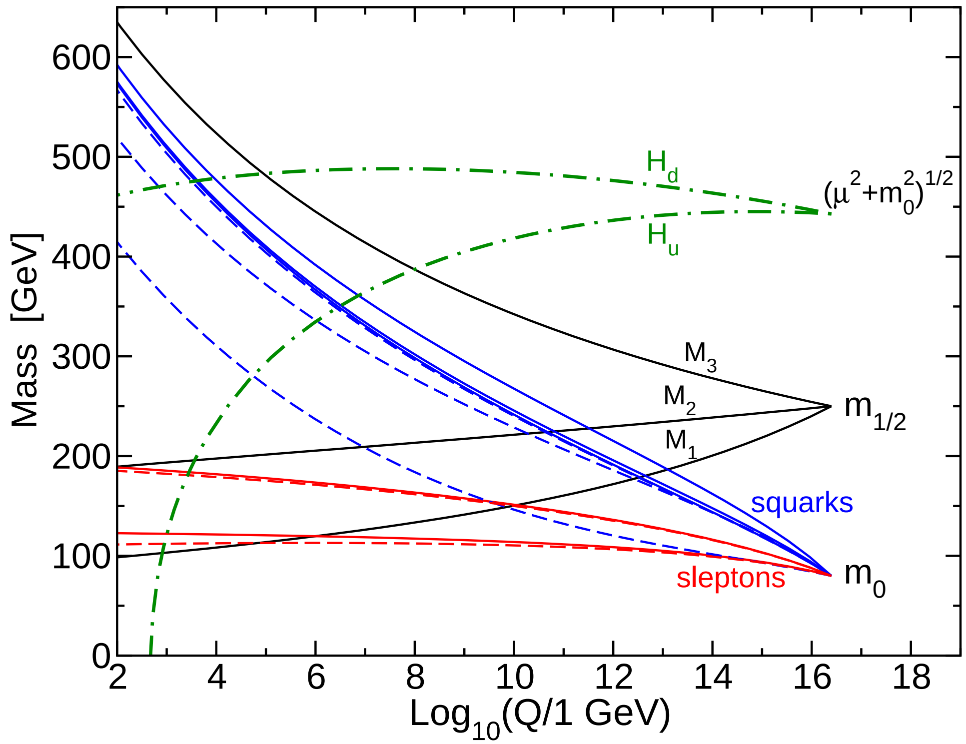
(scalar)^3 couplings unify: $\hat{A}_t = \hat{A}_b = \hat{A}_\tau = A_0$

Hats denote running couplings at the GUT scale.

This model parameterization is simple, but not otherwise well-motivated. It ignores running between GUT scale and Planck scale!

Abandoning mSUGRA allows interesting and qualitatively different phenomenology.

Assuming unified gaugino and scalar masses near the GUT scale predicts a hierarchical mass spectrum at the TeV scale:



This hierarchy is mostly the gluino’s fault.

More precisely. . .

Fine tuning of the electroweak scale is reduced if the pernicious influence of the gluino is suppressed. (G. Kane and S. King, `hep-ph/9810374`)

$$
\begin{aligned}
-m_{H_u}^2 \;&=\; 1.92\hat{M}_3^2 + 0.16\hat{M}_2\hat{M}_3 - 0.21\hat{M}_2^2 \\
&\quad -0.63\hat{m}_{H_u}^2 + 0.36\hat{m}_{t_L}^2 + 0.28\hat{m}_{t_R}^2 \\
&\quad + \text{many terms with tiny coefficients}
\end{aligned}
$$

The hatted parameters on the right are at the GUT scale, result is at the TeV scale.

If one takes a smaller gluino mass at the GUT scale, say $\hat{M}_3/\hat{M}_2 \sim 1/3$, then $-m_{H_u}^2$ will be much smaller.

As a result, $|\mu|^2$ will be smaller also.

Are there reasonable models in which $\hat{M}_3/\hat{M}_2$ is smaller?

Answer: yes, too many to count. String theories have no unified gauge group even though the gauge couplings unify, so the gluino/wino mass ratio can be anything you want.

I'll review my favorite mechanism, which even works if there is really a GUT theory like $SU(5)$ or $SO(10)$. But results below about dark matter don't depend crucially on this choice.

Usual assumption: the source of spontaneous SUSY breaking is a VEV

$$\langle F \rangle \neq 0$$

that is a singlet under the unified gauge group. This is implicit in the mSUGRA boundary condition.

But, more generally, F could transform as anything in the symmetric product of the adjoint rep of the unified gauge group with itself, as long as it is a Standard Model singlet.

For $SU(5)$:

$$(\mathbf{24}\times\mathbf{24})_S = \mathbf{1}+\mathbf{24}+\mathbf{75}+\mathbf{200}.$$

Suppose the F terms that break SUSY include both a singlet and an adjoint. Then at the GUT scale, one can parametrize:

$$\begin{aligned}\hat{M}_1 &= m_{1/2}(1+C_{24}),\\ \hat{M}_2 &= m_{1/2}(1+3C_{24}),\\ \hat{M}_3 &= m_{1/2}(1-2C_{24}).\end{aligned}$$

The special case $C_{24}=0$ recovers the usual mSUGRA model.

In $SU(5)$ GUT models, there is a superfield in the $\mathbf{24}$ (adjoint) representation anyway, used to break the group down to the Standard Model.

It is unnatural for the corresponding F field to not get a VEV, if the scalar component does.

To obtain $\hat{M}_3/\hat{M}_2 \sim 1/3$, one needs only $C_{24} \sim 0.2$.

$$\begin{aligned}
\hat{M}_1 &= m_{1/2}(1 + C_{24}), \\
\hat{M}_2 &= m_{1/2}(1 + 3C_{24}), \\
\hat{M}_3 &= m_{1/2}(1 - 2C_{24}).
\end{aligned}$$

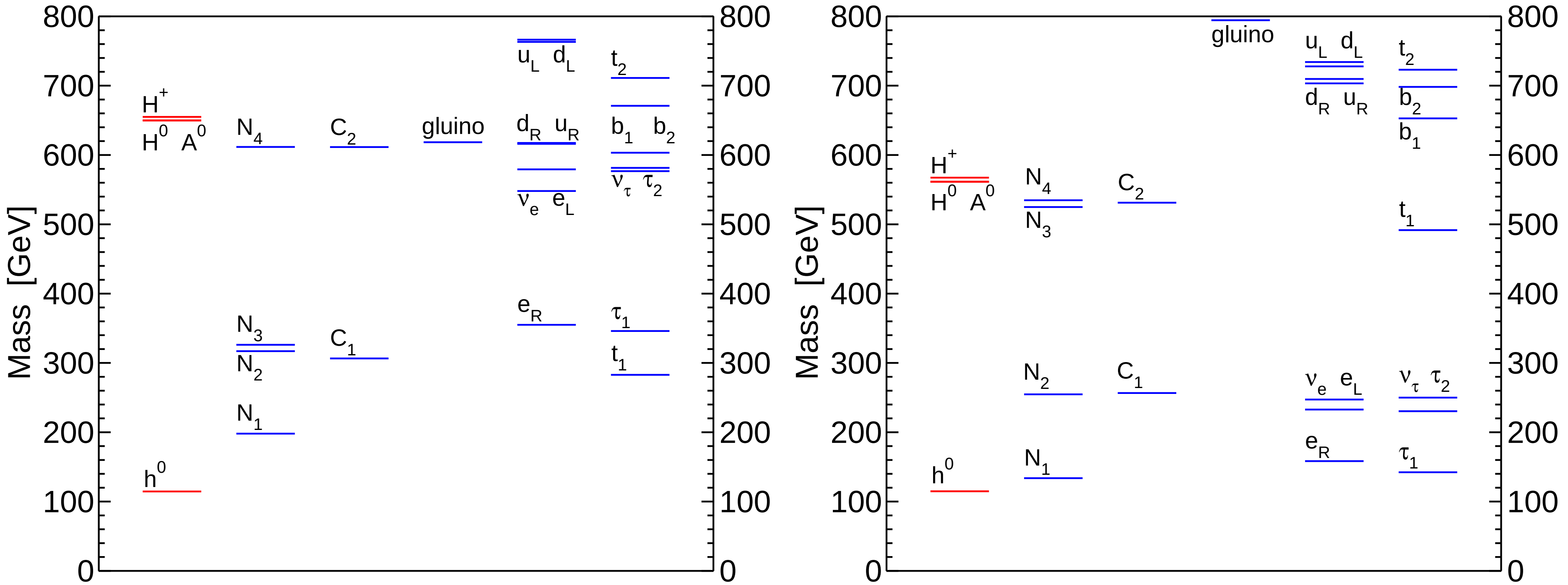
In the following, I will also assume a common scalar squared mass m_0^2, and a unified, sizable and negative (scalar)^3 coupling A_0.

The m_0 and A_0 unification assumptions are made for simplicity and convenience; the main qualitative feature of interest is the relative suppression of the gluino mass, which feeds less into squark masses as well.

The result is a “compressed” SUSY spectrum, with a smaller ratio of the masses of the heaviest SUSY particle and the LSP.

Comparison of Compressed SUSY and mSUGRA, for models with Higgs mass m_h just above LEP2 bound, and heaviest squarks in the 700-800 GeV range:

Compressed SUSY $\hat{M}_3/\hat{M}_2 = 1/3$ mSUGRA $\hat{M}_3/\hat{M}_2 = 1$



Note $|\mu|^2$ lower in Compressed SUSY; less cancellation needed.

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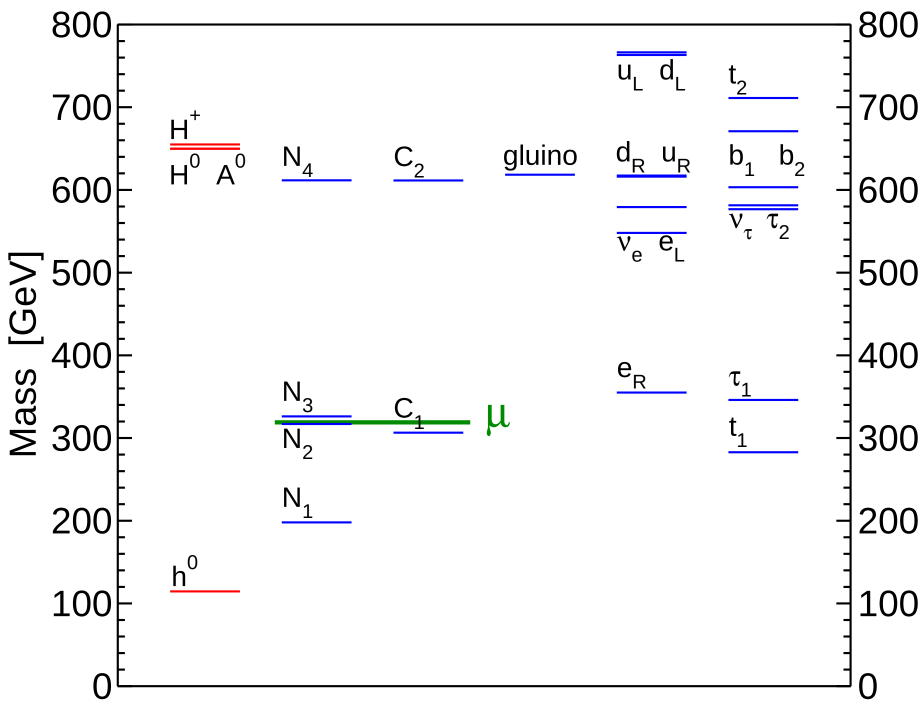
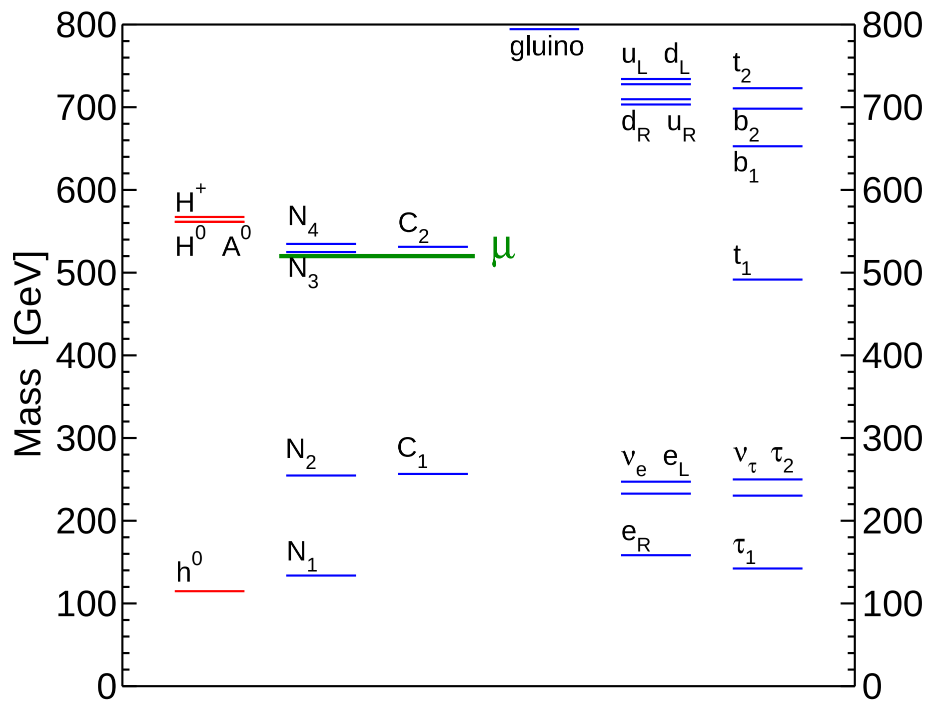


mSUGRA $\hat{M}_3/\hat{M}_2 = 1$



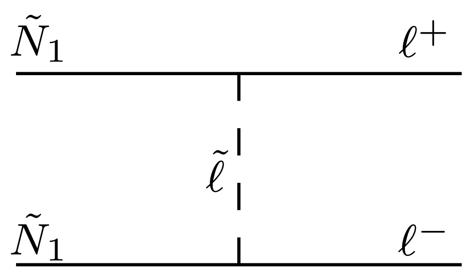
Note $|\mu|^2$ lower in Compressed SUSY; less cancellation needed.

Now let’s switch gears and consider the constraint from dark matter.

WMAP and other experiments have measured $\Omega_{\rm DM}h^2 \approx 0.11$

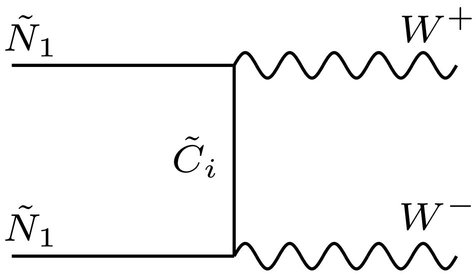
In much of the remaining SUSY parameter space, $\Omega_{\rm DM}h^2$ comes out too large. A mechanism for efficient annihilation of LSPs in the early universe is needed. The exceptions are usually classified qualitatively in 4 main categories:

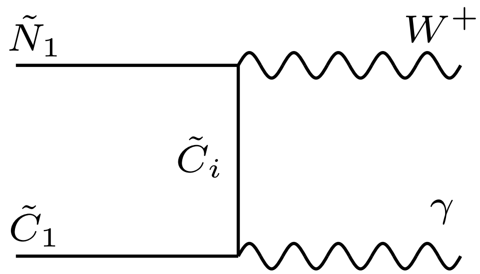
1) "Bulk region": LSPs annihilate through slepton exchange.

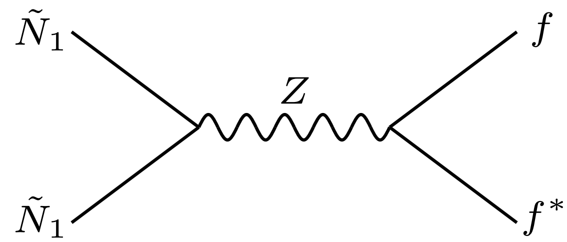


In mSUGRA, it is tough to accomodate this and LEP2 bounds at the same time.

2) "Focus point/Small μ": LSPs have enough higgsino content to annihilate or coannihilate to/through weak bosons

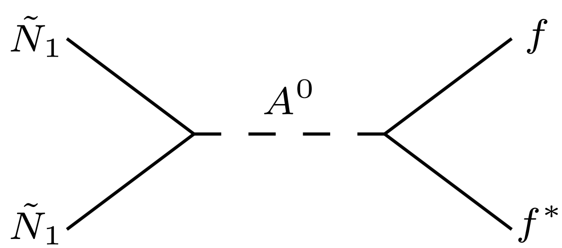






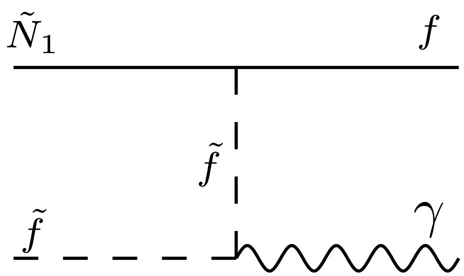
Need to get μ just right.

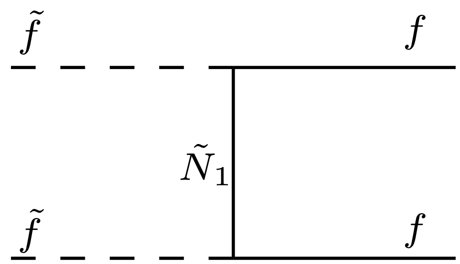
3) "Higgs resonance (funnel)": LSPs annihilate by s channel pseudoscalar Higgs exchange



Need LSP mass to be close to $m_{A^0}/2$, usually large $\tan\beta$.

4) “Co-annihilation region”: LSPs co-annihilate with sleptons (or top squarks) in thermal equilibrium



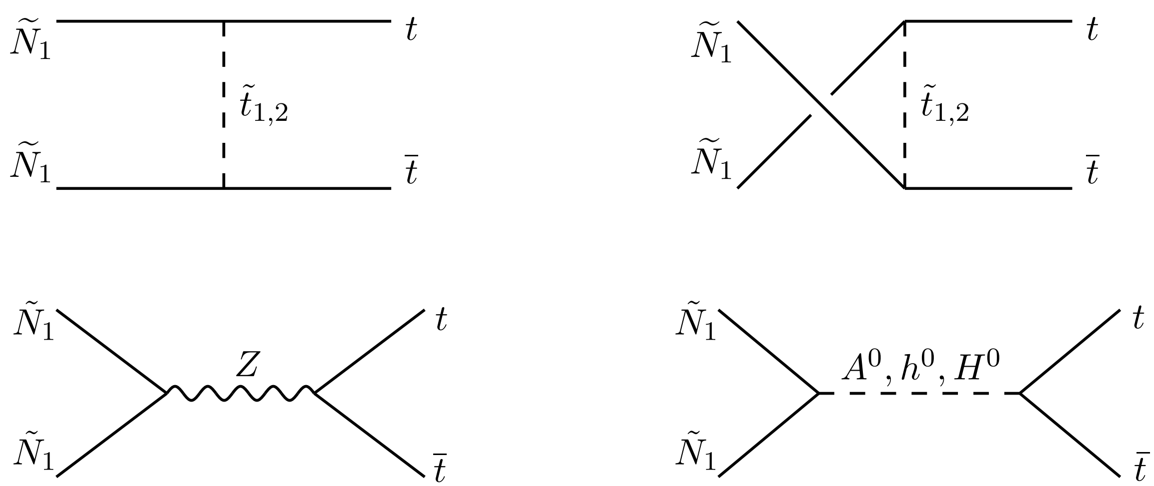


Need a small sfermion-LSP mass difference, tuned just right.

In Compressed Supersymmetry, I claim another scenario becomes natural, because the LSP is naturally heavier than the top quark, and the top squark is the next-lightest superpartner. . .

An alternative: Pair annihilation of LSPs to top quarks, mediated by top squark exchange.

Diagrams leading to $\tilde{N}_1 \tilde{N}_1 \to t\bar{t}$:



To get $\Omega_{\rm DM} h^2$ into the WMAP allowed range, need roughly:

$$m_t < m_{\tilde{N}_1} \lesssim m_t + 100 \text{ GeV},$$
$$m_{\tilde{N}_1} + 25 \text{ GeV} \lesssim m_{\tilde{t}_1} \lesssim m_{\tilde{N}_1} + 100 \text{ GeV}.$$

In Compressed Supersymmetry, the $\tilde{t}_1$ exchange can naturally dominate.

(The Z exchange diagram interferes destructively.)

In the following, I consider models with m_0 and $\hat{M}_1$ variable, with

$$C_{24} \neq 0, \qquad \tan\beta = 10, \qquad \mu > 0.$$

Imposed Higgs mass constraint (noting significant theoretical uncertainties):

$$M_h > 113\,\mathrm{GeV}.$$

Also, I assume

$$M_t = 172.7\,\mathrm{GeV}$$

which is the present one-sigma upper value.

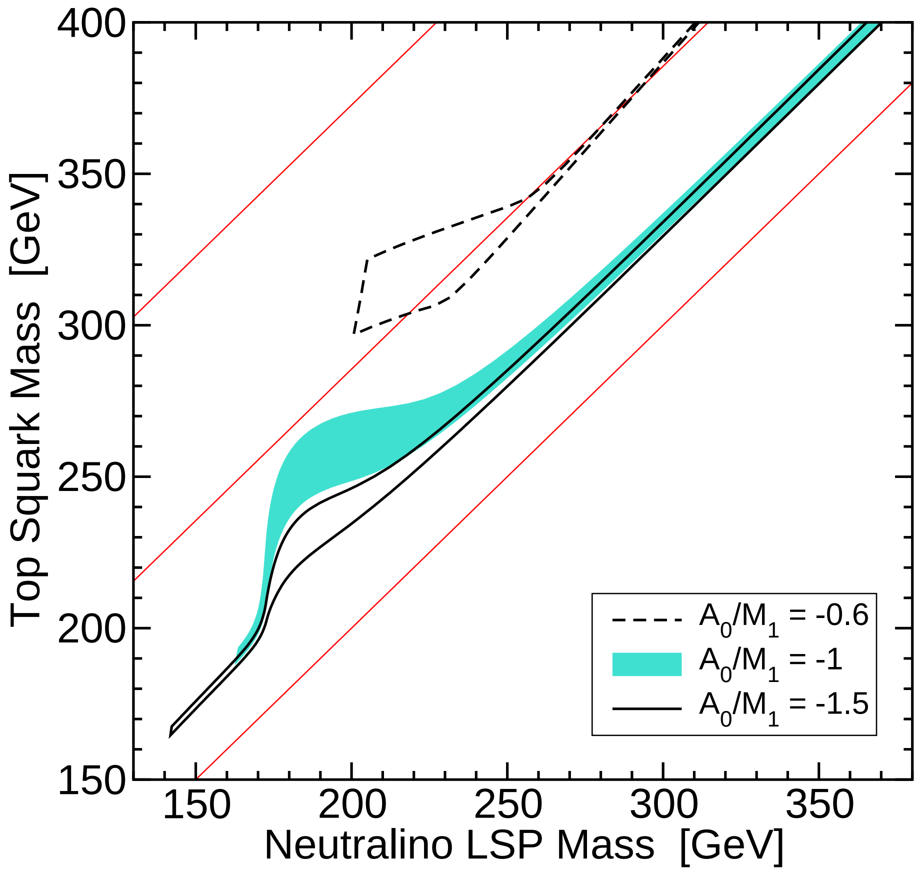
When the neutralino is the LSP, then all LEP2 and Tevatron bounds on superpartner masses are (easily) satisfied.

Finally, I impose the rather conservative constraint on dark matter:

$$0.09 < \Omega_{\mathrm{DM}} h^2 < 0.13$$

computed using `micrOMEGAs` (Belanger, Boudjema, Pukhov, Semenov).

Allowed regions in the $m_{\tilde{t}_1}$, $m_{\tilde{N}_1}$ plane for $C_{24} = 0.21$:



In the bulge regions, $\tilde{N}_1 \tilde{N}_1 \rightarrow t\bar{t}$ is mediated mostly by $\tilde{t}_1$ exchange.

Below upper red line, $\tilde{t}_1 \rightarrow t\tilde{N}_1$ is forbidden.

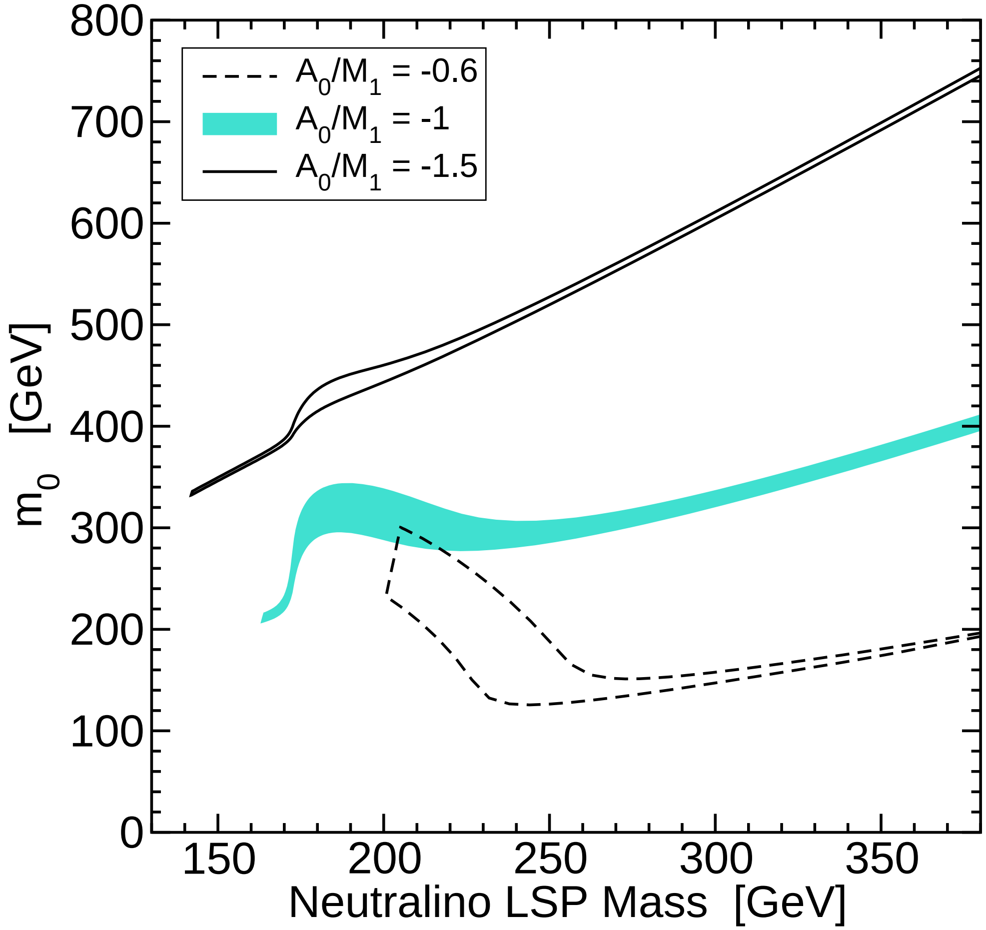
Below middle red line, $\tilde{t}_1 \rightarrow W b \tilde{N}_1$ is also forbidden.

Below lowest red line, $\tilde{t}_1$ is LSP.

Regions are cut off on the left by the M_h constraint.

Thin regions on either side of the bulge obtain correct dark matter density by co-annihilation with top-squark .

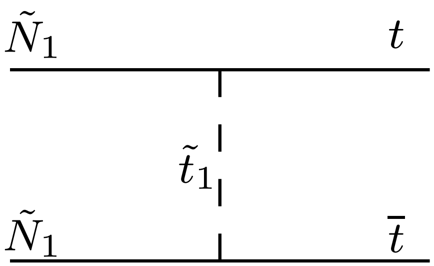
Common GUT-scale scalar mass m_0 for the same models:



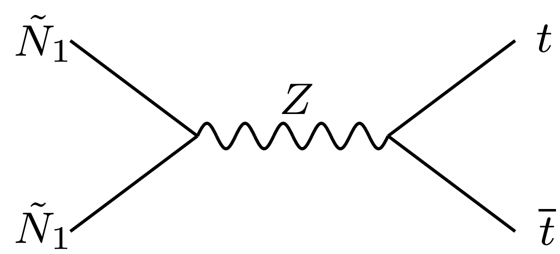
In these models, **all** soft SUSY-breaking mass parameters are less than $\hat{M}_1$, $\hat{M}_2$.

Beating the LEP Higgs constraint (almost) forces $m_{\tilde{N}_1}$ to be larger than m_t.

Why this happens: unlike other SM quark and lepton final states, $t\bar{t}$ does not have p-wave suppression.



(+ u-channel)



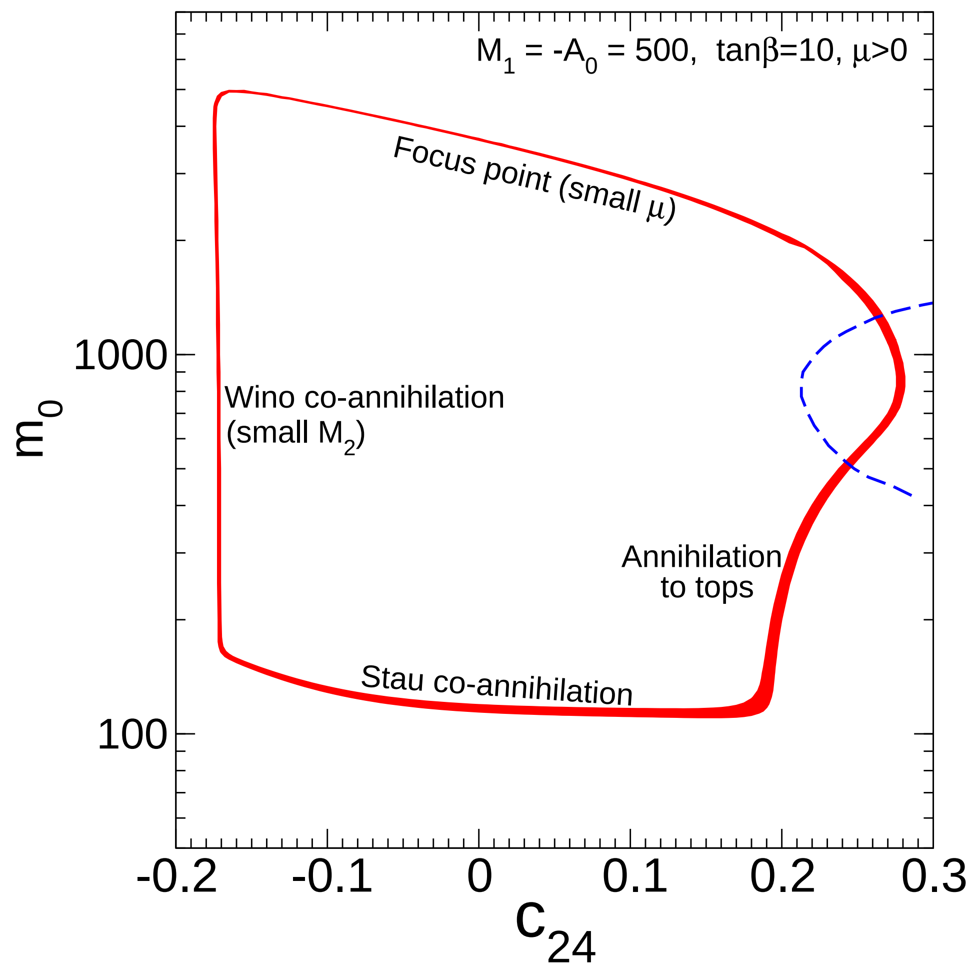
In most of the WMAP allowed region, the Z exchange diagram gives substantial destructive interference. The ratio of contributions to the initial state ${}^{2s+1}L_J = {}^1S_0$ amplitude is:

$$A_Z/A_{\tilde{t}_1} \approx -0.3$$

and other amplitudes are relatively minor.

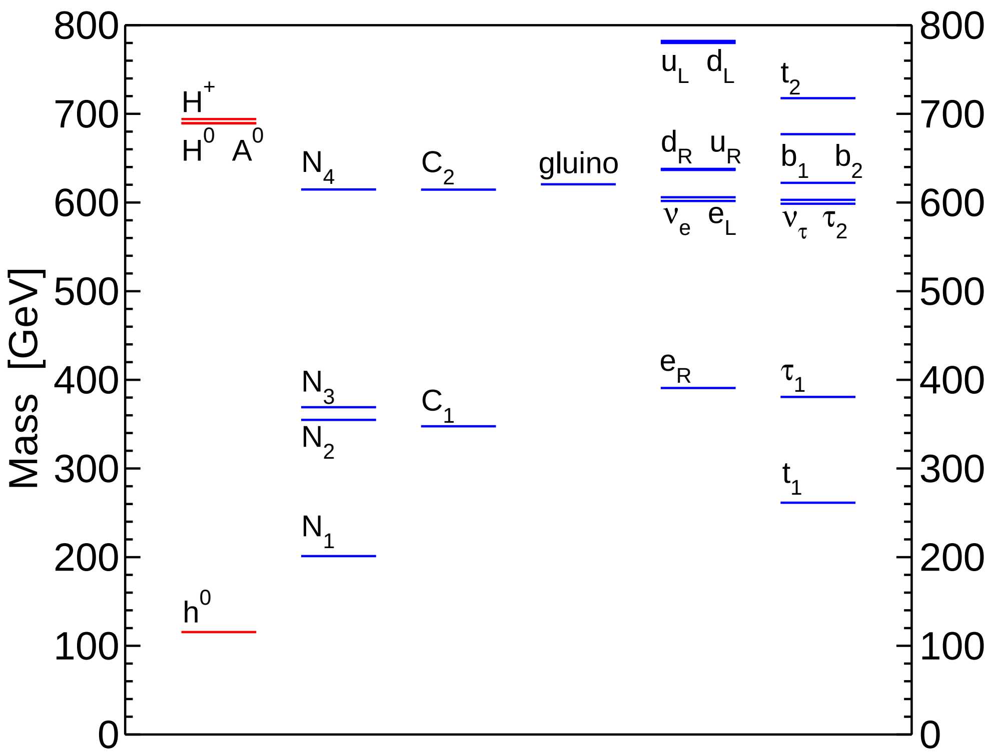
How is this related to other dark-matter allowed regions?

Hold $\hat{M}_1 = 500$ GeV fixed (so that $m_{\tilde{N}_1} \approx 200$ GeV). Then vary the gaugino non-universality parameter C_{24}, and m_0.



Points to the right of the dashed blue line have $M_h < 113$ GeV.

This scenario for SUSY dark matter has distinctive implications for colliders.



Important decays for hadron colliders:

$$\tilde{t}_1 \to c\tilde{N}_1 \qquad (100\%)$$

$$\tilde{g} \to \begin{cases} t\,\tilde{t}_1^* & (\sim 50\%) \\ \bar{t}\,\tilde{t}_1 & (\sim 50\%) \end{cases}$$

$$\tilde{q}_L \to \begin{cases} q\tilde{g} & (\sim 78\%) \\ q'\tilde{C}_2 & (\sim 11\%) \end{cases}$$

$$\tilde{q}_R \to q\tilde{N}_1 \qquad (\sim 90\%)$$

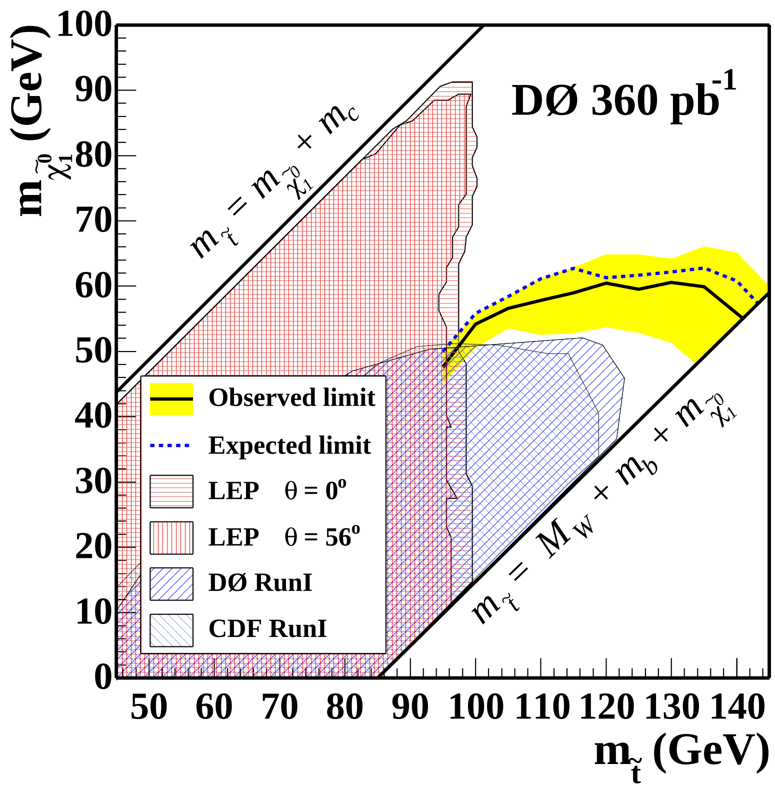
More generally, $\tilde{t}_1$ cannot decay to $t\tilde{N}_1$ in this scenario.

The spectrum is relatively heavy; the compression is upwards to make M_h heavy, so weakly-interacting superpartners are hard to see at hadron colliders.

The target is here!

In this scenario, superpartners are too heavy to give much hope at the Tevatron. One can look for the top squark $\tilde{t}_1$ by:

$$p\overline{p} \to \tilde{t}_1 \tilde{t}_1^* \to c\overline{c}\tilde{N}_1\tilde{N}_1 \to c\,\overline{c} + \not{E}_T$$



Usual Tevatron signals for SUSY, trileptons, like-sign dileptons, jets + $\not{E}_T$, all seem to be very hard or impossible. Not enough events.

LHC discovery signal

If $\tilde{t}_1 \to c\tilde{N}_1$, gluino pair production leads to:

$$pp \to \tilde{g}\tilde{g} \to \begin{cases} t\,\bar{t}\,\tilde{t}_1\,\tilde{t}_1^* \to t\,\bar{t}\,c\,\bar{c} + \not{E}_T & (50\%) \\ t\,t\,\tilde{t}_1^*\,\tilde{t}_1^* \to t\,t\,\bar{c}\,\bar{c} + \not{E}_T & (25\%) \\ \bar{t}\,\bar{t}\,\tilde{t}_1\,\tilde{t}_1 \to \bar{t}\,\bar{t}\,c\,c + \not{E}_T & (25\%) \end{cases}$$

Kraml and Raklev (2005) used like-charge leptonic decay modes for both top quarks. Both discovery and mass measurements are possible up to a gluino mass of about 900 GeV.

Most SUSY events at LHC will go through the gluino. (So add softer, light-flavor, jets from squark decays.)

LHC discovery signals (continued)

On the other hand, if $\tilde{t}_1 \to W b \tilde{N}_1$,

$$pp \to \tilde{g}\tilde{g} \to \begin{cases} t\,\overline{t}\,\tilde{t}_1\,\tilde{t}_1^* \to t\,\overline{t}\,b\,\overline{b}\,\ell^+\ell'^- + \not{E}_T & (50\%) \\ t\,t\,\tilde{t}_1^*\,\tilde{t}_1^* \to t\,t\,\overline{b}\,\overline{b}\,\ell^-\ell'^- + \not{E}_T & (25\%) \\ \overline{t}\,\overline{t}\,\tilde{t}_1\,\tilde{t}_1 \to \overline{t}\,\overline{t}\,b\,b\,\ell^+\ell'^+ + \not{E}_T & (25\%) \end{cases}$$

So 4 potential b tags, plus like sign dileptons, plus large $\not{E}_T$.

This tends to occur if $\hat{A}_t$ is smaller.

An unfortunate feature of Compressed Supersymmetry: sleptons decouple almost perfectly from the LHC.

They are too heavy to be found directly by Drell-Yan or Vector Boson Fusion, and do not appear in the cascade decay chains of squarks or gluinos in significant numbers.

After LHC, we may have discovered the gluino and many squarks, but not be able to say **anything** about the sleptons.

Charginos and neutralinos may be almost as bad.

The heavier, wino-like chargino $\tilde{C}_2$ can appear in the decays of left-handed squarks with $\sim 10\%$ branching fraction.

Then it decays to (in the example model shown earlier):

$$\tilde{C}_2 \to \begin{cases} \tilde{N}_3 W & (\sim 26\%) \\ \tilde{N}_2 W & (\sim 26\%) \\ \tilde{C}_1 Z & (\sim 25\%) \\ \tilde{C}_1 h & (\sim 22\%) \end{cases}$$

$$\tilde{N}_2 \to \begin{cases} \tilde{N}_1 h & (\sim 90\%) \\ \tilde{N}_1 Z & (\sim 10\%) \end{cases}$$

$$\tilde{N}_3 \to \tilde{N}_1 Z \qquad (\sim 97\%)$$

$$\tilde{C}_1 \to \tilde{t}_1 b \qquad (\sim 95\%)$$

Branching fractions are small; final states are varied. Good Luck sorting this out!

A good possibility: Stoponium?

Because the stop is long-lived, it can form bound states.

Consider the production of scalar stoponium at the LHC:

$$pp \longrightarrow \sigma_{\tilde{t}_1 \tilde{t}_1^*}$$

This was studied in a different context in 1993 by Drees and Nojiri, looking at the decays

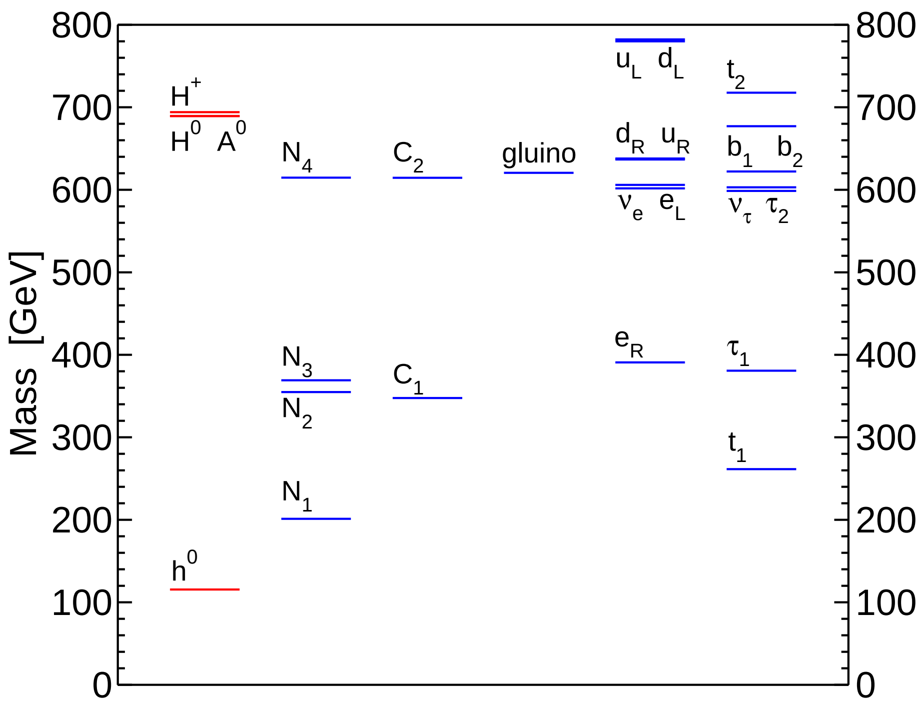
$$\sigma_{\tilde{t}_1 \tilde{t}_1^*} \longrightarrow \gamma\gamma, ZZ, WW.$$

They found that the $\gamma\gamma$ channel is the most promising at the LHC.

I believe this deserves further study. It would give a **precision** measurement of the stop mass, since stoponium is a very narrow resonance with no missing energy in its decays.

In progress. . .

A typical feature of Compressed SUSY with $\tilde{N}_1\tilde{N}_1 \to t\bar{t}$ dark matter: no visible superpartners at a $\sqrt{s} = 500$ GeV Linear Collider!



Only a light Higgs h^0 (nearly indistinguishable from Standard Model) will be directly visible. Also possible, initial state radiation:

$$e^+e^- \to \tilde{N}_1\tilde{N}_1\gamma$$

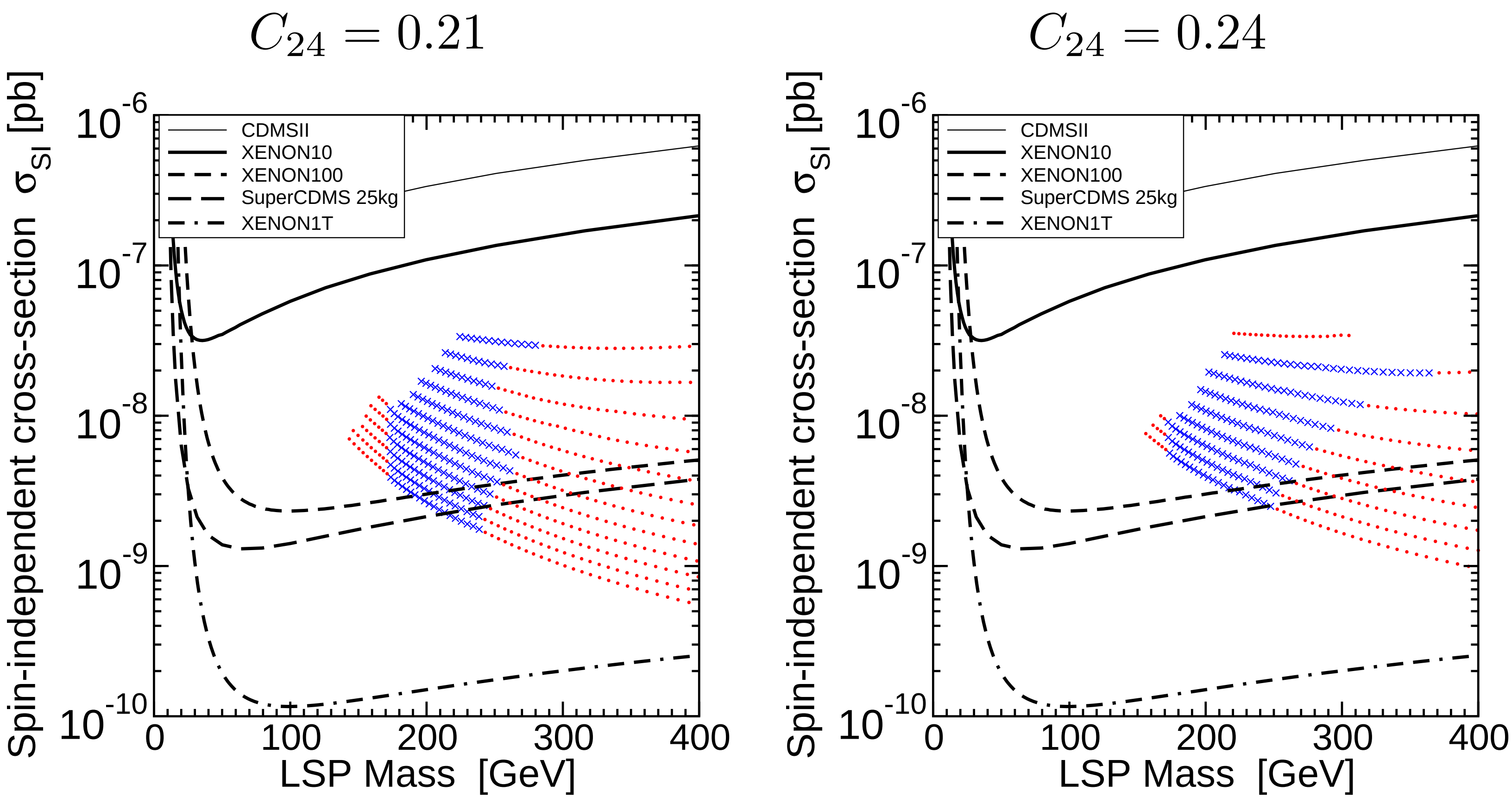
This scenario also leads to robust predictions for the direct detection of dark matter, thanks to the inherent range of LSP masses and mixings.

Neutralinos scatter off of heavy nuclei in low-background laboratory detectors. The standard figure of merit is the LSP-proton spin-independent elastic cross-section:

$$\sigma_{\rm SI}(\tilde{N}_1 p \rightarrow \tilde{N}_1 p)$$

In the following slide, I compare present experimental results (CDMSII, XENON10) and future projected sensitivities (XENON100, SuperCDMS 25kg at SNOLAB, and XENON1T) to the predictions of Compressed Supersymmetry.

(Baer, Box, Park, Tata have done a similar study in hep-ph/0707.0618)



Blue X's = "bulge" annihilation-to-top models,

Red dots = stop co-annihilation models

Present experiments do not constrain the scenario.

Future experiments should provide a definitive test.

Our choicest plans have fallen through,
our airiest castles tumbled over,
because of lines we neatly drew
and later neatly stumbled over.
– Piet Hein

Outlook

- Non-universal gaugino masses at the GUT scale with $\hat{M}_3/\hat{M}_2 \lesssim 1/3$ alleviate the fine-tuning problem of minimal SUSY, leading to a compressed mass spectrum
- A distinctive scenario for dark matter: $\tilde{N}_1\tilde{N}_1 \to t\bar{t}$, due mostly to $\tilde{t}_1$ exchange
- Discovery is impossible at the past LEP2 collider or the Tevatron, but is likely at the imminent LHC. (But sleptons decouple, and Higgsinos and Winos will be tough at best.)
- In this scenario, a much higher beam energy for a future ILC† may be required. We'll know better after the LHC.
- Direct dark matter detection prospects are very promising.

† Illinois Linear Collider