RIKEN PHENIX SPIN FEST @ WAKO

Lectures on Perturbative QCD
or
from basic principles to current applications

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Outline of the lectures

Lecture 1: basic ideas; exploring the QCD final state

Lecture 2: origin of singularities; infrared safety

Lecture 3: QCD initial-state; factorization; renormalization

Lecture 4: more on factorization & renormalization; pdfs

Lecture 5: applications in hadron-hadron collisions; spin

Literature & useful links

Lecture notes & write-ups:

Wu-Ki Tung: *Perturbative QCD and the Parton Structure of the Nucleon* (from www.cteq.org)

!!! Dave Soper: *Basics of QCD Perturbation Theory* (hep-ph/9702203)

J. Collins, D. Soper, G. Sterman: *Factorization of Hard Processes in QCD* (hep-ph/0409313)

CTEQ Collaboration: *Handbook of Perturbative QCD* (Rev. Mod. Phys. 67 (1995) 157 or from www.cteq.org)

Talks & lectures on the web:

!!! annual CTEQ summer schools (tons of material !): *www.cteq.org*

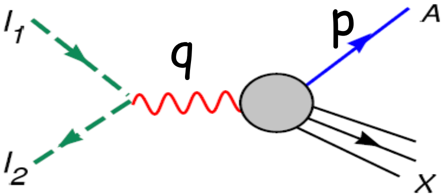
1st summer school on QCD Spin Physics @ BNL: *www.bnl.gov/qcdsp*

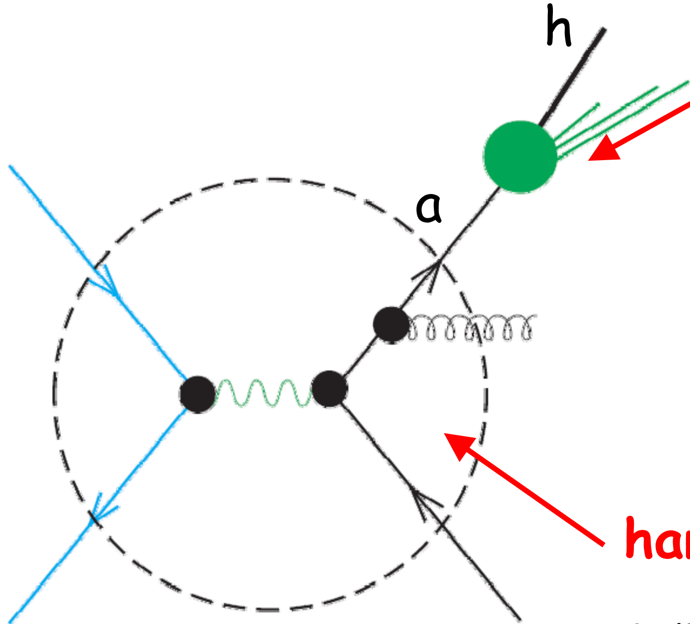
Lecture 3

QCD initial state, partons, DIS
factorization & renormalization

Electron-positron annihilation factorization (cont.)

let's see what factorization does for





fragmentation functions D_a^h

contains all long-distance interactions
hence not calculable but universal

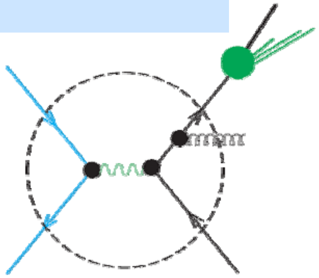
physical interpretation:

probability to find a hadron carrying
a certain momentum of parent parton

hard scattering $\widehat{F}_a$

contains only short-distance physics
amenable to pQCD calculations

Electron-positron annihilation factorization (cont.)



more explicitly we get

$$\frac{d\sigma}{dz\,d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[F_A^T(x,Q)(1+\cos^2\theta) + F_A^L(x,Q)\sin^2\theta\right]$$

where

$$\boxed{F_A^{T,L}(z,Q) = \sum_a \hat{F}_a^{T,L}\left(z,\frac{Q}{\mu_f}\right) \otimes D_a^h(z,\mu_f)}$$

"convolution"

$$f(x) \otimes g(x) \equiv \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$

factorization scale (arbitrary!)

characterizes the boundary between short and long-distance physics we postpone a closer look at the factorization scale dependence until we have introduced partons also in the initial state to cover

electron-hadron and

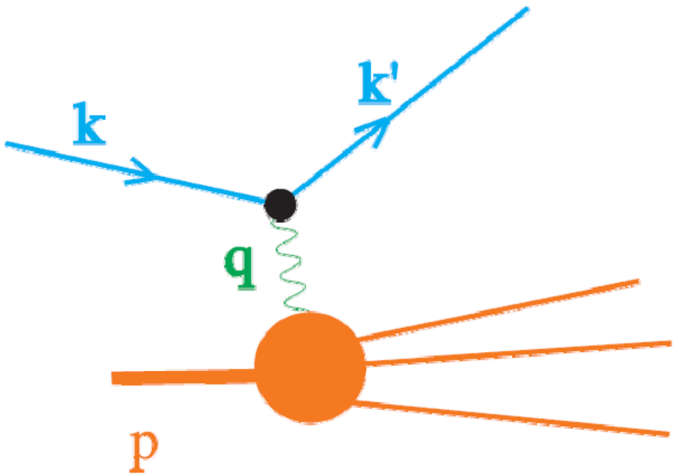
hadron-hadron interactions

the fact that experimental results do *not* depend on μ_f will then lead us to the famous

DGLAP evolution equations

Deep-inelastic scattering (DIS) kinematics

let us now consider the process $\boxed{l(k)\,h(p) \to l'(k') + X}$



relevant kinematics:

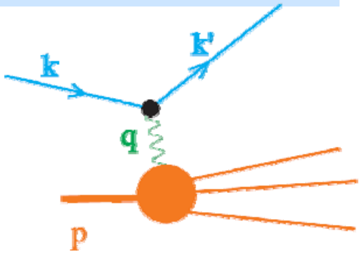
- momentum transfer $q^\mu = k^\mu - k'^\mu$
  $Q^2 = -q^2$
- "Bjorken"-x $\boxed{x = \dfrac{Q^2}{2p\cdot q}}$
- invariant mass
  $$W^2 = (p+q)^2 = m_h^2 + \frac{1-x}{x}Q^2$$
- fractional energy transfer
  $$y = \frac{p\cdot q}{p\cdot k} \overset{\text{lab}}{=} \frac{E-E'}{E}$$

"deep-inelastic": $Q^2 \gg 1\ \text{GeV}^2$

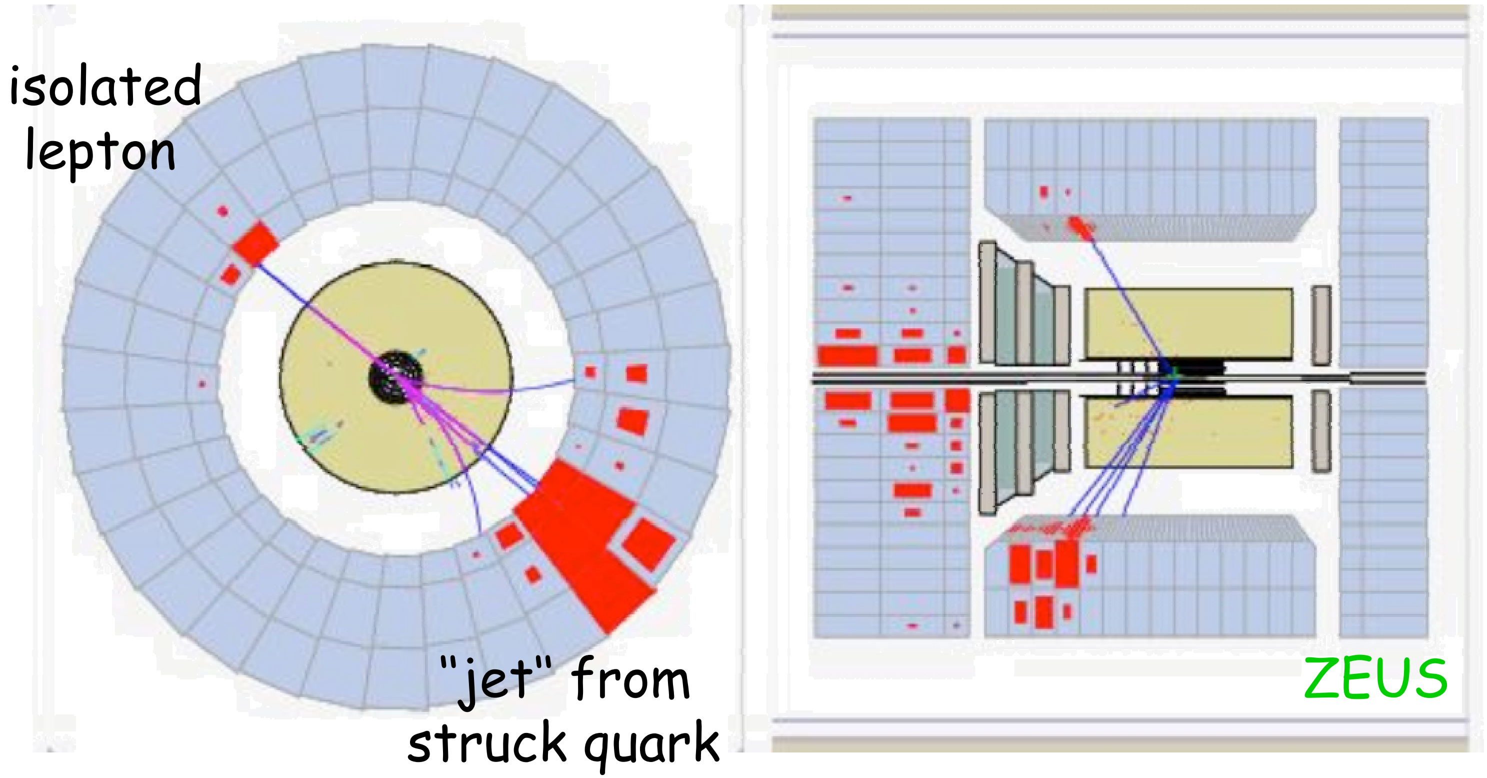
"scaling limit": $Q^2 \to \infty$, x fix

Deep-inelastic scattering (DIS) a neutral current event

here is how DIS looks like in "your" detector:



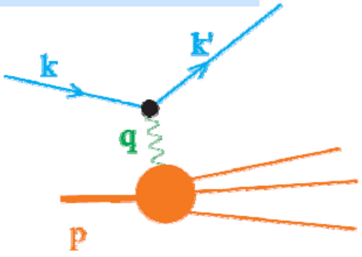
a neutral current event with photon-exchange

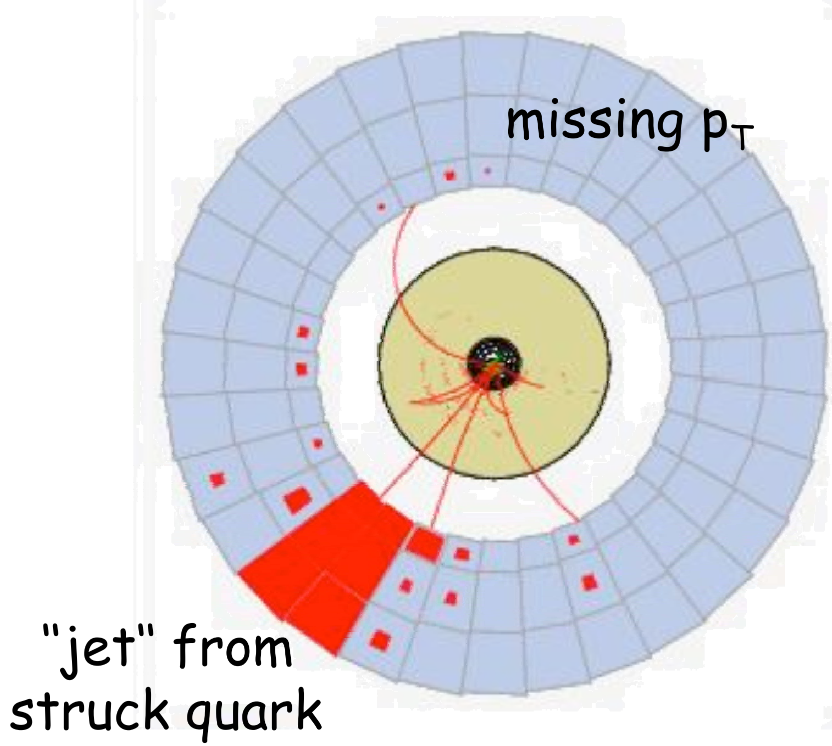


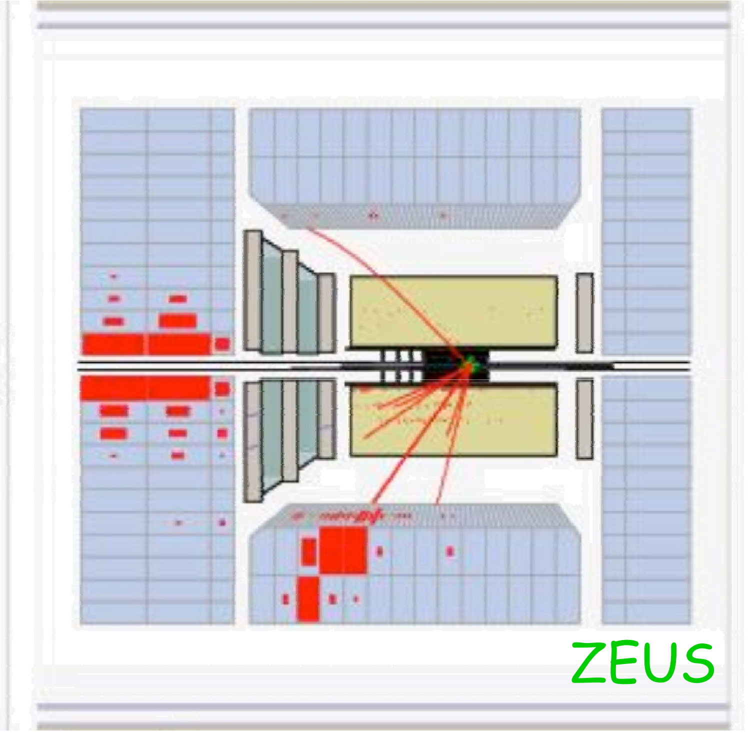
Deep-inelastic scattering (DIS) a charged current event

a charged current event with W-boson-exchange

(the electron turns into a neutrino which is "invisible")



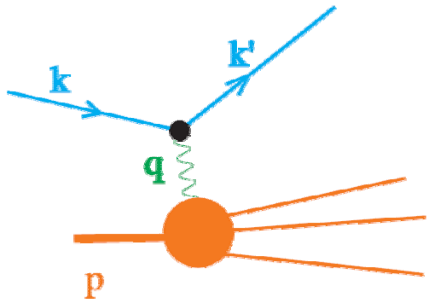




Deep-inelastic scattering (DIS) towards the parton model

first analysis of DIS does not require any knowledge about QCD

electroweak theory tells us how
the virtual vector boson couples:
(let's assume only photon exchange)



$$d\sigma = \frac{4\alpha^2}{s}\frac{d^3\vec{k}'}{2|\vec{k}'|}\frac{1}{Q^4}L^{\mu\nu}(k,q)W_{\mu\nu}(p,q)$$

phase space
scat. lepton

photon
propagator²

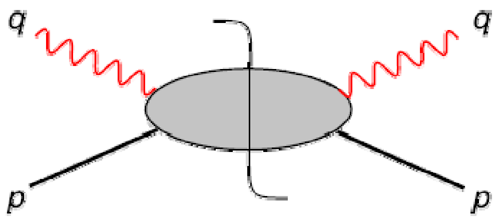
leptonic
tensor

hadronic tensor
contains information
about hadronic structure

(can be easily generalized to W/Z-boson exchange)

Deep-inelastic scattering (DIS) towards the parton model (cont.)

to all orders in the strong interaction $W_{\mu\nu}$ is given by the square of $\gamma^*(q)\, h(p) \rightarrow X$



symmetries (parity, Lorentz), hermiticity & current conservation tell us that $W^{\nu\mu} = W^{\mu\nu *}$ $q_\mu W^{\mu\nu} = 0$

$$W_{\mu\nu}(p,q) = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right)\frac{1}{p\cdot q} F_2(x,Q^2)$$

structure functions

can be easily combined with the "trivial" leptonic tensor (just QED)

$$L_{\mu\nu} = 2(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k\cdot k')$$

Deep-inelastic scattering (DIS) towards the parton model (cont.)

➡ **DIS cross section:**

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[[1+(1-y)^2]F_1(x,Q^2) + \frac{(1-y)}{x}[F_2(x,Q^2) - 2xF_1(x,Q^2)]\right]$$

different y-dep. can differentiate between F_1 and F_2-$2xF_1$

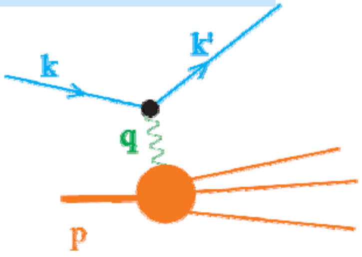
before we turn on the full glory of QCD dynamics
let's explore **DIS in the naive quark-parton model**:

Feynman; Bjorken, Paschos

Bjorken scaling limit: Q^2, $\nu = p \cdot q \to \infty$ with x fixed

- F_1, F_2 obey scaling law, i.e., they are indep. of Q^2
- virtual photon scatters off pointlike constituents
- DIS is like taking a snapshot of the hadron

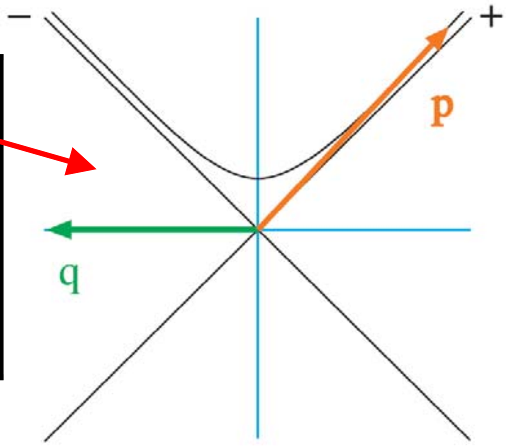
Deep-inelastic scattering (DIS) space-time structure



this can be best understood in a reference frame where the proton moves very fast and $Q \gg m_h$ is big

(recall light-cone kinematics from Lecture 1)

4-vector	hadron rest frame	Breit frame
$(p^+, p^-, \vec{p}_T)$	$\frac{1}{\sqrt{2}}(m_h, m_h, \vec{0})$	$\frac{1}{\sqrt{2}}(\frac{Q}{x}, \frac{x m_h^2}{Q}, \vec{0})$
$(q^+, q^-, \vec{q}_T)$	$\frac{1}{\sqrt{2}}(-m_h x, \frac{Q^2}{m_h x}, \vec{0})$	$\frac{1}{\sqrt{2}}(-Q, Q, \vec{0})$



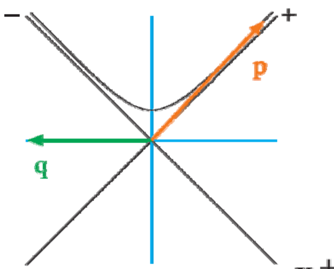
Lorentz boost

in general $(a^+, a^-, \vec{a}_T) \rightarrow (e^{\omega} a^+, e^{-\omega} a^-, \vec{a}_T) = (a'^+, a'^-, \vec{a}')$

here: $e^{\omega} = Q/(x m_h)$

Deep-inelastic scattering (DIS) space-time structure (cont.)

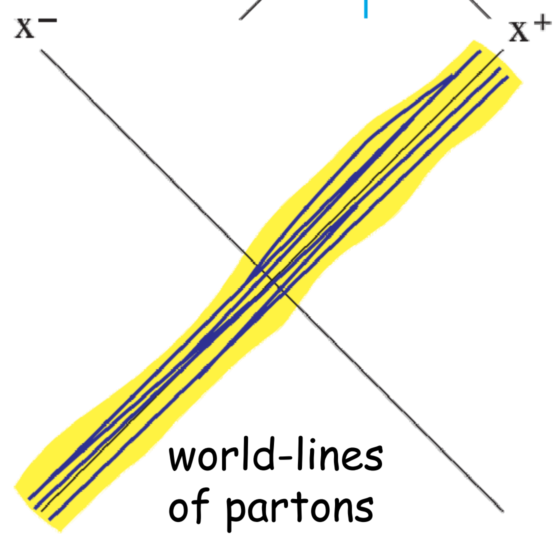
simple estimate for typical time-scale of interactions among the partons inside a fast-moving hadron:



rest frame: $\Delta x^{+} \sim \Delta x^{-} \sim \frac{1}{m}$

Breit frame: $\Delta x^{+} \sim \frac{1}{m}\frac{Q}{m} = \frac{Q}{m^2}$ large

$\Delta x^{-} \sim \frac{1}{m}\frac{m}{Q} = \frac{1}{Q}$ small

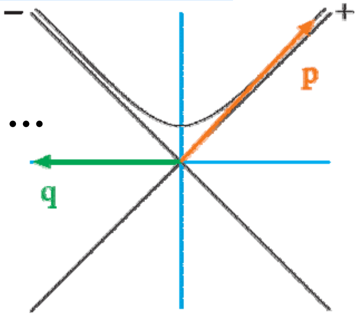


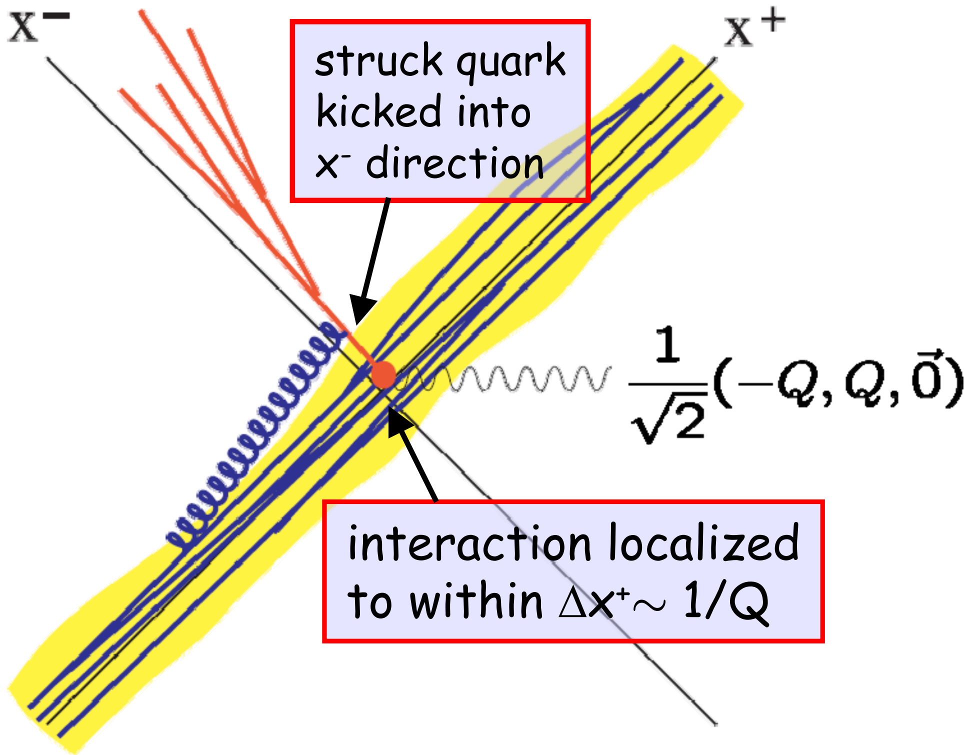
interactions between partons are spread out inside a fast moving hadron

How does this compare with the time-scale of the hard scattering?

Deep-inelastic scattering (DIS) space-time structure (cont.)

now let the virtual photon meet our fast moving hadron ...





upshot:

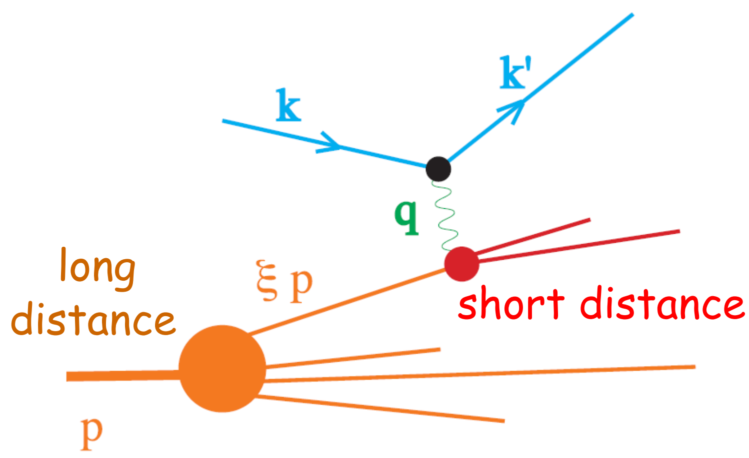
- partons are free during the hard interaction
- hadron effectively consists of partons that have momenta $(p_i^+, p_i^-, \vec{p}_i)$
- convenient to introduce momentum fractions
  $0 < \xi_i \equiv p_i^+/p^+ < 1$

Deep-inelastic scattering (DIS) according to the naive parton model

the space-time picture suggests the possibility of separating short and long-distance physics (=**factorization**!)

turned into the language of Feynman diagrams DIS looks like

$$\boxed{\frac{d^2\sigma}{dx\,dQ^2} \sim \int_0^1 d\xi \sum_a f_a^h(\xi) \frac{1}{\xi} \frac{d^2\hat{\sigma}}{d\hat{x}\,dQ^2}\Big|_{\hat{x}=x/\xi} + \mathcal{O}\left(\frac{m}{Q}\right)}$$



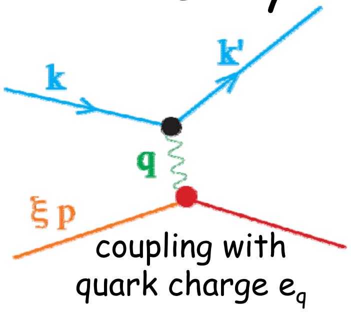
- $f_a^h(\xi)d\xi$ probability to find a parton with flavor a in a hadron h carrying light-cone momentum ξp^+
- $d^2\hat{\sigma}/d\hat{x}dQ^2$ cross section for electron-parton scattering

Deep-inelastic scattering (DIS) naive parton model (cont.)

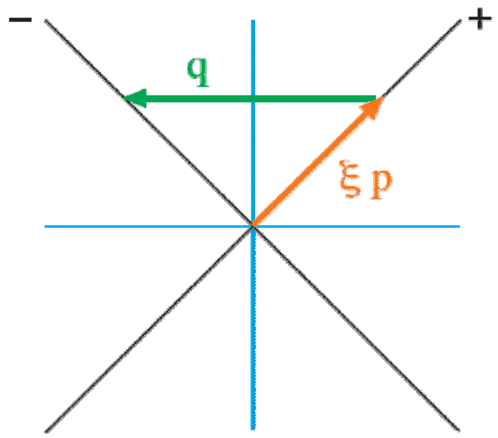
let's see how the **scaling property** comes about:

a quick computation of the hard scattering cross section at LO yields:

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}[1+(1-y)^2]$$



the scattered quark is on-mass shell:



$\xi\, p^+ + q^+ = 0$ implies that **ξ=x** at LO

$$\boxed{\frac{d^2\hat{\sigma}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\frac{1}{2}[1+(1-y)^2]e_q^2\delta(x-\xi)}$$

Deep-inelastic scattering (DIS) naive parton model (cont.)

compare with the definition of the structure functions!

find:

$$F_2(x) = 2xF_1(x) = \sum_{a=q,\bar{q}} \int_0^1 d\xi f_a(\xi) x e_a^2 \delta(x-\xi) = \sum_{a=q,\bar{q}} e_a^2 x f_a(x)$$

- the desired scaling property: no dependence on Q^2
- $F_L(x) \equiv F_2(x) - 2x\, F_1(x)$ vanishes! (Callan-Gross relation)

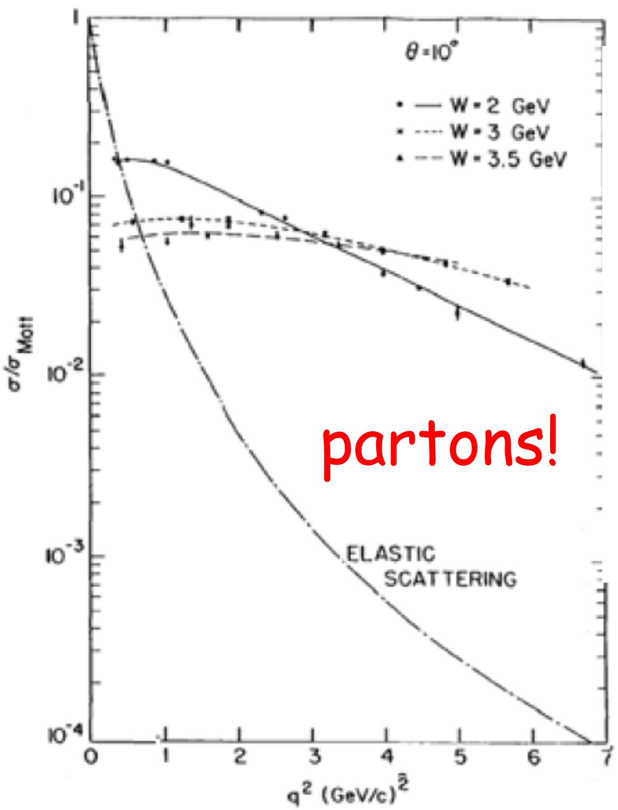
  (test that quarks are spin-1/2: they cannot absorb a long. pol. γ^*)

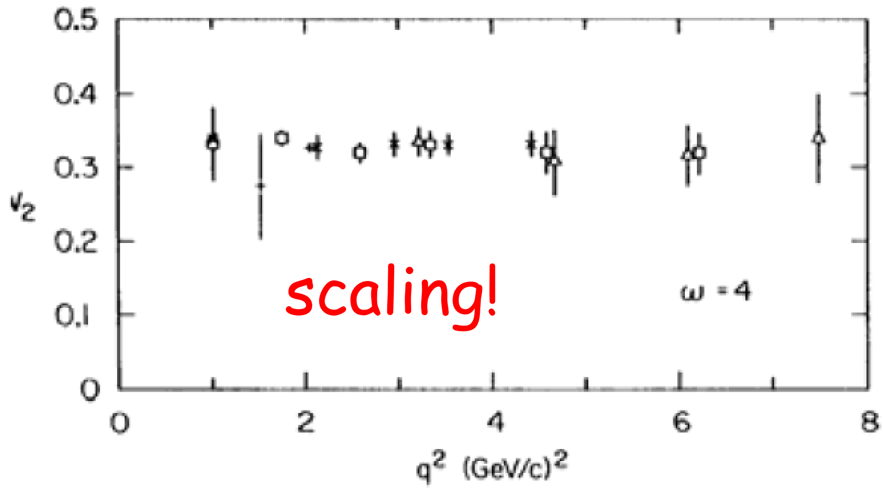
How does this compare with experiment?

Deep-inelastic scattering (DIS) SLAC-MIT experiment of 1969

1990

two unexpected results:

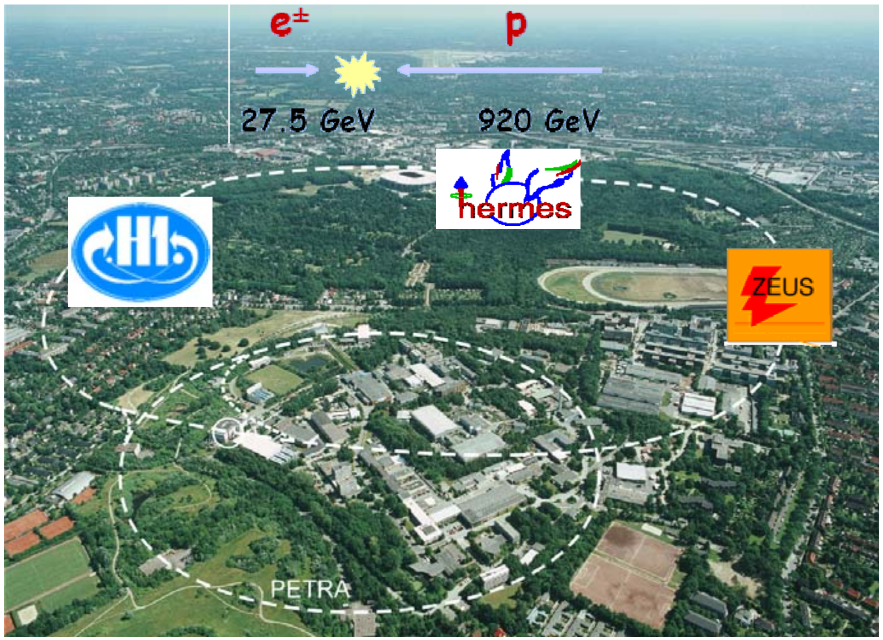




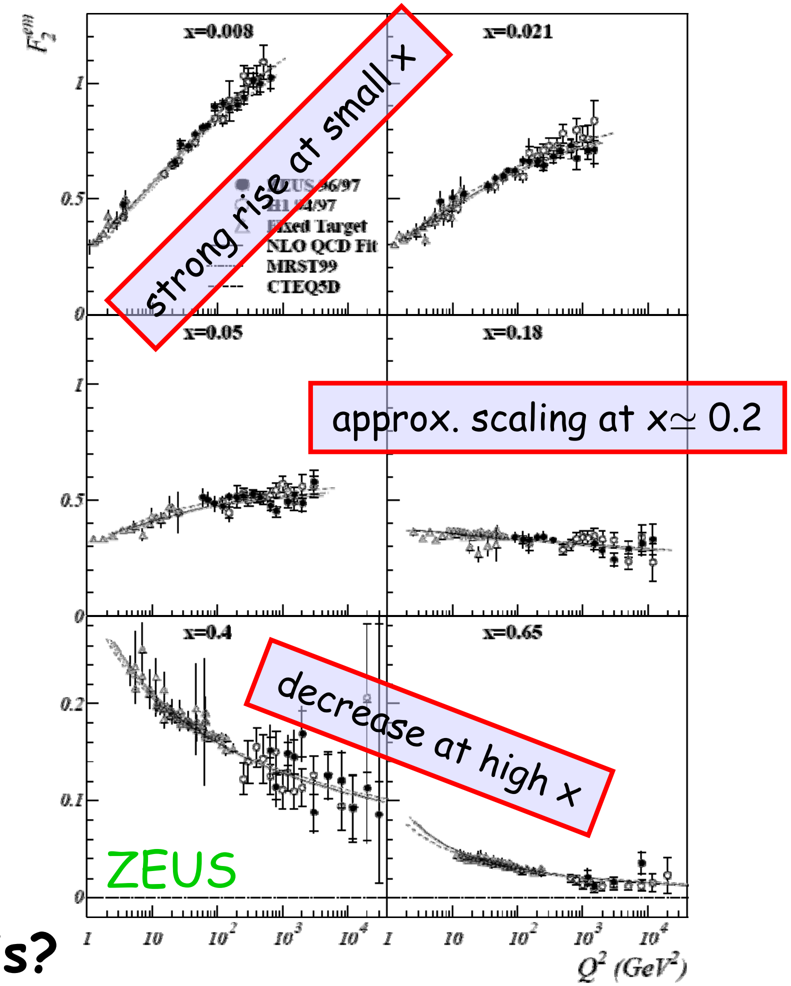
birth of the pre-QCD parton model

Deep-inelastic scattering (DIS)
HERA: scaling violations

the first (and only) ep-collider:



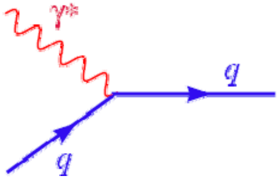
observe strong scaling violations



What does pQCD has to say about this?

Deep-inelastic scattering (DIS) according to pQCD

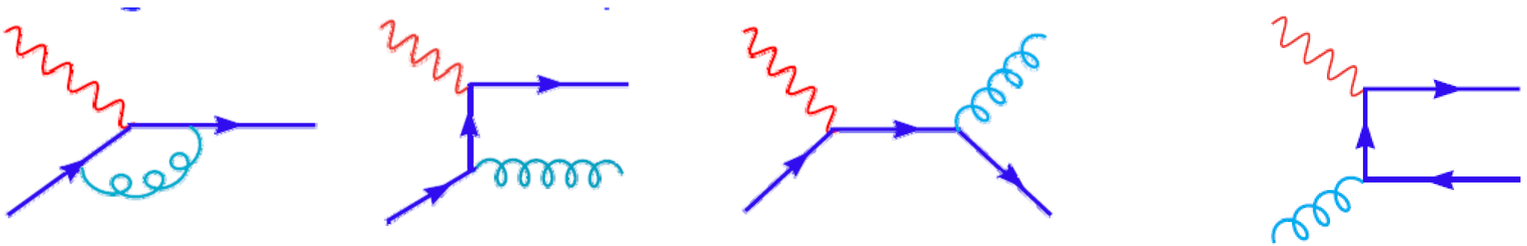
we got a long way (parton model) *without* invoking QCD



now we have to study QCD dynamics in DIS

– this leads to similar problems already encountered in e^+e^-

let's try to compute the QCD corrections to the parton model picture



α_s corrections to the LO process | photon-gluon fusion

our experience so far: have to expect divergencies!

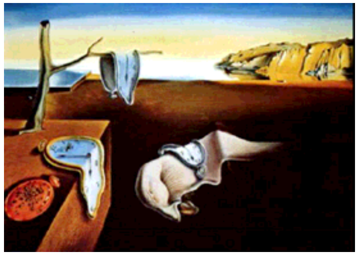
we cannot calculate with infinities ➡ introduce some regulator
remove it in the end

Deep-inelastic scattering (DIS) according to pQCD (cont.)

possible regulators:

- **small quark/gluon masses**
  intuitive and transparent
  but works only in NLO
- **dimensional regularization**
  = change dimension of space-time to $4-2\varepsilon$
  calculations more involved; works in general

let's choose this one



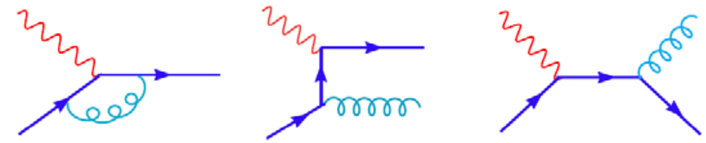
depending on your choice singularities will be hidden as

large logarithms, e.g., $\log(m^2/Q^2)$ or as $1/\varepsilon$

only if we have done everything consistently, including factorization, we can safely remove the regulator and can compare to experiment

Deep-inelastic scattering (DIS) according to pQCD (cont.)

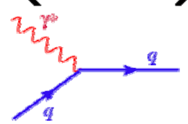
the general structure of the α_s corrections looks like this:



$$\left.\frac{d^2\hat{\sigma}}{dx\,dQ^2}\right|_{F_2} \equiv \hat{F}_2^q$$

LO

$$= e_q^2 x \left[\delta(1-x) + \frac{\alpha_s(\mu_r)}{4\pi}\left[P_{qq}(x)\ln\frac{Q^2}{m_g^2} + C_2^q(x)\right]\right]$$

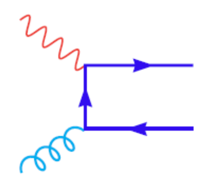


large logarithms (collinear emission)

finite coefficients

$$\left.\frac{d^2\hat{\sigma}}{dx\,dQ^2}\right|_{F_2} \equiv \hat{F}_2^g$$

$$= \sum_q e_q^2 x \left[0 + \frac{\alpha_s(\mu_r)}{4\pi}\left[P_{qg}(x)\ln\frac{Q^2}{m_q^2} + C_2^g(x)\right]\right]$$



the structure of the results

large logarithms + finite coefficients

already hints towards factorization ...

fasten your seatbelts and prepare for the "magic" of factorization

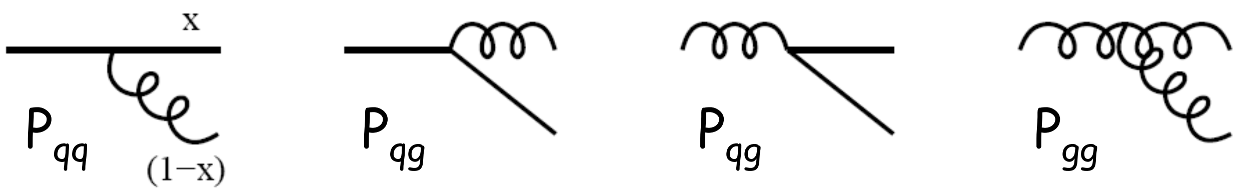
Deep-inelastic scattering (DIS) according to pQCD (cont.)

first it is important to notice that

- large logarithms (or $1/\varepsilon$) incorporate *all* long-distance physics (collinear emission)
- the coefficients $P_{ij}(x)$ multiplying the log's are *universal* and calculable (splitting functions)

the physical meaning of the splitting functions is easy:

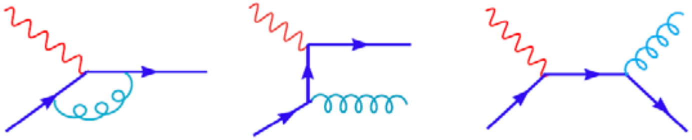
$P_{ij}(x)$: probability that a parton j splits collinearly into a parton i (and something) carrying a momentum fraction x



Deep-inelastic scattering (DIS) according to pQCD

to obtain the physical cross section we have convolute our partonic results with the parton densities (like in the parton model!):

e.g. for



and similar for the gluon part

$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_q^2 \Big[f_{a,0}(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \\ f_{a,0}(x) \Big[P_{qq}\left(\frac{x}{\xi}\right) \ln \frac{Q^2}{m_g^2} + C_2^q\left(\frac{x}{\xi}\right) \Big]\Big]$$

now ... here comes the "trick":

the $f_{a,0}(x)$ are unmeasurable bare (=infinite) densities and need to be re-defined (=renormalized) to make them physical

Deep-inelastic scattering (DIS) according to pQCD

the **renormalized quark densities** (at order α_s) are given by:

$$f_q(x,\mu_f^2) \equiv f_{q,0}(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi) P_{qq}\left(\frac{x}{\xi}\right) \ln\left(\frac{\mu_f^2}{m_g^2}\right) + z_{qq}$$

absorb *all* long-distance singularities at a factorization scale μ_f into $f_{q,0}$

physical densities: not calculable in pQCD but universal

This is our final result! Let's analyze it piece by piece!

insert back and keep only terms up to α_s:

$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(\mu_r)}{2\pi}\left[P_{qq}\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]\right]$$

Deep-inelastic scattering (DIS) according to pQCD

the physical structure fct. is independent of μ_f

(this will lead to the concept of renormalization group eqs.)

both, pdf's and the short-dist. coefficient depend on μ_f

(choice of μ_f: shifting terms between long- and short-distance parts)

$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(\mu_r)}{2\pi}\underbrace{\left[P_{qq}\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]}_{\text{short-distance "Wilson coefficient"}}\right]$$

yet another scale: μ_r
due to the renormalization
of ultraviolet divergencies

choice of the factorization scheme

that was a lot of material and perhaps hard to swallow

let us postpone questions like

- What the hell does renormalization?
- Pdfs are universal, so what is their formal definition?
- What should I do with all these arbitrary scales?
- What is a factorization scheme?

until tomorrow!