Gauge Theories & Particle Physics

Physics 539 | Spring Semester 1997 | Chris Quigg

May 1, 1997

Recommended Reading:

Gauge Theories, Chapter 9.
Aitchison and Hey, §15.1–15.2.
K. Nakamura, "Note on Nucleon Decay," in 1994 Review of Particle Properties, *Phys. Rev. D***50**, 1173 (1994), pp. 1673–1675.

Cultural Reading:

P. Langacker, "Proton Decay" (electronic archive: hep–ph/9210238).

Deep Background:

G. G. Ross, *Grand Unified Theories* (Addison-Wesley, Reading, Massachusetts, 1984).
W. McKay and J. Patera, *Tables of Dimensions, Indices, and Branching Rules for Representations of Simple Algebras* (Dekker, New York, 1981).

Problems (due May 6, 1997)

43. Let us investigate the coupling-constant unification predicted by the $SU(5)$ unified theory.

(a) Using your favorite values of the fine structure constant and the strong coupling constant at the Z^0 mass [for example, $1/\alpha(M_Z^2) = 129.89 \pm 0.09$ from H. Burkhardt and B. Pietrzyk, *Phys. Lett.* **B356**, 398 (1995) or S. Eidelman and F. Jegerlehner, *Z. Phys. C***67**, 1585 (1995), and $\alpha_s(M_Z^2) = 0.118 \pm 0.003$ from the 1996 Review of Particle Physics], calculate the unification energy u and the value $1/\alpha_u$ of the $SU(5)$ coupling constant at the unification scale.

(b) Using the value of the unification scale you found in (a), compute the weak mixing parameter $x_W \equiv \sin^2\theta_W$ at the Z^0 mass and compare with the value that you found in Problem 28 using the LEP data.

(c) Now use the connection between the measured value of x_W, α_2, and α to determine $1/\alpha_2(M_Z^2)$. Compute $1/\alpha_Y(M_Z^2)$ and, from it, $1/\alpha_1(M_Z^2)$. Evolve $1/\alpha_1$, $1/\alpha_2$, and $1/\alpha_3$ to high energies and test whether they meet at a single unification point.

44. Consider a unified theory based on the gauge group $SO(10)$.

(a) By referring to the $SU(5) \otimes U(1)$ decomposition of the representations of $SO(10)$, show that each fermion generation can be accommodated in an irreducible **16**-dimensional representation, which also has a place for a left-handed antineutrino.

(b) Show that the adjoint **45** representation contains the gauge bosons of the $SU(5)$ theory.

(c) Now examine the branching of $SO(10)$ into $SU(4) \otimes SU(2) \otimes SU(2)$, and the subsequent branching of $SU(4)$ into $SU(3) \otimes U(1)$. Use the $SU(3) \otimes SU(2) \otimes SU(2)$ decomposition of the fermion representation to show that $SO(10)$ contains the left-right-symmetric electroweak group as a subgroup.

(d) Give the transformation properties of the **45** gauge bosons under $SU(3) \otimes SU(2) \otimes SU(2)$, and identify the $SU(5)$ gauge bosons among them. [Reference: H. Fritzsch and P. Minkowski, *Ann. Phys. (NY)* **93**, 193 (1975).]

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Course information, including class schedules and assignments, is available on the World Wide Web. Point your browser to http://lutece.fnal.gov/Princeton/.