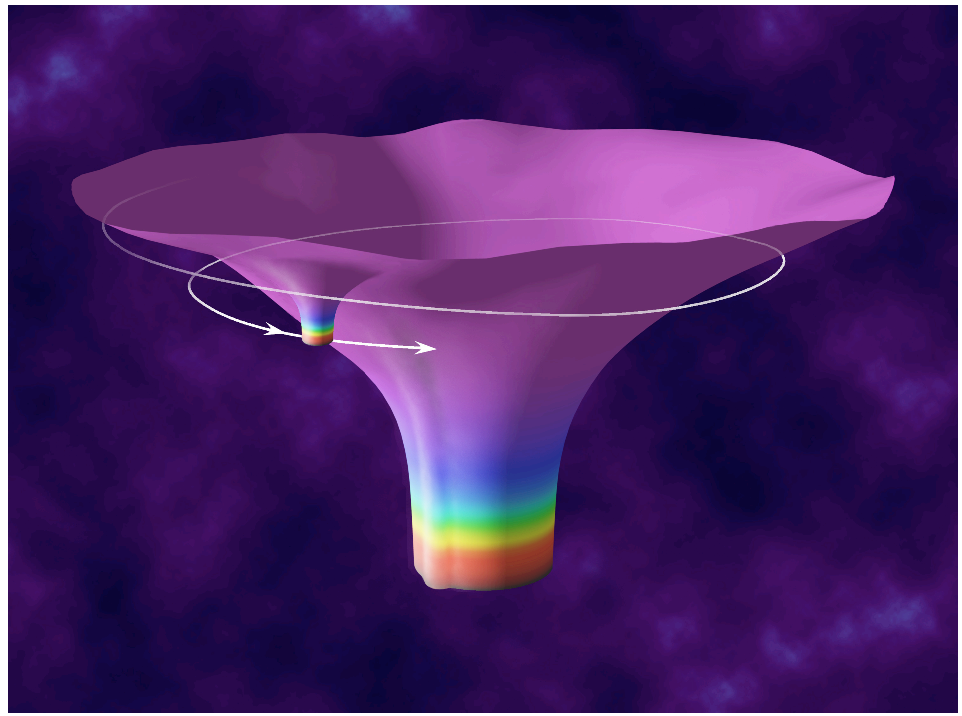
EMRI source modelling and data analysis

¬EMRI waveforms
- --snapshots
- --w/ radiation reaction
- --kludged waveforms

¬EMRI searches
- -- t-f tracks
- -- semi-coherent

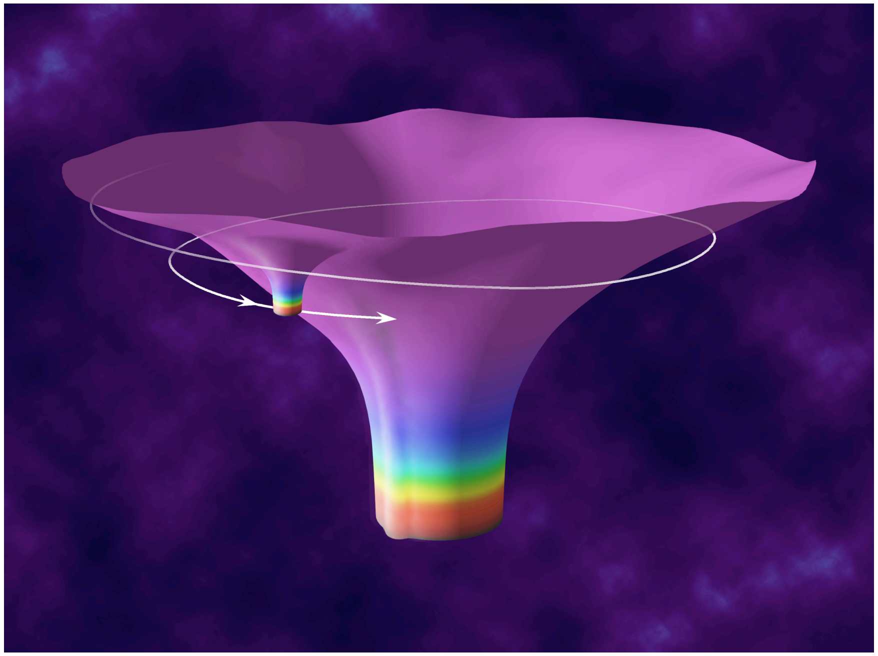
¬EMRIs and confusion noise



Calculating EMRI waveforms

¬ Matched filtering required to dig EMRIs out of the noisy data, but full numerical relativity NOT required to produce the waveforms: one can do perturbation theory in the mass ratio $m/M \sim 10^{-5}$, with small body treated as point particle.

¬ Basically, the CO travels nearly on a geodesic, but radiation reaction causes a slow inspiral. The radiation reaction force diverges at the point particle, and so must be regularized.



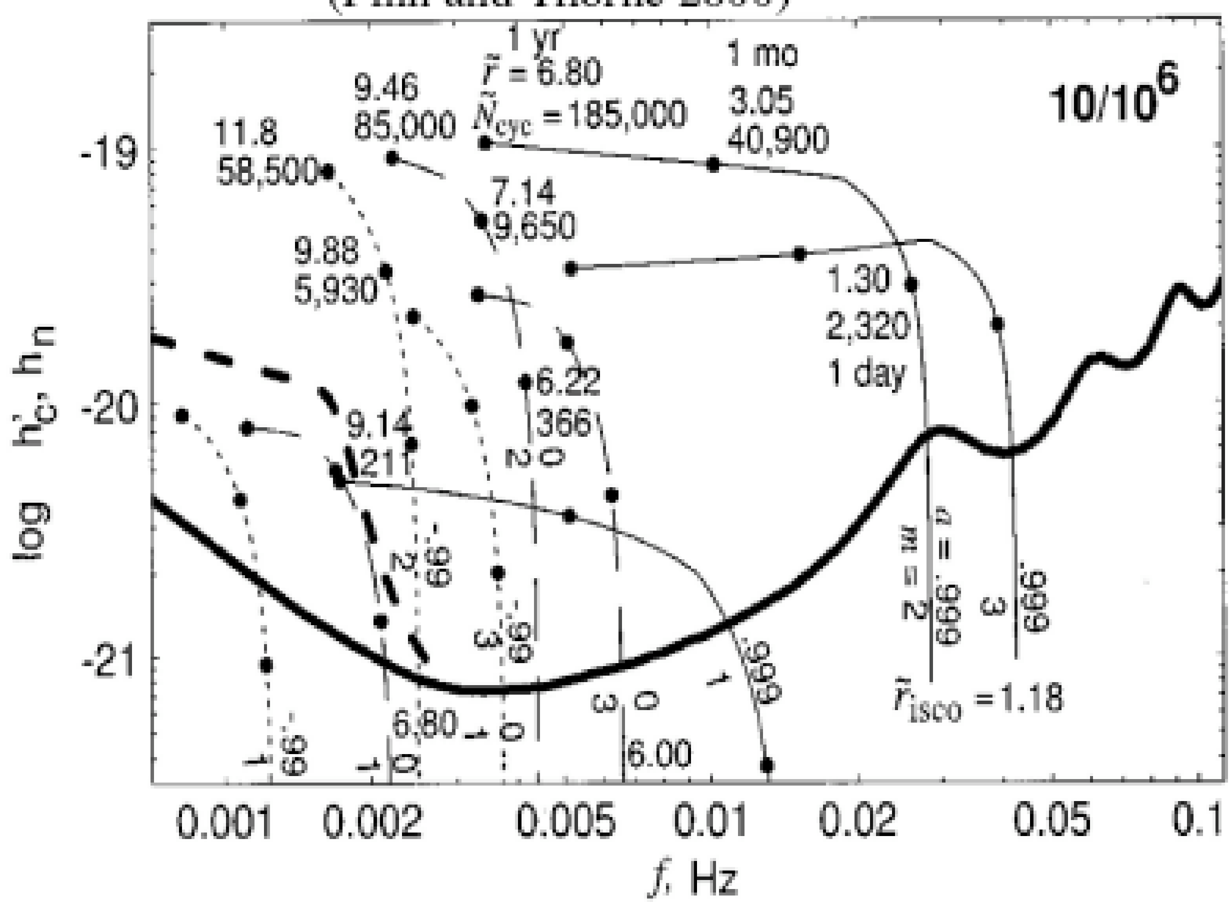
A prescription for doing the regularization was given by Wald&Quinn ('97) and Mino, Sasaki, &Tanaka ('97), but developing a practical numerical implementation remains an active area of research. An approximate, adiabatic approach was developed by Mino('03).

How EMRI SNR builds up

$10 M_{\odot}, 10^6 M_{\odot}$

Different BH spins, but circular orbits and $\dot{S}_{BH} \parallel \dot{L}_{orb}$

(Finn and Thorne 2000)

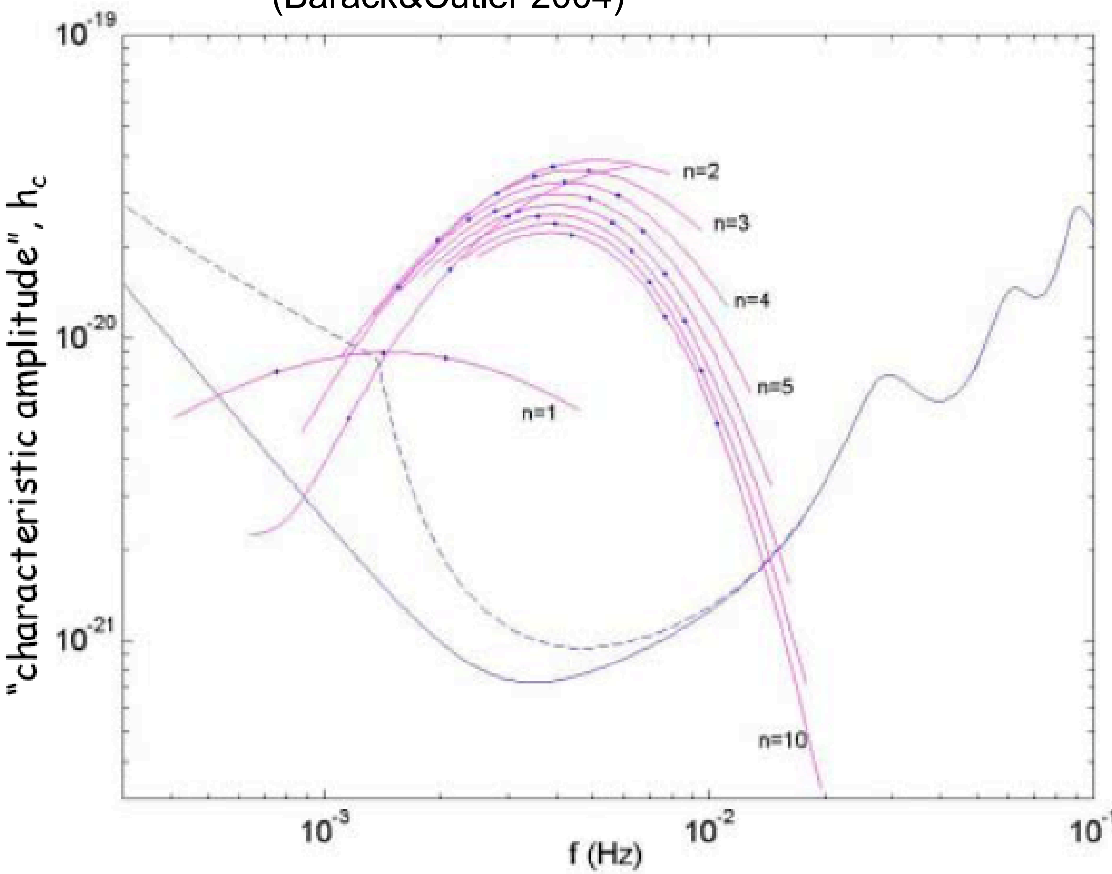


How EMRI SNR builds up

$10 M_\odot, 10^6 M_\odot$

Eccentric orbit, but BH spin = 0

(Barack&Cutler 2004)

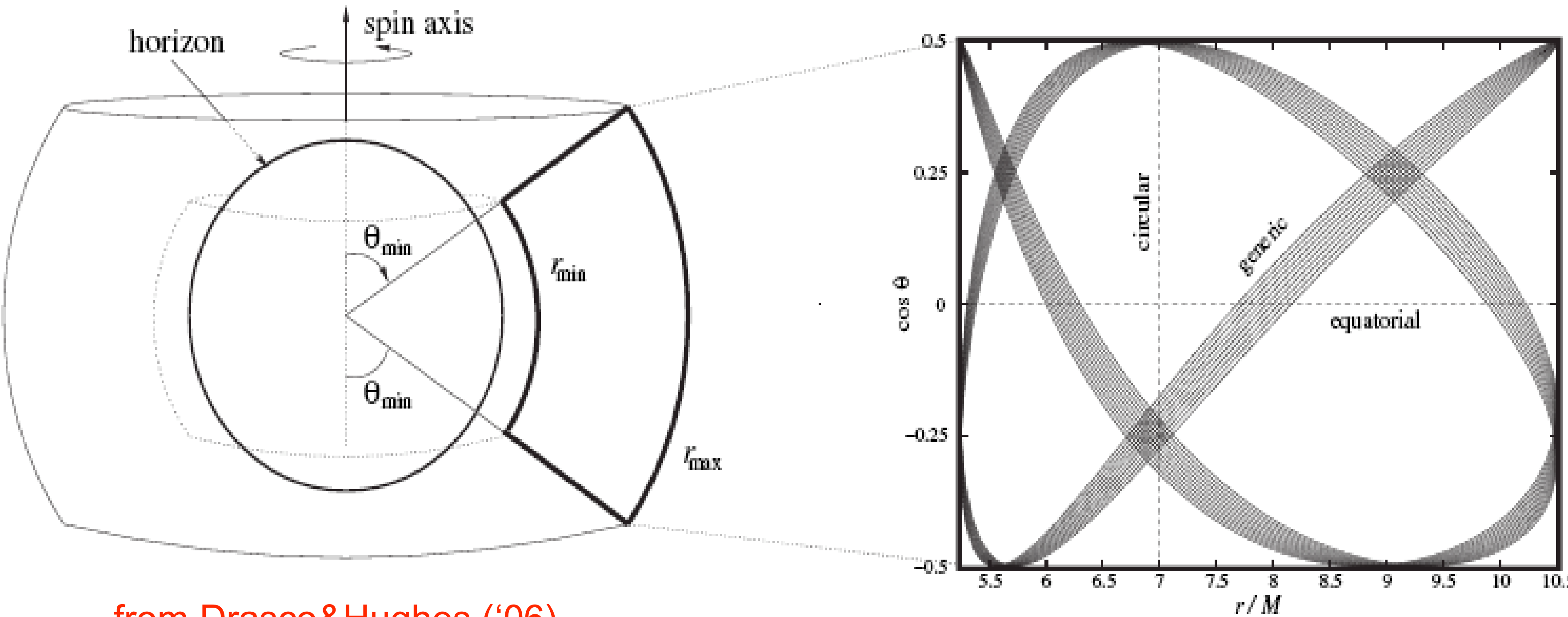


e(plunge)=0.3

e(plunge-10yr)=0.77

- Curves represent 10 yrs of source evolution
- Dots indicate (from left to right) state of system 5, 2, and 1 years before plunge.

geodesic orbits in Kerr



--from Drasco&Hughes ('06)

There are 3 basic frequencies: f_ϕ , f_θ , f_r

Gravitational waves measured at infinity have a discrete spectrum made up of harmonics of just these 3 frequencies:

$$f_{mkn} = m f_\phi + k f_\theta + n f_r$$

with m,k,n integers.

Geodesics in Kerr

In Boyer-Lindquist coords:

$$\rho^4\left(\frac{dr}{d\tau}\right)^2 = \left[E(r^2+a^2)-aL_z\right]^2-\Delta\left[r^2+(L_z-aE)^2+Q\right]\equiv R(r)\ ,$$

$$\rho^4\left(\frac{d\theta}{d\tau}\right)^2 = Q-\cot^2\theta L_z^2-a^2\cos^2\theta(1-E^2)\equiv\Theta(\theta)\ ,$$

$$\rho^2\left(\frac{d\phi}{d\tau}\right) = \csc^2\theta L_z+aE\left(\frac{r^2+a^2}{\Delta}-1\right)-\frac{a^2L_z}{\Delta}\equiv\Phi(r,\theta)\ ,$$

$$\rho^2\left(\frac{dt}{d\tau}\right) = E\left[\frac{(r^2+a^2)^2}{\Delta}-a^2\sin^2\theta\right]+aL_z\left(1-\frac{r^2+a^2}{\Delta}\right)\equiv T(r,\theta)\ .$$

where $\rho^2=r^2+a^2\cos^2\theta$ and $\Delta=r^2-2Mr+a^2$.

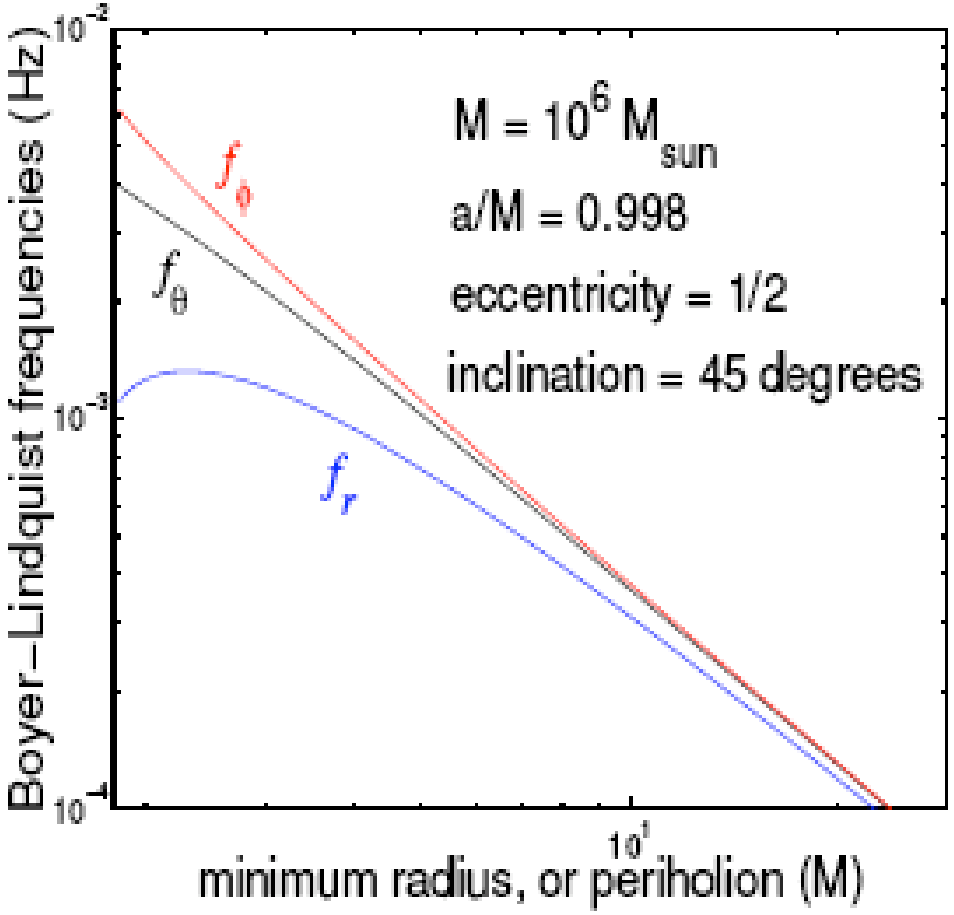
and E, Lz, Q are the energy, ang. momentum, and Carter's const. Can transform to "Mino time" λ using $d\tau/d\lambda=\rho^{-2}$. Then r,θ are both periodic functions of λ, and

$$\frac{dt}{d\lambda}\equiv T(r,\theta) = \sum_{kn}T_{kn}e^{-i(k\Upsilon_\theta+n\Upsilon_r)\lambda}\ ,$$

$$\frac{d\phi}{d\lambda}\equiv\Phi(r,\theta) = \sum_{kn}\Phi_{kn}e^{-i(k\Upsilon_\theta+n\Upsilon_r)\lambda}\ ,$$

--see Drasco&Hughes, astro-ph/0308479, for more details

The 3 basic frequencies vs perihelion distance



--from E.E. Flanagan

Snapshot waveforms

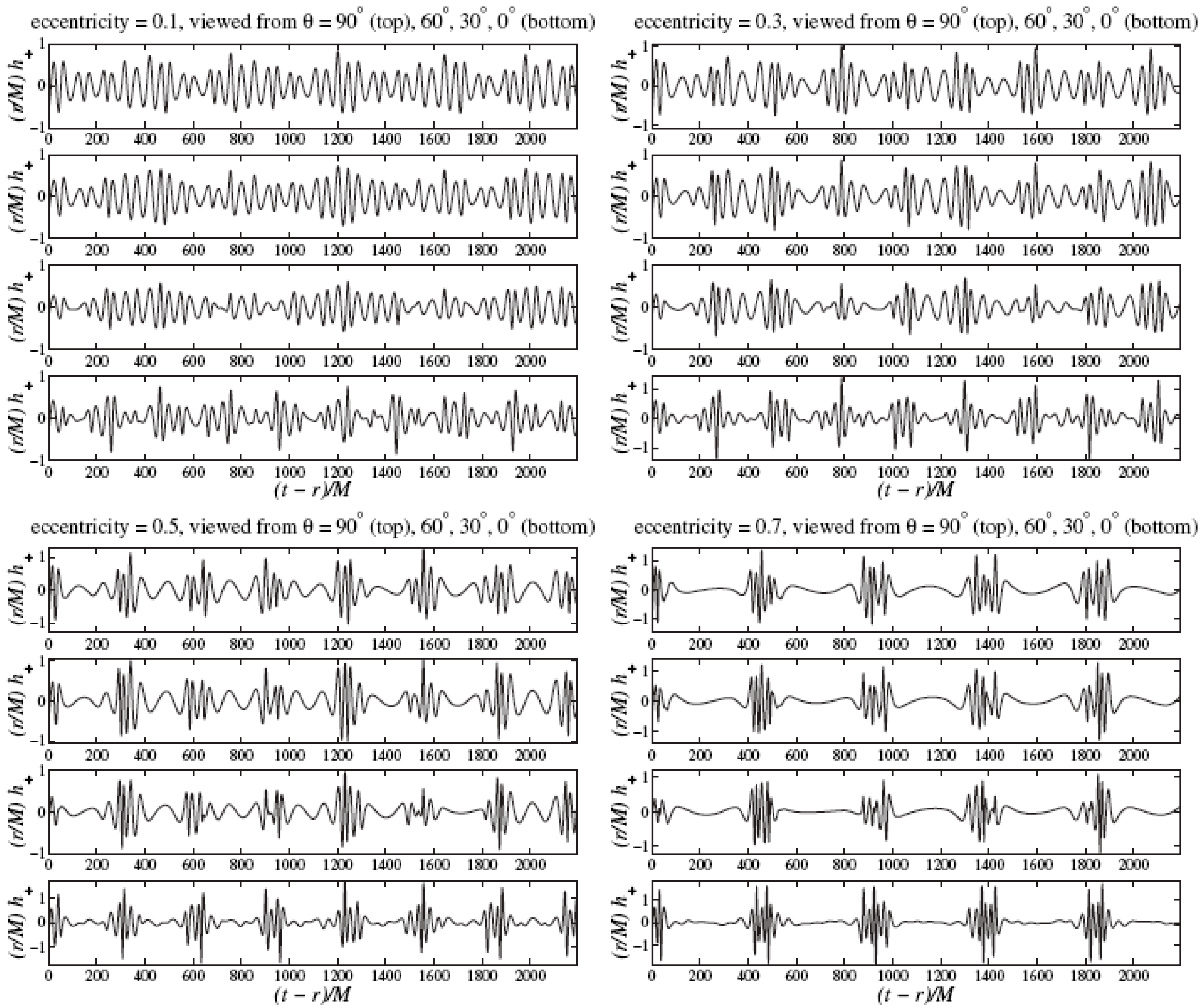
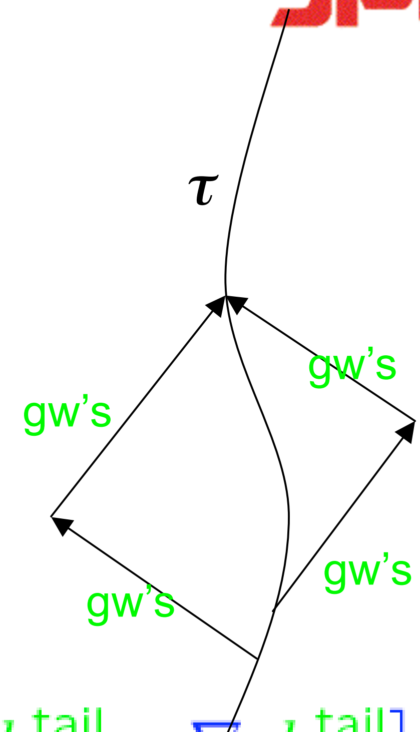


FIG. 6. Snapshot waveforms for orbits with inclination $\theta_{\rm inc} = 80°$, semilatus rectum $p = 6$, and eccentricities $e = 0.1, 0.3, 0.5, 0.7$. The magnitude of the black hole's spin angular momentum is $aM = 0.9M^2$.

--from Drasco&Hughes ('06)

Gravitational self-force

For pt particle or small BH traveling on (near) geodesic, gravitational self-force is entirely due to back-scattered radiation, or “tail” terms.



General result due to Mino,Sasaki &Tanaka(‘97) and Wald&Quinn(‘97) is:

$$F^{\alpha}(\tau) = \frac{1}{2}\mu\,(g^{\alpha\beta} + u^{\alpha}u^{\beta})\,u^{\lambda}u^{\sigma}\left[\nabla_{\beta}h^{\text{tail}}_{\lambda\sigma} - \nabla_{\lambda}h^{\text{tail}}_{\beta\sigma} - \nabla_{\sigma}h^{\text{tail}}_{\beta\lambda}\right]$$

$$h^{\text{tail}}_{\alpha\beta} = \mu \lim_{\varepsilon\to 0}\int_{-\infty}^{\tau(x)-\varepsilon} d\tau' G^{\text{ret}}_{\alpha\beta,\alpha'\beta'}[\vec{x},\vec{z}(\tau)]u^{\alpha'}(\tau')u^{\beta'}(\tau') = h^{\text{ret}}_{\alpha\beta} - h^{\text{sing}}_{\alpha\beta}$$

A great deal of effort has gone into using this equation to calculate the self-force, but still no satisfactory implementation for Kerr. (Successfully done by Barack&Lousto(‘05) for circular orbits in Schwarzschild.)

Adiabatic Eqs. of Motion

Inspiraling trajectory osculates through a series of geodesics, with slowly time varying E, Lz, Q:

$$\rho^4\left(\frac{dr}{d\tau}\right)^2 = \left[E(r^2+a^2)-aL_z\right]^2-\Delta\left[r^2+(L_z-aE)^2+Q\right]\equiv R(r)\,,$$
$$\rho^4\left(\frac{d\theta}{d\tau}\right)^2 = Q-\cot^2\theta L_z^2-a^2\cos^2\theta(1-E^2)\equiv\Theta(\theta)\,,$$
$$\rho^2\left(\frac{d\phi}{d\tau}\right) = \csc^2\theta L_z+aE\left(\frac{r^2+a^2}{\Delta}-1\right)-\frac{a^2L_z}{\Delta}\equiv\Phi(r,\theta)\,,$$
$$\rho^2\left(\frac{dt}{d\tau}\right) = E\left[\frac{(r^2+a^2)^2}{\Delta}-a^2\sin^2\theta\right]+aL_z\left(1-\frac{r^2+a^2}{\Delta}\right)\equiv T(r,\theta)\,.$$

$$\left\langle\frac{dE}{dt}\right\rangle=\sum_{l=2}^{\infty}\sum_{m=-l}^{l}\sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{1}{4\pi\omega_{mkn}^2}\left(\left|Z_{lmkn}^{\mathrm{H}}\right|^2+\alpha_{lmkn}\left|Z_{lmkn}^{\infty}\right|^2\right)\,,$$

$$\left\langle\frac{dL_z}{dt}\right\rangle=\ldots$$

$$\left\langle\frac{dQ}{dt}\right\rangle=\ldots$$

Key observation by Mino: can solve for the average value $\left\langle\frac{dQ}{dt}\right\rangle$, etc. using half-retarded minus half-advanced solution for metric perturbation $h_{\alpha\beta}$, which is regular on particle's wordline.

Controversy re adiabatic radiation reaction

But……

in principle self-force can have a piece that does not show up in $\left\langle \frac{dE}{dt} \right\rangle$, $\left\langle \frac{dL_z}{dt} \right\rangle$, or $\left\langle \frac{dQ}{dt} \right\rangle$

e.g., for a circular orbit in Schwarzschild, consider a radial force of constant magnitude. Obviously it has no effect on the particle's angular momentum or energy.

Pound, Poisson & Nickel (2005) claim that this "conservative piece" of the self-force can have important secular effects (based on a toy model where they calculate the effect of E&M radiation reaction on a charged particle on a nearly Newtonian orbit, in weak-field gravity). This claim is still controversial.

Eventual need for 2nd-order perturbation theory

Expected phase error from 1st-order perturbation theory is:

$$\sim \Phi_{tot}(m/M) \sim 10^{6}(10^{-5}) \sim 10 \text{ radians}$$

Therefore to extract all the information from the signal, one probably has to go beyond 1st-order perturbation theory.

“Kludge” waveforms

Lacking very accurate waveforms, different “kludge” waveforms have been developed
1) to help scope out data analysis strategies,
2) for preliminary analyses of parameter estimation accuracy,
3) to perhaps serve in initial stages of actual searches, since they’re relatively cheap to calculate.
E.g.:

Analytic “kludge” waveforms (Barack and Cutler):

Description: At any instant, binary described as an eccentric, Keplerian orbit emitting a (lowest-order) quadrupolar waveform—given analytically by Peters and Matthews (1963). However the orbital parameters evolve according to post-Newtonian equations of motion. Perihelion precession, Lense-Thirring precession, and orbital decay are all included. Easy to calculate, so are being used in Mock LISA Data Challenges.

“Kludge” waveforms

Numerical “kludge:” waveforms (Gair et al.):

Description: At any instant, CO follows actual geodesic of Kerr metric. E, L_z, Q evolve using post-Newtonian equations. Waveform calculated from quadrupole formula. Fairly straightforward to calculate and quite accurate on short timescales:

Comparsion of numerical kludge and Teukolsky “snapshot” waveforms:

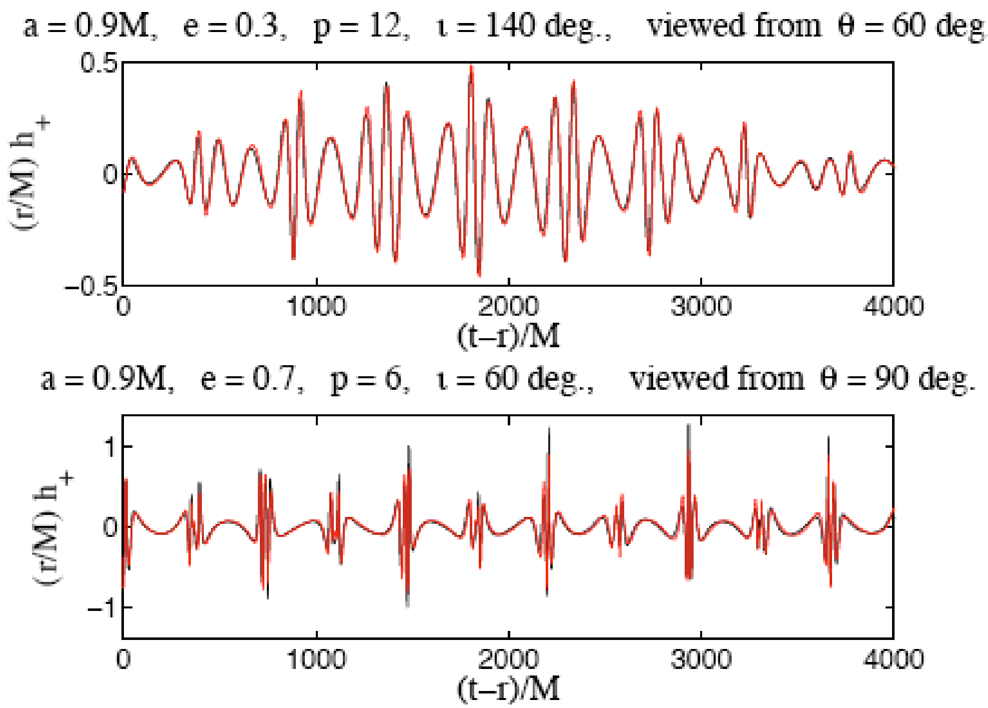
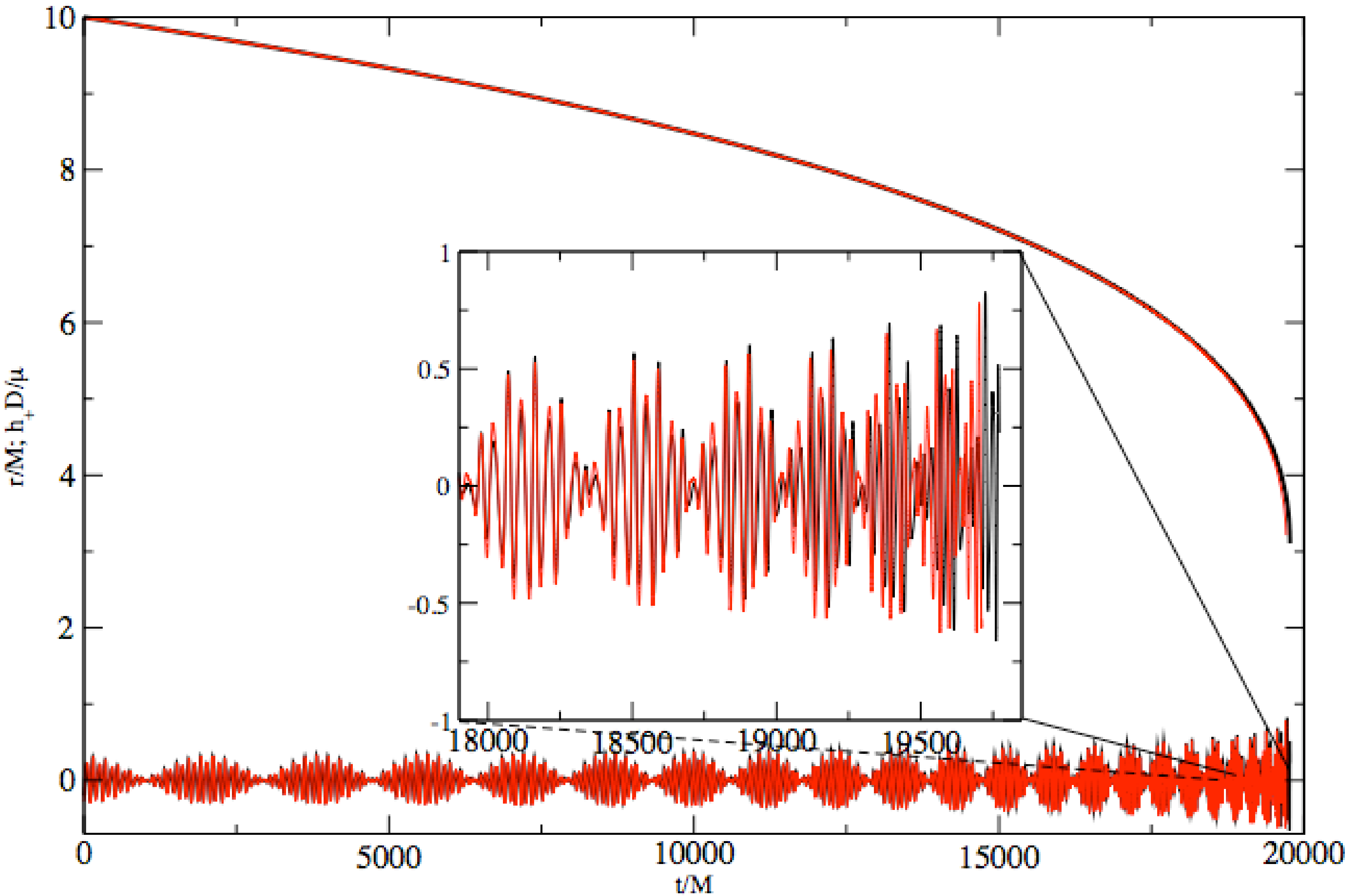


Figure 2. Comparison of Teukolsky [8] (black) and numeric kludge [41] (red) waveform snapshots for two different generic black hole orbits. The parameters of the black hole and the orbits are shown in the titles above the plots. This level

Comparison of numerical kludge w/ Teukolsky waveform for circular, non-equatorial inspiral in Kerr

(thanks to S. Babak, H. Fang, J. Gair, K. Glampedakis, & S. Hughes)



SNR Threshold for Detection

Let ρ = total matched-filtering SNR for both A and E channels.

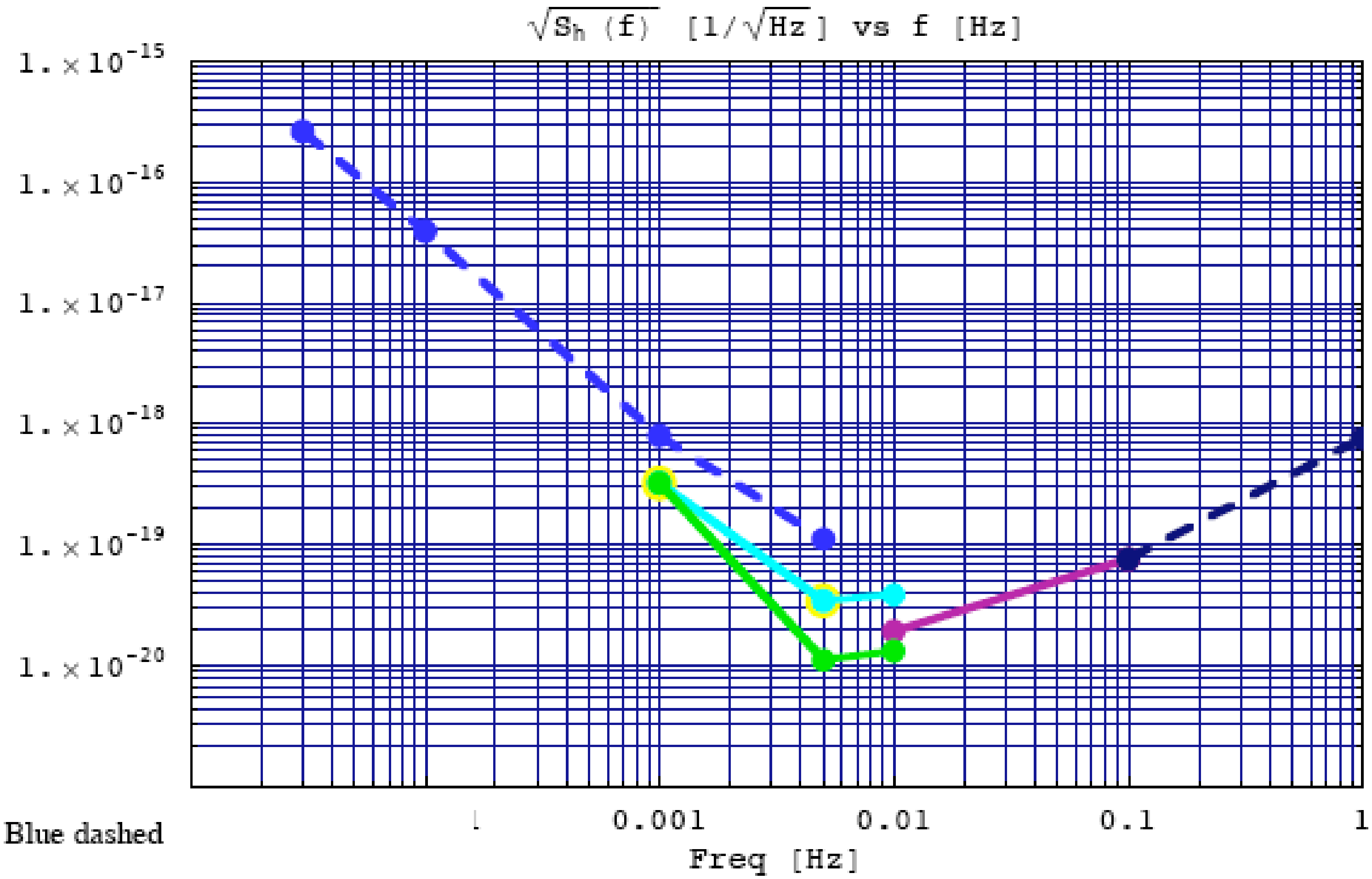
¬There are (very roughly) $N_t \sim 10^{35-40}$ independent, ~year-long templates. Significant detection requires $N_t e^{-\rho^2/2} << 1$, or $\rho \geq 14$

¬Gair & Wen have developed a search method based on looking for excess power in a rectangular region in the time-frequency plane. Requires $\rho \geq 60$ for detection. Not yet fully optimized.

¬Barack, Creighton, Cutler & Gair have developed a semi-coherent search method based on doing fully coherent searches for ~3-week segments (the most one can afford) and then adding the powers from different segments. Estimated to require $\rho \geq 30-35$ for detection. Also not yet optimized (could be made hierarchical).

¬Markov Chain Monte Carlo, Genetic Algorithms not yet investigated, but they will be.

EMRIs set minimum of LISA noise curve



SMBH - Blue dashed

Galactic Binaries - Thick Light Blue

Verification Binaries - Yellow Large Points

EMRI - Thick Green

IMBH - Magenta Point

High Frequency - Dark Blue dashed

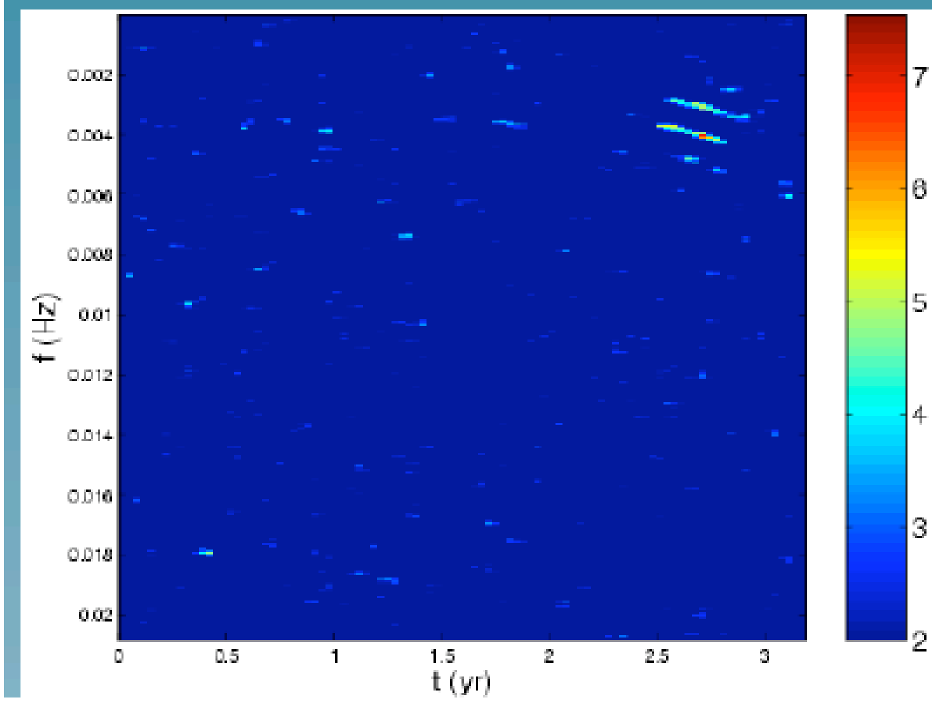
Search for excess power in t-f plane

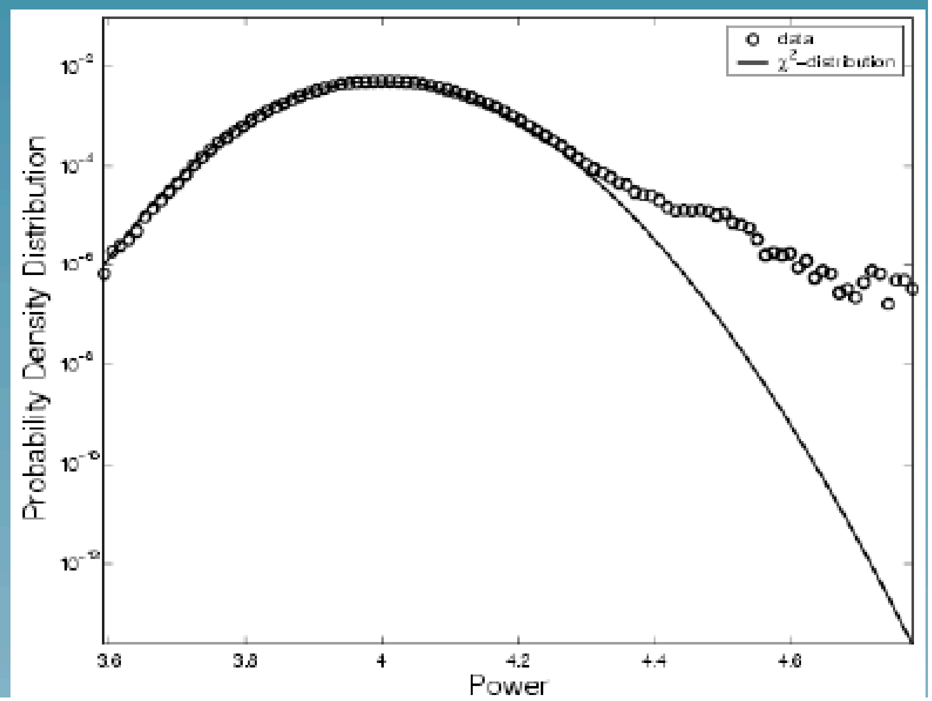
See talk by Wen, Fri @10:30.

Results - source at d = 1.4 Gpc

Last 3 yrs of inspiral for case 10 + 10^6; SNR = 85

- Expect 5-40 events with $d < 1.4$ Gpc over three years. Detected with reasonable confidence.



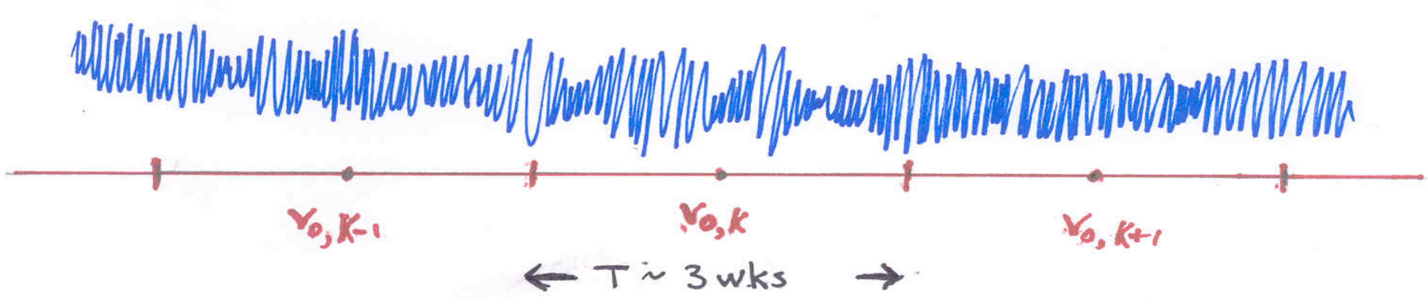


Stack-Slide Search Technique

(Barack, Creighton, Cutler, Gair)

Basic idea:

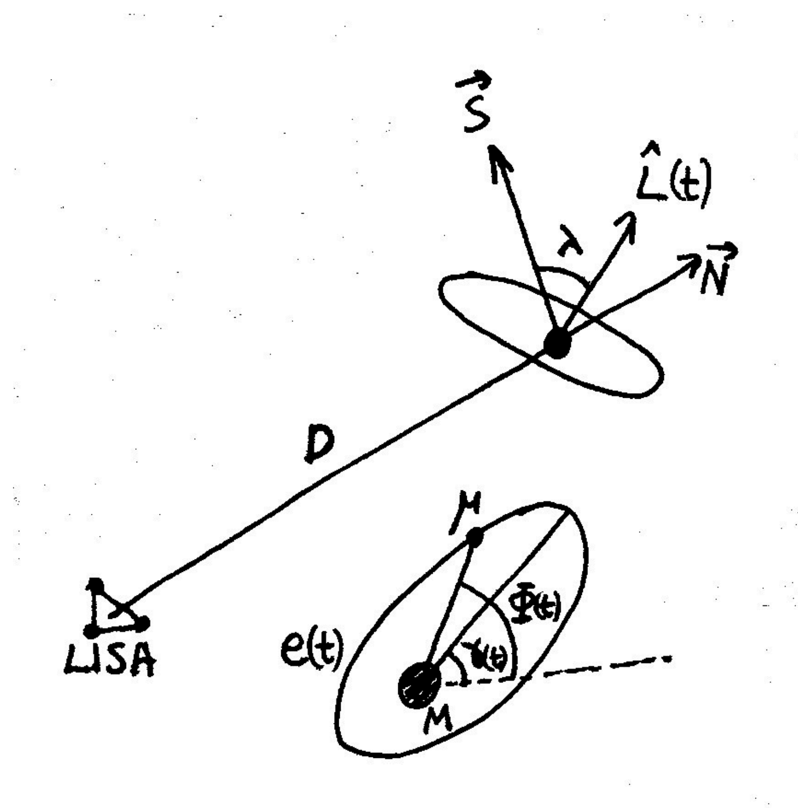
- Break two-yr waveform into ~3-wk segments; implement coherent, matched-filter search for segments.



- Add up the power from different segments, along tracks determined corresponding to different sets of physical parameters.

Parameter Space for inspiral problem (neglecting spin of CO)

M, μ	2
$\vec{N} = (D, \theta_S, \varphi_S)$	3
$\vec{S} = (S, \theta_K, \varphi_K)$	3
$t_0 \equiv t(\nu_0),$	1
$e(t_0), \Phi(t_0), \gamma(t_0),$	3
$\hat{L}_0 = [\lambda, \alpha(t_0)]$	2
	14



Template Counting:

Parameters can be divided into **extrinsic** (hard) and **intrinsic** (easy)

- Use Buonanno-Chen-Vallisneri (2003) trick to search quickly over 5 extrinsic angles (2 sky positions and 3 Euler angles) giving orientation of source at some $\mathbf{t_0}$

- Use FFT trick to search quickly over all $\mathbf{t_0}$

Counting Templates: Short Segments

13-dim

all params (except D)

→

7-dim

intrinsic params

$$g_{ab} = \frac{1}{2}\Gamma_{ab}$$

$$\gamma_{AB} = \left\langle \mathbf{Proj}\{ g_{ab} \} \right\rangle_{\mathbf{ext}}$$

Computational Cost: What can we afford?

Total cost (in floating point operations) of coherent filtering for T-length segments in data of total length τ is

$$\sim 10 N_{int.temp}(\tau/T)[3 f_{max}\tau \log_2(f_{max}\tau)]$$

Assuming we have 50 Teraflop machine , 1/3 of it goes into our coherent segment search, have 2 years of data, f_max is 30 mHz, and guess $\tau/T \sim 10^2$, then

$$N_{\mathrm{int}.temp} \leq 10^{10}$$

Number of (intrinsic-space templates) for 3-week segments entered at ν_0 = 1 mHz.

	$1.0e5 < M < 2.7e5$	$2.7e5 < M < 7.4e5$	$7.4e5 < M < 2.0e6$	$2.0e6 < M < 5.5e6$
$0.5 < \mu < 1.3$	$2.5 \cdot 10^5$	$5.8 \cdot 10^6$	$1.3 \cdot 10^8$	$2.9 \cdot 10^9$
$1.3 < \mu < 3.7$	$7.6 \cdot 10^5$	$1.7 \cdot 10^7$	$3.8 \cdot 10^8$	$8.7 \cdot 10^9$
$3.7 < \mu < 10$	$2.0 \cdot 10^6$	$4.6 \cdot 10^7$	$1.0 \cdot 10^9$	$2.3 \cdot 10^{10}$
$10 < \mu < 27$	$5.4 \cdot 10^6$	$1.2 \cdot 10^8$	$2.7 \cdot 10^9$	$6.2 \cdot 10^{10}$
$27 < \mu < 74$	$1.5 \cdot 10^7$	$3.4 \cdot 10^8$	$7.4 \cdot 10^9$	$1.7 \cdot 10^{11}$

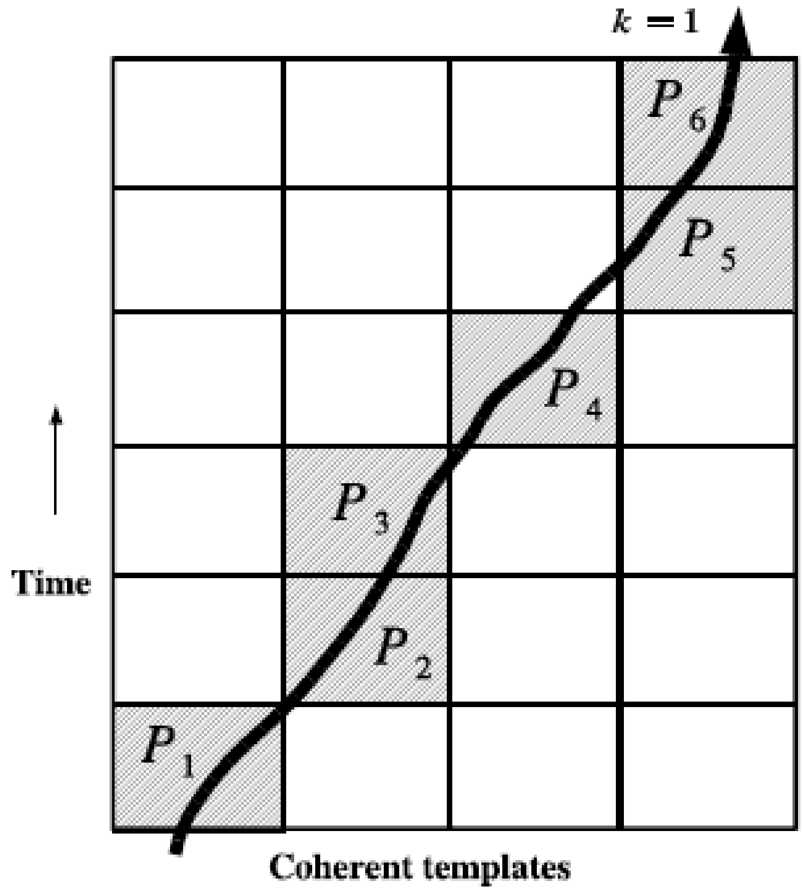
TABLE I. Number of templates required for 3 week long integration centered at $\nu_0 = 1$ mHz, for various CO and MBH mass ranges, and with both data channels. All masses are in $M_\odot$. The average match has been set here to 0.9. With $\mathcal{A} = 0.8$, one need only divide all entries by ~ 10. To re-scale for other T and ν_0, use the approximated scaling factors in Eq. (12)

$$N_{temp} \propto (\nu_0 M)^{3.1} (\nu_0 T)^{4.3}$$

Incoherent step: Stack together the power from short segments

There are 3 rapidly varying phase angles in trajectory/waveform. One saves computational cost by NOT requiring phase coherence from one stack to the next.

$$P = \sum_{k=1}^{N} P_k$$



Detection Threshold for our Semi-coherent Search

By combination of analytic arguments and Monte Carlo simulations, we have estimated that for a source to be detectable with our semi-coherent ("stacked") search algorithm, its matched-filter SNR (for both synthetic detectors combined) must be:

$$SNR_{thresh} = \sqrt{\left(8 N_{stack} / M\right)\left(1 + 4.5 N_{stack}^{-1/2}\right)}$$

where $N_{stack} = \tau / \mathrm{T}$

and M (approx 0.8-0.9) is the average match factor (overlap) between segment templates and actual waveform templates.

Estimate of Detection Threshold, w/ 50 Tflops

$$N_{stack} = 50 - 75 \quad \left(\tau = 3yr/\mathrm{T} = 2 - 3wk\right)$$

and M = 0.8 implies

$$SNR_{thresh} \approx 30 - 35$$

Even with infinite computing power, we'd need SNR approx 14 to insure a small false alarm rate—given the vast number of possible templates. So LISA loses only a factor of about 2 in sensitivity because of limitations of realistic computing power.

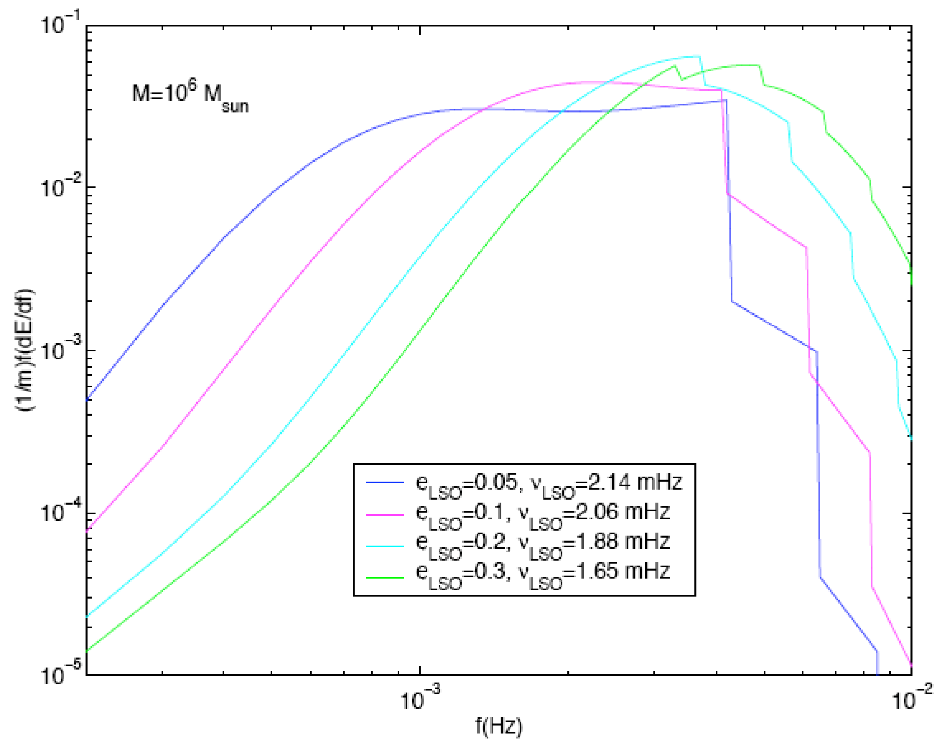
Confusion Noise from EMRIs

(Barack&Cutler 2004)

We know "confusion noise" from WD binaries dominates the LISA noise curve at f < 2-3 mHz. What about EMRIs?

Thousands of EMRI sources are "on" at any instant, so to initial approx. they sum to a Gaussian noise source. The spectral density is the weighted average of spectral densities of all the individual EMRIs:

$$S_h^{emri}(f) \propto f^{-27/8}$$



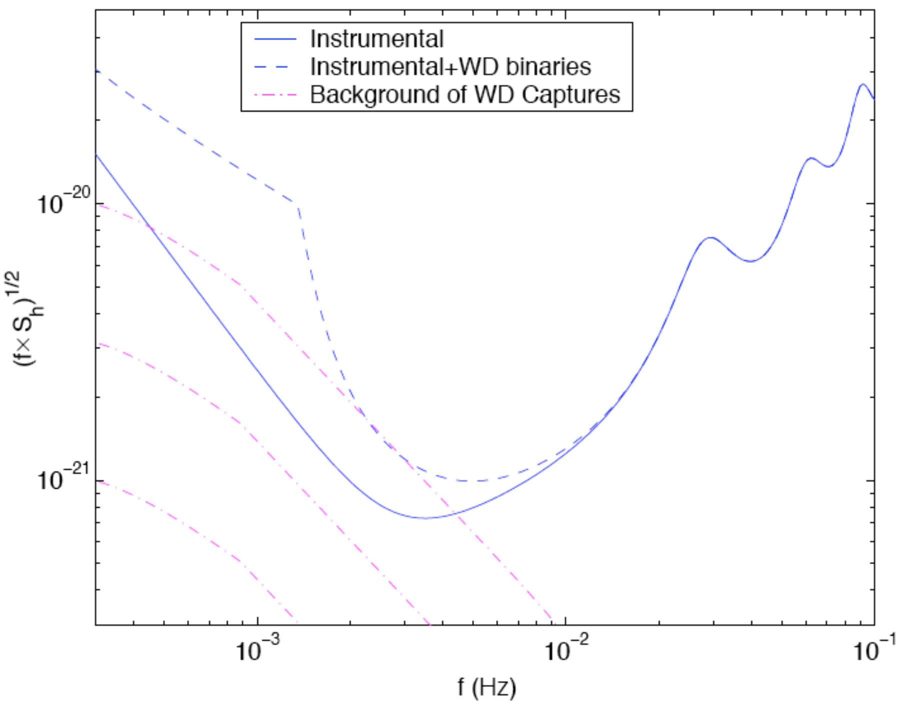
Spectrum for 10^6 Msun BH

Confusion background from EMRIs

(before subtraction of resolvable sources)

κ= ave. capture rate/yr for 10^6 Msun BHs

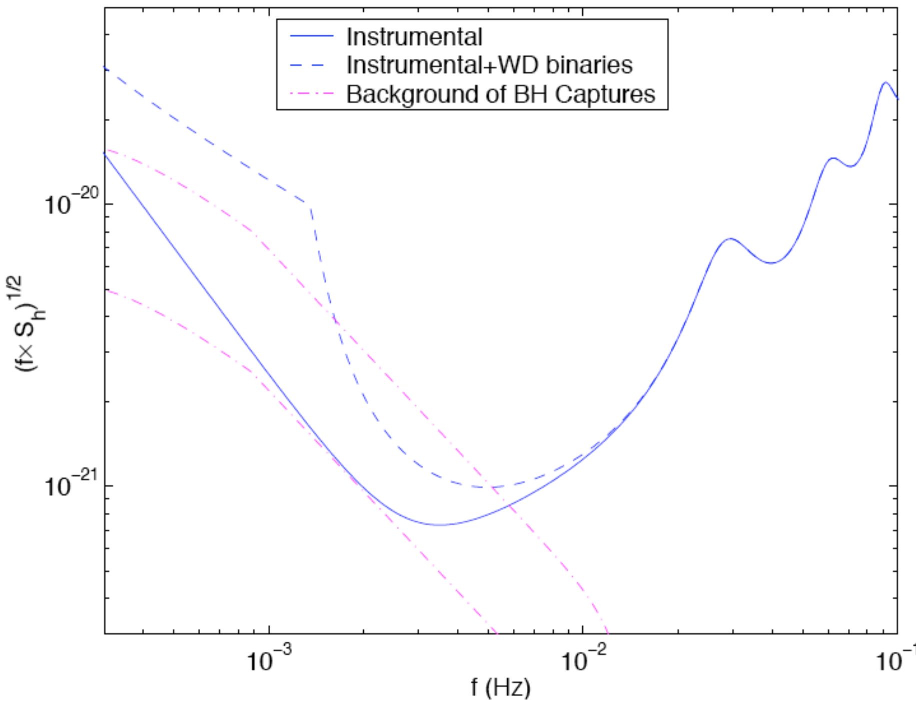
WDs



$$\kappa^{WD} = 4\cdot10^{-8},\ 4\cdot10^{-7},\ 4\cdot10^{-6}$$

~97% unresolvable

BHs



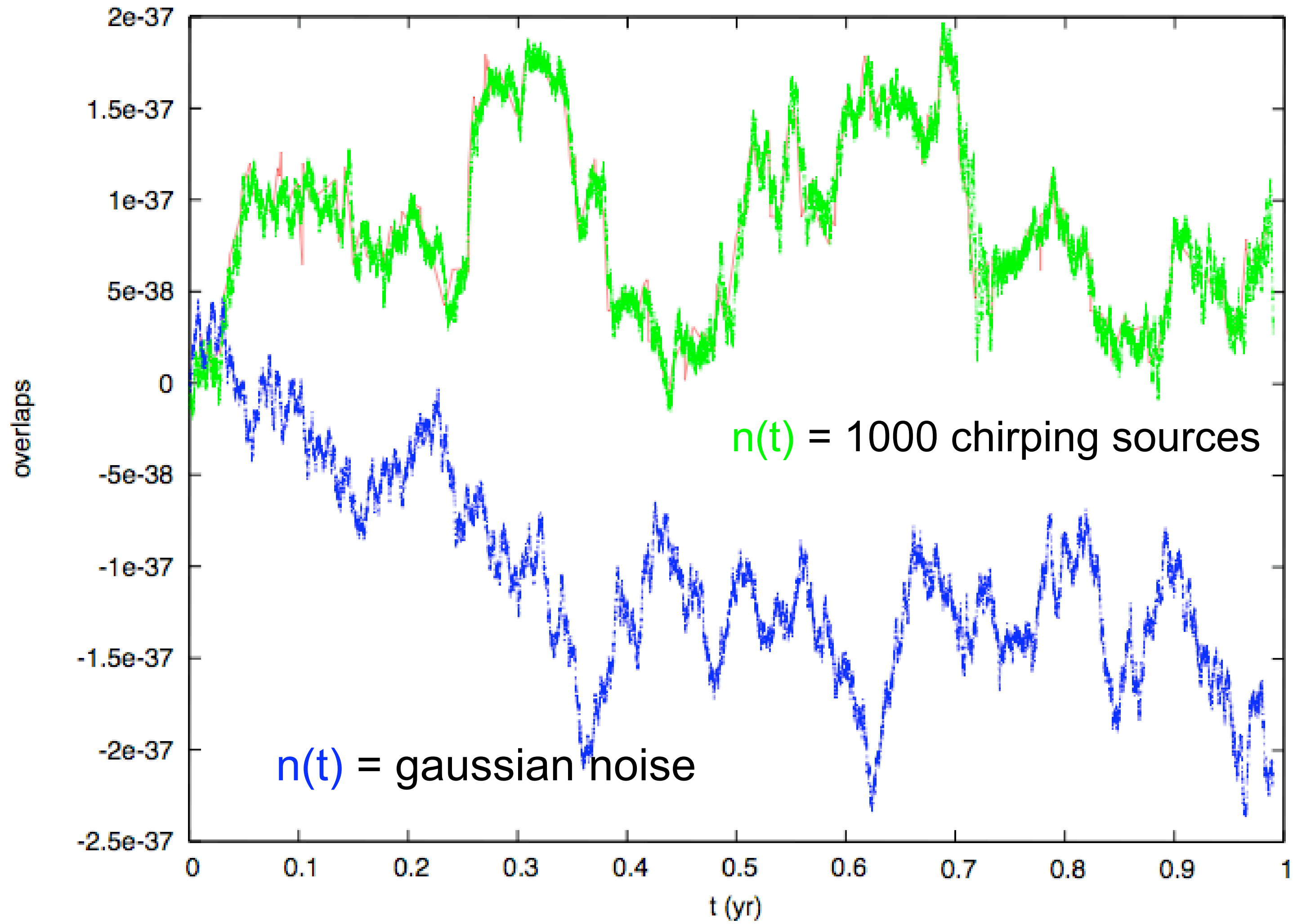
$$\kappa^{BH} = 6\cdot10^{-8},\ 6\cdot10^{-7}$$

~30-90% unresolvable

EMRI background vs. Gaussian noise w/ same spectral density

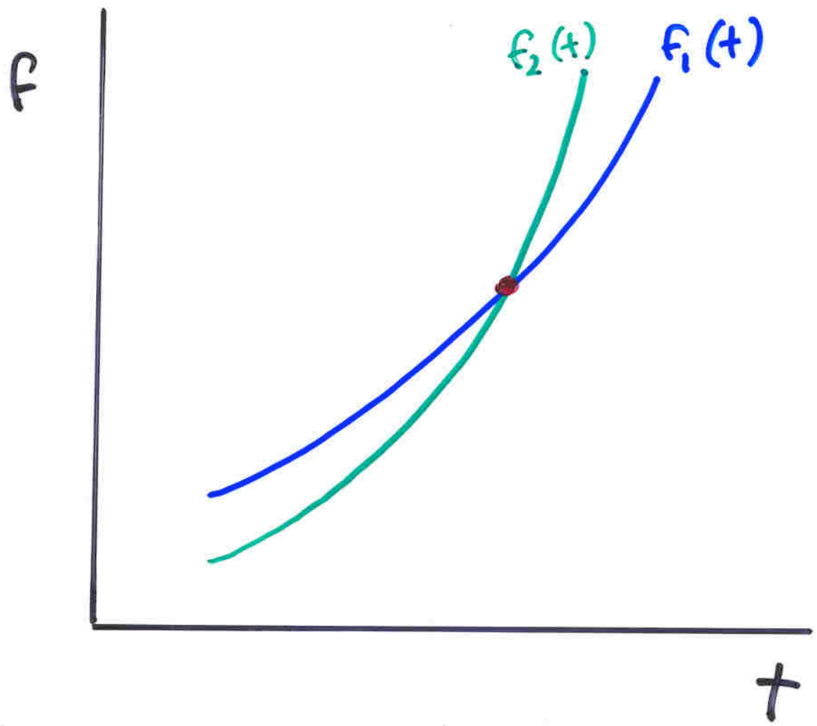
(work in progress by Racine, Cutler, Drasco, Babak)

Plot of $\int_0^t h(t')n(t')dt'$



How 2 different chirping waveforms "interfere with" each other:

$\int h_1(t)\,h_2(t)\,dt$ ---integral dominated by contribution from short time around crossing of $f_1(t)$ and $f_2(t)$



t-f tracks for 2 merging NS binaries at different z