## CHAPTER 6 <br> Similarities and Differences in Eighth-Grade Mathematics Teaching Across Seven Countries

0ne of the primary questions left unanswered by the TIMSS 1995 Video Study, and a question that motivated the current study, was whether teachers in all countries whose students achieve well in mathematics teach the subject in a similar way. It was tempting for some people who were familiar with the 1995 study to draw the conclusion that the method of teaching mathematics seen in the Japanese videotapes was necessary for high achievement. But in the 1995 study of teaching in Germany, Japan, and the United States, there was only one high-achieving country as measured through TIMSS-Japan. As the TIMSS 1999 Video Study includes a number of high-achieving countries, this study can shed more light on whether high-achieving countries teach in a similar manner.

The TIMSS 1999 Video Study aimed to reveal similarities and differences in teaching practices among all seven countries in the sample and to consider whether distinctive patterns of eighth-grade mathematics teaching can be detected in each country. This chapter summarizes the results presented in the earlier chapters and presents some new within-country analyses in order to address the descriptive and comparative issues that motivated this study.

For reader convenience, table 6.1 repeats the eighth-grade achievement results presented in chapter 1 for the seven participating countries on the TIMSS 1995 and the TIMSS 1999 mathematics assessments. As stated earlier, based on results from TIMSS 1995 and 1999, Hong Kong SAR and Japan were consistently high achieving relative to other countries in the TIMSS 1999 Video Study and the United States was lower achieving in mathematics than all the other countries in 1995 and lower than all but the Czech Republic of those countries that participated in 1999 (Gonzales et al. 2000).

TABLE 6.1. Average scores on the TIMSS mathematics assessment, grade 8, by country: 1995 and 1999

| Country | TIMSS 1995 mathematics score ${ }^{1}$ |  | TIMSS 1999 mathematics score ${ }^{2}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Average | Standard error | Average | Standard error |
| Australia ${ }^{3}$ (AU) | 519 | 3.8 | 525 | 4.8 |
| Czech Republic (CZ) | 546 | 4.5 | 520 | 4.2 |
| Hong Kong SAR (HK) | 569 | 6.1 | 582 | 4.3 |
| Japan (JP) | 581 | 1.6 | 579 | 1.7 |
| Netherlands ${ }^{3}$ (NL) | 529 | 6.1 | 540 | 7.1 |
| Switzerland (SW) | 534 | 2.7 | - | - |
| United States (US) | 492 | 4.7 | 502 | 4.0 |
| International average ${ }^{4}$ | - | - | 487 | 0.7 |

-Not available.
${ }^{1}$ TIMSS 1995: AU>US; HK, JP>AU, NL, SW, US; JP>CZ; CZ, SW>AU, US; NL>US.
${ }^{2}$ TIMSS 1999: AU, NL>US; HK, JP>AU, CZ, NL, US.
${ }^{3}$ Nation did not meet international sampling and/or other guidelines in 1995. See Beaton et al. (1996) for details.
${ }^{4}$ International average: AU, CZ, HK, JP, NL, US>international average.
NOTE: Rescaled TIMSS 1995 mathematics scores are reported here (Gonzales et al. 2000). Due to rescaling of 1995 data, international average not available. Switzerland did not participate in the TIMSS 1999 assessment.
SOURCE: Gonzales, P., Calsyn, C., Jocelyn, L., Mak, K., Kastberg, D., Arafeh, S., Williams, T., and Tsen, W. (2000). Pursuing Excellence: Comparisons of International Eighth-Grade Mathematics and Science Achievement From a U.S. Perspective, 1995 and 1999 (NCES 2001028). U.S. Department of Education. Washington, DC: National Center for Education Statistics.

## Locating Similarities and Differences in Eighth-Grade Mathematics Teaching

Two lenses can be used to provide two different views: one lens provides a wide-angle shot and considers more general features of teaching; the other lens moves in closer and focuses on the nature of the content and the way in which students and teachers interact with respect to mathematics. Each lens provides a different perspective on the data.

## A Wide-Angle Lens: General Features of Teaching

From a wide-angle view, the analyses contained in this report reveal that eighth-grade mathematics lessons across the seven countries share some general features. For example, mathematics teachers in all the countries organized the average lesson to include some public whole-class work and some private individual or small-group work (table 3.6). Teachers in all the countries talked more than students, at a ratio of at least 8:1 teacher to student words (figure 5.15). At least 90 percent of lessons in all the countries used a textbook or worksheet of some kind (table 5.6). Teachers in all the countries taught mathematics largely through the solving of mathematics problems (at least 80 percent of lesson time, on average, figure 3.3). And, on average, lessons in all the countries included some review of previous content as well as some attention to new content (figure 3.8).

The suggestion that the countries share some common ways of teaching eighth-grade mathematics is consistent with an interpretation that emphasizes the similarities of teaching practices across countries because of global institutional trends (LeTendre et al. 2001). By focusing on variables that reflect general features of teaching and instructional resources, the similarities
among countries in this study can be highlighted. This wide-angle lens suggests that many of the same basic ingredients were used to construct eighth-grade mathematics lessons in all of the participating countries (Anderson-Levitt 2002).

## A Close-Up Lens: Mathematics Problems and How They Are Worked On

A second, close-up lens brings a different picture into focus. This closer view considers how the general features of teaching were combined and carried out during the lesson. It reveals particular differences among countries in mathematics problems and how they are worked on.

One way of examining differences among countries is to ask whether countries showed distinctive patterns of teaching. Did the mathematics lessons in one country differ from the lessons in all the other countries on particular features? This criteria sets the bar quite high, but provides one way of looking across countries for unique approaches to the teaching of mathematics. Looking across the results presented in this report, there is no country among those that participated in the study that is distinct from all the other countries on all the features examined in this study. The 1995 study seemed to point to Japan as having a more distinct way of teaching eighth-grade mathematics when compared to the other two countries in that study. Based on the results of the 1999 study, all countries exhibited some differences from the other countries on features of eighth-grade mathematics teaching. However, Japanese eighth-grade mathematics lessons differed from all the other countries in the study on 17 of the analyses related to the mathematics lessons, or 15 percent of the analyses conducted for this report. ${ }^{1}$ Among the other countries, the Netherlands was found to be distinct from all the other participating countries on 10 analyses, or 9 percent, the next highest frequency. Among the other five countries, three differed from every other country where reliable estimates could be calculated from 1 to 3 percent of the analyses (the Czech Republic, Hong Kong SAR, and the United States), and the other two were not found to differ from all the other countries on any feature examined in this study (Australia and Switzerland).

Among the 17 analyses on which Japan differed from all of the other countries, most were related to the mathematical problems that were worked on during the lesson and to instructional practices. For example, Japanese eighth-grade lessons were found to be higher than all the other countries where reliable estimates could be calculated on the percentage of lesson time spent introducing new content ( 60 percent, figure 3.8), the percentage of high complexity problems (39 percent, figure 4.1), mathematically related problems ( 42 percent, figure 4.6), problems that were repetitions ( 40 percent, figure 4.6), problems that were summarized ( 27 percent, table 5.4), and the average time spent on a problem (15 minutes, figure 3.5). In some cases, Japanese lessons were found to be higher than lessons in the other countries on a feature, and in other cases, lower.

Dutch eighth-grade mathematics lessons, on the other hand, differed from all the other countries on 8 analyses, mostly related to the structure of the lesson and particular instructional practices. For example, Dutch lessons were higher than all the other countries where reliable estimates could be calculated on the percentage of lessons that contained goal statements ( 21 percent, figure 3.12) and in which a calculator was used ( 91 percent, figure 5.18), the percentage of problems per lesson that were set up using only mathematical language or symbols ( 40 percent, figure

[^0]5.1), and the estimated average time per lesson spent on future homework problems ( 10 min utes, table 3.8). Like Japan, in some cases, Dutch eighth-grade mathematics lessons were found to be higher than lessons in the other countries on a feature, and in other cases, lower.

These results show that some features of eighth-grade mathematics teaching in several countries, particularly in Japan and the Netherlands, were not shared by the other countries. Although these differences are limited to a few features relative to the number analyzed for this report, the criterion for being "unique" is set quite high by requiring a significant difference from all the other countries. The number of features on which differences were found rises if the question becomes whether an individual country differed from the majority of the other countries. Looking at the issue of relative differences another way, it was found that on 5 percent of the analyses conducted no differences were found among any of the participating countries. When a close-up lens is used, differences in teaching practices become evident, especially on features that describe the mathematics problems presented and the ways in which they were worked on during the lesson.

## Looking Through Both Lenses, and Asking More Questions

Is there a way to reconcile these findings that suggest there are both broad similarities in eighthgrade mathematics lessons across the countries as well as particular differences across the countries in how mathematics problems are presented and solved? One approach is to notice that similar ingredients or building blocks can be combined in different ways to create different kinds of lessons. Teaching across countries simultaneously can look both similar and different, (Anderson-Levitt 2002).

It is often the case that when mathematics teaching and learning are compared across countries, it is the differences that receive particular attention (Leung 1995; Manaster 1998; Schmidt et al. 1999; Stigler and Hiebert 1999). In this study, as well, the differences among the countries are intriguing because, in part, they are the kinds of differences that might relate to different learning experiences for students (Clarke 2003; National Research Council 2001). In particular, differences in the kinds of mathematics problems presented and how lessons are constructed to engage students in working on the problems can yield differences in the kinds of learning opportunities available to students (National Research Council 2001; Stein and Lane 1996).

How can the differences in teaching among the countries be explored further? The previous chapters, and the examples reviewed above, reported differences between pairs of countries feature by feature. Beyond comparing countries on individual features, there are at least two approaches that can provide additional insights: one approach is to look inside each country, at constellations of features, that describe the way in which lessons were constructed; and a second approach is to re-examine individual features within the context of each country's "system" of teaching. Both of these approaches are explored in the next section.

# What Are the Relationships Among Features of Mathematics Teaching in Each Country? 

## Lesson Signatures in Each Country

If there are features that characterize teaching in a particular country, there should be enough similarities across lessons within the country to reveal a particular pattern to the lessons in each country. If this were the case, then overlaying the features of all of the lessons within a country would reveal a pattern or, as labeled here, a "lesson signature."

The analyses presented to this point in the report focused on the presence of particular lesson features. In contrast, lesson signatures take into account when features occurred in the course of a lesson and consider whether and how basic lesson features co-occurred. The lesson signatures shown below were created by considering three dimensions that provide a dynamic structure to lessons: the purpose of the activities, the type of classroom interaction, and the nature of the content activity. These three dimensions were comprised of selected features analyzed for this study, and generally follow the organization of the three main chapters in this report (chapters 3 through 5). To create a lesson signature, each eighth-grade mathematics lesson was exhaustively subdivided along each of the three dimensions by marking the beginning and ending times for any shifts in the features. In this way, the dimensions could be linked by time through the lesson. This allowed an investigation into the ways in which the purpose segments, classroom interaction phases, and content activities appeared, co-occurred, and changed as the lessons proceeded.

The lesson signatures presented on the following pages were constructed by asking what was happening along the three dimensions during each minute of every eighth-grade mathematics lesson. ${ }^{2}$ Each variable or feature within a dimension is listed separately and is accompanied by its own histogram which represents the frequency of occurrence across all the lessons in that country, expressed as a percentage of the eighth-grade mathematics lessons. The histogram increases in height by one pixel ${ }^{3}$ for every 5 percent of lessons marked positively for a feature at any given moment during the lesson time, and disappears when fewer than 5 percent of lessons were marked positively (due to technological limitations).

Along the horizontal axis of each lesson signature is a time scale that represents the percentage of time that has elapsed in a given lesson, from the beginning to the end of a lesson. The percentage of lesson time was used to standardize the passing of time across lessons which can vary widely in length, from as little as 28 minutes to as much as 119 minutes (see table 3.1). Representing the passing of time in this way provides a sense of the point in a lesson that an activity or event occurred relative to the point in another lesson that the same activity or event occurred. For example, if lesson $A$ was twice as long as lesson $B$, and the first mathematics problem in lesson $A$ was presented 6 minutes into the lesson and the first mathematics problem in lesson $B$ was presented 3 minutes into the lesson, the lesson signature would show that the first mathematics problem in both lessons occurred at the same relative time.

[^1]To assist the reader in gauging the passing of time in the lessons, each lesson signature has vertical lines marking the beginning (zero), middle ( 50 percent), and end ( 100 percent) of the lesson time, moving from left to right. The 20, 40, 60, and 80 percent marks are indicated as well. By following the histogram of a particular feature from the zero to the 100 percent of time markings, one can get a rough idea of the percentage of lessons that included the feature at various moments throughout the lesson. For example, a lesson signature may show that 100 percent of lessons begin with review, but by the midpoint of a lesson, the percentage of lessons that are focused on review has decreased. As an additional aid to the reader, tables that list the percentage of lessons that included each feature from the zero to 100 percent time marks (in increments of 20) are included in appendix F.

Comparing the histograms of features within or across dimensions provides a sense of how those features were implemented as lesson time elapsed. Patterns may or may not be easily identified. Where patterns are readily apparent, this suggests that many lessons contain the same sequence of features. Where patterns are not readily apparent, this suggests variability within a country, either in terms of the presence of particular features or in terms of their sequencing. Furthermore, if the histograms of particular features are all relatively high at the same time in the lesson, this suggests that these features are likely to co-occur. However, in any single lesson observed in a country, this may or may not be the case. Thus, the histograms provide a general sense of what occurs as lesson time passes rather than explicitly documenting how each lesson moved from one feature to the next.

As noted above, a set of features within each of three dimensions are displayed along the left side of the lesson signatures (i.e., purpose, classroom interaction, and content activity). Within each dimension, the features that are used to represent each dimension are mutually exclusive (that is, a lesson was coded as exhibiting only one of the features at any point in time). However, in the interest of space, some low frequency features in two of the dimensions are not shown. For classroom interaction, the features not shown are "optional, teacher presents information" and "mixed private and public interaction" (these two features, combined, accounted for no more than 2 percent of lesson time in each country; see table 3.6). For content activity, the feature not shown is "non-mathematical work" (accounting for no more than 2 percent of lesson time in each country; see figure 3.2).

All of the features presented in the lesson signatures are defined and described in detail in chapters 3,4 , and 5 . The lessons signatures show additional detail about independent and concurrent problems. As stated earlier, independent problems were presented as single problems and worked on for a clearly recognizable period of time. Concurrent problems were presented as a set of problems that were worked on privately for a time. To provide the reader with a sense of the utilization of independent problems in the lessons, independent problems are grouped into 4 categories: the first independent problem worked on in the lesson, the second through fifth independent problems worked on in the lesson, the sixth through tenth independent problems worked on in the lesson, and the eleventh and higher independent problems worked on in the lesson. For concurrent problems, it was possible to distinguish between times when they were worked on through whole-class, public discussion (concurrent problems, classwork) and times when they were worked on through individual or small-group work (concurrent problems, seatwork). These two features are displayed in the lesson signatures as well.

Each lesson signature provides a view, at a glance, of how the lessons from a country were coded for each of the three dimensions (i.e., purpose, classroom interaction, and content activities).

The lesson signature for each country will be discussed in turn, and will also be supplemented by findings presented in prior chapters. In this way, the lesson signatures become a vehicle for pulling together the many pieces of information contained in this report. Because each signature displays 15 histograms, it is often difficult to assess the exact frequency of a given code at a particular moment in the lesson. As stated earlier, percentages for each feature, organized by country, are included in tables in appendix F.

## The lesson signature for Australia

Purpose
As stated earlier, 89 percent of eighth-grade Australian mathematics lessons included some portion of time during the class period devoted to review (table 3.4), representing an average of 36 percent of time per lesson (figure 3.8). Moreover, 28 percent of mathematics lessons were found to spend the entire lesson time in review of previously learned content, among the highest percentages of all the countries (figure 3.9). As visible on the lesson signature (figure 6.1), 87 percent of the Australian eighth-grade mathematics lessons began with a review of previously learned content. A majority of Australian lessons focused on review through the first 20 percent of lesson time, with a decreasing percentage of lessons going over previously learned content through the remainder of the lesson (figure 6.1 and table F.1, appendix F). Starting at about onethird of the way into the lesson and continuing to the end, a majority of Australian lessons engaged students with new content, representing an average of 56 percent of time per lesson (figure 3.8), with the practicing of new content becoming an increasing focus in the latter parts of the lesson (figure 6.1 and table F.1, appendix F).

## Classroom interaction

In terms of the interaction format in which eighth-grade students and teachers worked on mathematics, Australian lessons were found to show no detectable difference in the percentage of lesson time devoted to whole-class, public interaction versus private, individual or small-group interaction ( 52 and 48 percent, on average, respectively, table 3.6). The majority of Australian lessons were conducted through whole-class, public interaction during roughly the first third of lesson time, and again at the very end of the lesson (figure 6.1 and table F.1, appendix F). In between those two periods of time in the lesson, eighth-grade Australian students were found to be engaged in private work in a majority of lessons, usually with students working individually on problems that asked students to repeat procedures that had been demonstrated earlier in the lesson ( 75 percent of private interaction time per lesson was spent working individually, on average, figure 3.10 ; 65 percent of student private work time is spent repeating procedures that had been demonstrated earlier in the lesson, figure 5.13). In the hypothesized Australian country model, experts posited that there would be a "practice/application" period in the typical eighthgrade Australian mathematics lesson during which students would often be assigned a task to complete privately while the teacher moved about the class assisting students (figure E.2, appendix E). As the lesson signature shows, at a time in the lesson when the largest percentage of Australian lessons were focused on the practice of new content (roughly during most of the last third of the lesson), a majority of Australian lessons were found to have students working individually or in small groups (private interaction; figure 6.1 and table F.1, appendix F).

## Content activities

During the first half of the Australian mathematics lessons, there does not appear to be any consistent pattern in the types of problems that are presented to students (figure 6.1 and table F.1, appendix F). That is, teachers conveyed previously learned or new content to students by working on independent problems, sets of problems, either as a whole class or as seatwork (concurrent problems), or on mathematics outside the context of a problem (e.g., presenting definitions or concepts, pointing out relationships among ideas, or providing an overview of the lesson). During most of the last half of the Australian eighth-grade mathematics lessons, however, a majority of lessons were found to employ sets of problems (concurrent problems) as a way to focus on new content.

The delivery of content in Australian lessons is also revealed in analyses presented earlier in the report but not readily evident in the lesson signature. For example, when taking into consideration all of the problems presented in the eighth-grade Australian mathematics lessons, except for answered-only problems, 61 percent of problems per lesson were found to be posed by the teacher with the apparent intent of using procedures-problems that are typically solved by applying a procedure or set of procedures. This is a higher percentage than the percentage posed by the teacher with the apparent intent of either making connections between ideas, facts, or procedures, or to elicit a mathematical convention or concept (stating concepts; 24 and 15 percent, respectively, figure 5.8). Furthermore, when the problems introduced in the lesson were examined a second time for processes made public while working through the problems, 77 percent of the problems per lesson in Australia were found to have been solved by focusing on the procedures necessary to solve the problem or by simply giving results only without discussion of how the answer was obtained (figure 5.9). Moreover, when the 15 percent of problems per lesson that were posed to make mathematical connections were followed through to see whether the connections were stated or discussed publicly, 8 percent per lesson were solved by explicitly and publicly making the connections (figure 5.12). Finally, when experts examined the problems worked on or assigned during each lesson for the level of procedural complexity-based on the number of steps it takes to solve a problem using common solution methods- 77 percent of the problems per eighth-grade mathematics lesson in Australia were found to be of low complexity, among the highest percentages in all the countries (figure 4.1).

These observations and findings suggest that, on average, eighth-grade Australian mathematics lessons were conducted through a combination of whole-class, public discussion and private, individual student work, with an increasing focus on students working individually on sets of problems that were solved by using similar procedures as new content was introduced into the lesson and practiced.
FIGURE 6.1. Australian eighth-grade mathematics lesson signature: 1999

NOTE: The graph represents both the frequency of occurrence of a feature and the elapsing of time throughout a lesson. For each feature listed along the left side of the graph, the histogram (or bar) represents the percentage of eighth-grade mathematics lessons that exhibited the feature-the thicker the histogram, the larger the percentage of lessons that exhibited the feature. From left to right, the ang the percentage of lessons that included the feature as lesson time elapsed. A listing of the percentage of lessons that included each feature by the elapsing of time is included in appendix F. To create each histogram, each lesson was divided into 250 segments, each representing 0.4 percent of lesson time. The codes applied to each lesson at the start of each segment were tabulated, using weighted data, and

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## The lesson signature for the Czech Republic

Purpose
One of the distinguishing characteristics of eighth-grade mathematics teaching in the Czech Republic was the relatively prominent role of review, as illustrated in the lesson signature (figure 6.2). Support for this statement is based on the observation that a majority of Czech mathematics lessons focused on the review of previously learned content throughout the first half of the lesson, particularly during roughly the first third of the lesson (table F.2, appendix F). Findings presented in prior chapters also indicate that 100 percent of eighth-grade Czech mathematics lessons contain at least some portion of the lesson devoted to review (table 3.4), consuming an average of 58 percent of time per lesson, surpassing the average in all the countries except the United States (figure 3.8). Moreover, one-quarter of Czech lessons were found to spend the entire lesson time in review (figure 3.9).

The hypothesized Czech country model (figure E.3, appendix E), as compiled by country experts, suggests that review is not intended as a time to go over homework. Indeed, an earlier analysis shows that, on average, less than one minute per lesson was spent reviewing homework (table 3.9). Rather, according to the hypothesized country model, review in Czech lessons includes such goals as re-instruction and securing old knowledge, and serves as an opportunity for teachers to evaluate students. As noted in chapter 3, this latter point was observed in the Czech mathematics lessons during the interaction pattern of students presenting information that was optional for other students, an interaction pattern that distinguished the eighth-grade Czech mathematics lessons from lessons in all the other countries where reliable estimates could be calculated (table 3.6). As shown in the hypothesized Czech country model and as observed in the videotaped lessons, oral examinations could be given at the beginning of the lesson. In these cases, one or two students were called to the board to solve a mathematical problem presented by the teacher, for which the students were then publicly graded on their performance. As also observed in the videotapes, the students could be asked to describe in detail the steps to solve the problem.

Though review appears to have played a prominent role in eighth-grade Czech mathematics lessons, as can be observed in the lesson signature, the focus in a majority of the lessons turned to the introduction and practice of new content after the midpoint of the lesson (figure 6.2 and table F.2, appendix F). Nonetheless, an examination of the way in which lesson time is divided among the three purposes defined for this study revealed that eighth-grade Czech lessons devoted a greater proportion of lesson time to review than to introducing and practicing new content combined ( 58 percent compared to 42 percent, on average, figure 3.8).

## Classroom interaction

The way in which students and teachers interacted during the lesson in eighth-grade Czech mathematics lessons was characterized by the predominance of whole-class, public interaction (figure 6.2 and table F.2, appendix F). Indeed, 61 percent of lesson time, on average, was spent in public interaction (table 3.6). Conversely, the lesson signature shows the relative infrequency of students working individually or in small groups (private interaction). During the 21 percent of average lesson time that students spent in private interaction (table 3.6), it was overwhelmingly organized for students to work individually rather than in small groups or pairs ( 92 percent of private interaction time per lesson, on average, was spent working individually, figure 3.10).

As already noted above, the eighth-grade Czech mathematics lessons exhibited a third interaction pattern that was unique in comparison to the other countries: the public presentation of information by a student in response to the teacher's request, during which time the other students could interact with that student and teacher, or work on an assignment at their desk (optional, student presents information). On average, 18 percent of lesson time in Czech lessons was devoted to this form of interaction (table 3.6), though as is shown in the lesson signature, this form of classroom interaction was not characteristic to any particular point in a lesson (figure 6.2 and table F.2, appendix F).

## Content activities

As seen in the Czech lesson signature (figure 6.2), a noticeable percentage of eighth-grade Czech mathematics lessons began and ended with non-problem segments-that is, segments in which mathematical information was presented but problems were not worked on (see also table F.2, appendix F). Given that 91 percent of Czech mathematics lessons contained a goal statement by the teacher that made clear what would be covered during the lesson (figure 3.12), this may account, in part, for the 45 percent of lessons that began with non-problem segments (table F.2, appendix F).

When considering the ways in which mathematics problems were worked on during the lesson, the lesson signature reveals that a majority of lessons utilized independent problems to engage students in mathematics, throughout most of the lesson time (except at the very beginning and end of the lessons; table F.2, appendix F). Though the use of sets of problems (concurrent problems) is not uncommon in eighth-grade Czech mathematics lessons, analyses revealed that, on average, Czech lessons included 13 discrete, independent problems, among the highest number in all the countries (table 3.3) and constituting an average of 52 percent of the total lesson time (figure 3.4). Moreover, when problems were introduced into the lesson, 81 percent of problems per lesson were set up using mathematical language or symbols only (figure 5.1) and 77 percent were posed by the teacher with the apparent intent of using procedures-problems that are typically solved by applying a procedure or set of procedures (figure 5.8). This latter finding was found to be higher than the percentage of problems per lesson that were posed to make connections between ideas, facts, or procedures, or problems that were posed to elicit a mathematical convention or concept (stating concepts; 16 and 7 percent, on average, respectively, figure 5.8). When the problems introduced in the lesson were examined a second time for the processes made public while working through the problems, 71 percent of problems per lesson were found to have been solved by simply giving results only without discussion of how the answer was obtained or by focusing on the procedures necessary to solve the problem (figure 5.9). Moreover, when the 16 percent of problems per lesson that were posed to make mathematical connections were followed through to see whether the connections were stated or discussed publicly, 52 percent per lesson were solved by explicitly and publicly making the connections (figure 5.12). Finally, when experts examined the problems worked on or assigned during each lesson for the level of procedural complexity-based on the number of steps it takes to solve a problem using common solution methods- 64 percent of the problems per eighth-grade mathematics lesson in the Czech Republic were found to be of low complexity, 25 percent of moderate complexity, and 11 percent of high complexity (figure 4.1).

Looking back across these findings suggests that, on average, eighth-grade Czech mathematics lessons emphasized review to, in part, check students' knowledge and conveyed new content by having students work on a relatively high number of independent problems that required a focus on using procedures, all of which was conducted largely through whole-class, public interactions.
FIGURE 6.2. Czech eighth-grade mathematics lesson signature: 1999


## The lesson signature for Hong Kong SAR

Purpose
In Hong Kong SAR, 24 percent of eighth-grade mathematics lesson time, on average, was devoted to review, among the lowest percentages of all the countries (figure 3.8). Nonetheless, analysis of the data shows that 82 percent of lessons contained a review segment (table 3.4). The lesson signature (figure 6.3) shows that when review was included as part of the lesson, it tended to occur in the early moments of the lesson (see also table F.3, appendix F). Indeed, although about threequarters ( 77 percent) of the lessons initially began with a review of previously learned content, one-third ( 33 percent) focused on review at the 20 percent mark of elapsed time (table F.3, appendix F). In the hypothesized Hong Kong SAR country model developed by country experts, review time was mentioned as preparation for the new content to be presented later in the lesson (figure E.4, appendix E). A specific routine described by Hong Kong SAR experts was the process of going over content presented in past lessons that was relevant to the day's new procedures or concepts.

Eighth-grade mathematics lessons in Hong Kong SAR were among those lessons that, on average, spent the largest percentage of time on introducing and practicing new content ( 76 percent, figure 3.8), which is visible in the lesson signature (figure 6.3). Indeed, 92 percent of eighthgrade mathematics lessons in Hong Kong SAR were found to contain a segment in which new content was introduced in the lesson, among the highest percentages of all the countries (table 3.4). Conversely, the percentage of lessons that were devoted entirely to review in Hong Kong SAR was among the lowest of the countries ( 8 percent, figure 3.9). All of this would seem to suggest that, although a large percentage of eighth-grade Hong Kong SAR mathematics lessons included review, it played a relatively lesser role in the lessons in terms of the proportion of lesson time devoted to the activity.

Finally, although about three-quarters of lesson time in Hong Kong SAR mathematics classes was devoted to new content, beginning around the midpoint of the lesson, the practice of new content introduced in the lesson became an increasing focus of activity (figure 6.3 and table F.3, appendix F). Country experts highlighted the role of practicing new content in the hypothesized Hong Kong SAR country model (figure E.4, appendix E). The term used by experts to label the practicing phase was "consolidation." According to the experts, consolidation is accomplished through private work, the assessment of student work (i.e., some students working at the board), or homework. Country experts indicated that through the practice phase, teachers aim to increase students' confidence that new problems can be completed. According to these experts, the practice phase ensures that homework problems can be worked successfully and that the skills can be applied accurately on future examinations. It should be recalled that the average duration of a mathematics lesson in Hong Kong SAR was the shortest among all the countries (table 3.1), suggesting that activities are likely to have been compressed and focused.

Classroom interaction
As supported by the nearly solid band that represents public interaction in figure 6.3, an average of three-quarters of mathematics lesson time in Hong Kong SAR was spent in public interaction, a greater proportion of lesson time than in all the other countries except the United States (table 3.6). During this time, the teacher talked much more than the students, in a ratio of teacher to student words of 16:1 (figure 5.15). When students contributed verbally, it was often limited to brief utterances (figure 5.17). Country experts suggested that one possible interpretation of the
high percentage of teacher talk, along with the emphasis on introducing and practicing new content, is a press for efficiency. The experts suggested that teachers and students share an expectation that the relatively short lesson time (an average of 41 minutes and a median of 36 minutes, table 3.1) is designed to cover new content for which students will be accountable in the future.

That three-quarters of lesson time, on average, was spent in whole-class, public discussion means that an average of one-quarter of lesson time was spent in some other form of classroom interaction. As reported earlier, an average of 20 percent of lesson time in eighth-grade Hong Kong SAR mathematics lessons was devoted to students working largely independently (table 3.6 and figure 3.10). Unlike in some of the other countries in the study in which a majority of mathematics lessons turned toward individual or small-group work during some continuous portion of the lesson (i.e., Australia, the Netherlands, and Switzerland), Hong Kong SAR was among the countries that engaged students largely through whole-class, public interaction (i.e., the Czech Republic, Japan, and the United States; see tables 3.6 and F. 1 through F.7, appendix F).

## Content activities

An examination of the lesson signature shows that in addition to a number of eighth-grade mathematics lessons in Hong Kong SAR starting out with a review of previously learned content (77 percent of lessons start out this way), a number also initially focused on content activities such as non-problem-based mathematical work (43 percent): presenting definitions, pointing out relationships among ideas, or providing an overview or summary of the major points in a lesson. Moreover, although 20 percent of Hong Kong SAR lessons began by working on an independent mathematics problem, by roughly one fifth of the way through the lesson 67 percent focused on independent problems (figure 6.3 and table F.3, appendix F). This point in the lesson coincided with a shift from review of previously learned content to the introduction of new content in a majority of the lessons ( 58 percent of lessons were found to focus on the introduction of new content at around the same time point; figure 6.3 and table F.3, appendix F). As students and teachers moved through the second half of the lesson, Hong Kong SAR mathematics lessons also began to focus on the practice of new content through a mix of independent problems and sets of problems (concurrent problems) assigned to students as whole-class or seatwork (figure 6.3 and table F.3, appendix F).

The content of the mathematics introduced into eighth-grade Hong Kong SAR lessons was also shaped, in part, by the format of the mathematics problems and the ways in which the students worked on the problems. For example, 83 percent of problems per lesson in Hong Kong SAR were conveyed through the use of mathematical language or symbols only (figure 5.1), among the highest percentages of all the countries. When the problems introduced into eighth-grade mathematics lesson in Hong Kong SAR were examined by experts for the level of procedural complexity-based on the number of steps it takes to solve a problem using common solution methods- 63 percent of problems per lesson were found to be of low procedural complexity, 29 percent of moderate complexity, and 8 percent of high complexity (figure 4.1). Moreover, analyses revealed that 84 percent of problems per lesson were posed by the teacher with an apparent intent of using procedures-problems that are typically solved by applying a procedure or set of procedures-among the highest percentages of all the countries (figure 5.8). When all the mathematics problems were examined a second time to understand the processes publicly discussed by teachers and students as they worked through the problems, 48 percent of problems per eighth-grade mathematics lesson in Hong Kong SAR were found to have focused publicly on the procedures needed to solve the problem and 15 percent were solved by giving results only
without discussion of how the answer was obtained (figure 5.9). Finally, analyses revealed that 13 percent of problems in Hong Kong SAR mathematics lessons were posed by the teacher with an apparent intent of making connections-problems that are typically solved by constructing relationships between mathematical ideas (figure 5.8). When these problems were examined a second time to understand the processes publicly discussed by teachers and students as they worked through them, on average, 46 percent were found to have focused publicly on making connections (figure 5.12).

These findings suggest that during the relatively short period of time spent on mathematics in Hong Kong SAR, eighth-grade lessons, on average, emphasized the introduction and practice of new content through whole-class, public discussion and working on problems that focused on using procedures.
FIGURE 6.3. Hong Kong SAR eighth-grade mathematics lesson signature: 1999
 ce of a feature and the el NOTE: The graph represents both the frequency of occurrence of a feature and the elapsing of time throughout a lesson. For each feature listed along the left side of the graph, the histogram (or bar) represents the percentage of eighth-grade mathematics lessons that exhibited the feature-the thicker the histogram, the larger the percentage of lessons that exhibited the feature. From left to right, the percentage of elapsed time in a lesson is mare along the bottom of graph. The histogram increases by one pixe (or printable dot) for every 5 percent of lessons marked for a feature at any given of moment during the lesson time, and disappears when fewer than 5 percent of lessons were marked (due to technological limitations). By following each histogram from left to right, one can get an idea of
the percentage of lessons that included the feature as lesson time elapsed. A listing of the percentage of lessons that included each feature by the elapsing of time is included in appendix F . To create each the percentage of lessons that included the feature as lesson time elapsed. A listing of the percentage of lessons that included each feature by the elapsing of time is included in appendix F . To create each reported as the percentage of lessons exhibiting each feature at particular moments in time.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.



## The lesson signature for Japan

## Purpose

As noted earlier, some features of eighth-grade mathematics teaching in Japan were different from all the other participating countries. The lesson signature for Japan displays another way of looking at some of these features. Although 73 percent of Japanese eighth-grade mathematics lessons began with a review of previously learned content (figure 6.4 and table F.4, appendix F), the review of previously learned content constituted an average of 24 percent of lesson time (figure 3.8), among the smallest percentages in all of the countries. Indeed, as the lesson signature shows, by the time that 20 percent of the lesson time had passed, a majority of Japanese mathematics lessons were focused on the introduction of new content (figure 6.4 and table F.4, appendix F), which continued throughout the remaining lesson time. This relative emphasis on the introduction of new content in Japanese lessons is supported by two previously reported analyses: 95 percent of lessons included some portion of lesson time to introduce new content (table 3.4), averaging 60 percent of time per lesson (figure 3.8), the largest proportion of time among all the countries. Although the introduction of new content seemed to be a primary focus in the Japanese lessons, starting at the midpoint of the lesson, an increasing percentage of lessons moved toward the practice of the new content, constituting an average of 16 percent of time per lesson (figure 3.8).

## Classroom interaction

Like lessons in several of the other countries that participated in this study, eighth-grade Japanese mathematics lessons were conducted largely through whole-class, public discussions (figure 6.4 and table F.4, appendix F). A majority of Japanese mathematics lessons were carried out in this way throughout most of the lesson time, averaging 63 percent of time per lesson to this form of classroom interaction (table 3.6). During the roughly one-third of lesson time that was not characterized by whole-class instruction, students worked largely on their own (table 3.6 and figure 3.10). Throughout the course of a lesson, eighth-grade Japanese mathematics lessons were found to shift back and forth between whole-class instruction and individual student work an average of 8 times per lesson, more often than any of the other countries except the Czech Republic (table 3.7). Thus, Japanese mathematics teachers varied the type of interaction taking place in the classroom more often than in any of the other countries except the Czech Republic.

## Content activities

Observation of the eighth-grade Japanese mathematics lessons suggests that the teachers conduct the lessons by focusing on a relatively few number of independent problems related to a single topic for extended periods of time. This can be seen in the Japanese lesson signature in the focus of a majority of lessons on one to five problems over the course of a lesson (figure 6.4 and table F.4, appendix F). On average, eighth-grade Japanese mathematics lessons introduced three independent problems per lesson, fewer than all of the other countries except Australia (table 3.3), taking up an average of 64 percent of lesson time (figure 3.4). Moreover, devoting a large percentage of time to work on a relatively few problems also meant that the average time spent per problem was uniquely long in Japan-15 minutes on average (figure 3.5). Ninety-four percent of eighth-grade mathematics lessons in Japan included problems that related to a single topic, as opposed to more than one topic (figure 4.8). When problems were introduced into the lesson, 89 percent of the problems per lesson were set up using only mathematical language or
symbols whereas 9 percent of the problems contained references to real-life contexts, among the smallest percentages of the countries (figure 5.1). Of all the problems introduced into the eighthgrade Japanese mathematics lessons, 41 percent per lesson were posed with the apparent intent of using procedures, among the smallest percentages of all the countries (figure 5.8). The majority of problems per lesson were posed with the apparent intent of making connections between ideas, facts, or procedures ( 54 percent, figure 5.8). When the problems introduced in the lessons were examined a second time for the processes that were made public while working through the problems, 27 percent of problems per lesson were solved by explicitly using procedures and 37 percent were solved by making explicit the relevant mathematical connections (figure 5.9). Moreover, when the 54 percent of problems per lesson that were posed to make mathematical connections were followed through to see whether the connections were stated or discussed publicly, almost half of the problems per lesson ( 48 percent) were solved by explicitly making the connections (figure 5.12).

Other indications of how eighth-grade mathematics lessons in Japan are conducted show that 17 percent of problems per lesson, on average, included a public discussion and presentation of alternative solution methods, among the highest percentages of all the countries (table 5.1). Fifteen percent of problems per mathematics lesson in Japan were accompanied with a clear indication by the teacher that students could choose their own method for solving the problem (table 5.2). About one-quarter ( 27 percent) of mathematics problems per lesson were summarized by the teacher to clarify the mathematical point illustrated by the problem (table 5.4), more than in any of the other countries. Finally, based on an examination by experts into the level of procedural complexity of the problems introduced into the Japanese lesson-by looking at the number of steps it takes to solve a problem using common solution methods- 17 percent of problems per lesson were found to be of low procedural complexity, 45 percent of moderate complexity, and 39 percent of high complexity (figure 4.1). ${ }^{4}$

Looking back across these observations and findings suggests that on average eighth-grade Japanese mathematics lessons were conducted largely through whole-class, public discussion during which the emphasis was on the introduction of new content, conveyed to students by focusing on a few number of independent problems related to a single topic over a relatively extended period of time, with the goal of making connections among mathematical facts, ideas, and procedures.

[^3]FIGURE 6.4. Japanese eighth-grade mathematics lesson signature: 1995

NOTE: The graph represents both the frequency of occurrence of a feature and the elapsing of time throughout a lesson. For each feature listed along the left side of the graph, the histogram (or bar) repre-
sents the percentage of eighth-grade mathematics lessons that exhibited the feature-the thicker the histogram, the larger the percentage of lessons that exhibited the feature. From left to right, the sents the percentage of eighth-grade mathematics lessons that exhibited the feature-the thicker the histogram, the larger the percentage of lessons that exhibited the feature. From left to right, the l the percentage of lessons that included the feature as lesson time elapsed. A listing of the percentage of lessons that included each feature by the elapsing of time is included in appendix F. To create each histogram, each lesson was divided into 250 segments, each representing 0.4 percent of lesson time. The codes applied to each lesson at the start of each segment were tabulated, using weighted data, and

[^4]The lesson signature for the Netherlands
Purpose
As seen in other lesson signatures, a majority of eighth-grade Dutch mathematics lessons began with a review of previously learned content ( 64 percent), though a noticeable percentage of lessons began directly with the introduction of new content ( 29 percent, figure 6.5 and table F.5, appendix F). By the midpoint of the lesson, the percentage of lessons that were focused on review, introducing new content, or practicing new content, were relatively evenly divided (30, 34 , and 29 percent, respectively, table F.5, appendix F). Overall, the majority of lesson time was spent on new content (either through the introduction of new content or its practice, 63 percent, figure 3.8) and nearly one-quarter of lessons were devoted entirely to review (24 percent, figure 3.9). The midpoint of the lesson is also the time when a majority of Dutch lessons moved into private interaction, wherein students worked individually or in small groups, and focused on sets of problems (concurrent problems) completed as seatwork. As suggested by country experts (see the country model, figure E.5, appendix E), Dutch students' first experiences with new concepts or procedures might come directly from the textbook, perhaps while working on homework assigned for the next day (on average, 10 problems started during the lesson were assigned to be completed as homework, table 3.8). In those instances, according to the experts, students usually are responsible for reading over the text sections as they work privately on completing problems. This view is consistent with analyses that show a high percentage of Dutch lessons were taught by mathematics teachers who cited the textbook as a major determinant of the lesson content (97 percent, table 2.6). Finally, as stated earlier, analyses revealed that it was not uncommon in Dutch eighth-grade mathematics lessons for the practice work begun by students in class to be continued at home. A relatively large portion of lesson time in comparison to the other coun-tries-estimated to be 10 minutes of a 45-minute lesson, on average-was spent on problems that were assigned for future homework (table 3.8).

## Classroom interaction

A majority of eighth-grade Dutch mathematics lessons began with whole-class, public discussion and this form of classroom interaction continued until almost the midpoint of the lesson (figure 6.5 and table F.5, appendix F). This period of time coincided with the time when a majority of the lessons were focused largely on a review of previously learned content. Based on an earlier analysis, Dutch lessons were estimated to spend among the largest amount of time on the public discussion of previously assigned homework in all the countries ( 16 minutes, on average, table 3.9). As observed in the videotapes, these discussions appeared to take place during the beginning of lessons, when a majority of lessons focused on review. As noted above, a majority of Dutch lessons moved into the introduction of new content or its practice at around 40 percent of the way through a lesson (figure 6.4 and table F.5, appendix F), which is close to the time in the lesson when a majority of Dutch lessons moved into individual or small-group work (private interaction). Among all the countries, eighth-grade Dutch mathematics lessons dedicated a greater percentage of lesson time to students working privately ( 55 percent, on average, table 3.6), largely individually rather than in small groups (on average, 90 percent of private interaction time per lesson was spent working individually, figure 3.10).

## Content activities

The introduction and practice of new content in eighth-grade Dutch lessons coincided with an increasing percentage of lessons focused on individual student work on sets of problems (concurrent problems). As discussed above, and as suggested by country experts, eighth-grade Dutch students were observed to spend around half of the lesson time working individually. While working at their desks, eighth-grade Dutch students spent an average of 61 percent of lesson time on sets of problems (concurrent problems) rather than on independent problems (figure 3.4), among the highest percentages of all the countries. As displayed in the lesson signature (figure F.5), working on sets of problems commonly occurred during the second half of the lesson (table F.5, appendix F). Though all the countries utilized mathematics problems as the primary vehicle through which students came to acquire mathematical knowledge, eighth-grade Dutch lessons were found to devote a greater percentage of lesson time to working on problems than all of the countries except the United States ( 91 percent of lesson time, on average, figure 3.3). Mathematics problems in Dutch lessons were among the most frequent to be set up using a reallife connection ( 42 percent, figure 5.1) and to use calculators ( 91 percent of the lessons, figure 5.18). When experts reviewed the mathematics problems introduced into Dutch lessons for the level of procedural complexity-by looking at the number of steps it takes to solve a problem using common solution methods- 69 percent of problems per lesson were deemed of low procedural complexity, 25 percent of moderate complexity, and 6 percent of high complexity (figure 4.1). Moreover, of all the problems introduced into the lessons, the majority of problems per lesson ( 57 percent) were posed with the apparent intent of using procedures, with another quarter posed with the apparent intent of making connections between ideas, facts, or procedures (24 percent, figure 5.8). When the problems introduced in the lessons were examined a second time for the processes made public while working through the problems, 36 percent of problems per lesson were found to be solved by explicitly using procedures and 22 percent were solved by actually making mathematical connections (figure 5.9). Of the 24 percent of problems per lesson that were posed to make mathematical connections, 37 percent per lesson were explicitly and publicly making the connections. Furthermore, examination of the sets of problems (concurrent problems) assigned to students to work on individually at their desks showed that almost threequarters of the lesson time devoted to working privately focused on repeating procedures that had been demonstrated earlier in the lesson ( 74 percent, on average, figure 5.12).

Finally, homework appeared to play a role in the learning of content in Dutch lessons, as evidenced by the estimated time spent in discussion of previously assigned homework ( 16 minutes, on average, table 3.9) and the estimated percentage of lesson time spent on problems that were assigned for future homework ( 10 minutes, table 3.8). Country experts suggested that this relative emphasis on homework placed some responsibility on students for selecting what needed to be discussed at the beginning of the lesson and for working through the new content toward the end of the lesson (see figure E.5, appendix E).

All of these observations about eighth-grade Dutch mathematics lessons suggest that on average the introduction and practice of new content in eighth-grade Dutch lessons is often accomplished through students working individually on sets of mathematics problems that focus on using procedures. This is consistent with the country experts' assertion that Dutch students are expected to take responsibility for their own learning and are therefore given more independence or freedom to work on problems on their own or with others (figure E.5, appendix E).
FIGURE 6.5. Dutch eighth-grade mathematics lesson signature: 1999

NOTE: The graph represents both the frequency of occurrence of a feature and the elapsing of time throughout a lesson. For each feature listed along the left side of the graph, the histogram (or bar) represents the percentage of eighth-grade mathematics lessons that exhibited the feature-the thicker the histogram, the larger the percentage of lessons that exhibited the feature. From left to right, the moment during the lesson time, and disappears when fewer than 5 percent of lessons were marked (due to technological limitations). By following each histogram from left to right, one can get an idea of the percentage of lessons that included the feature as lesson time elapsed. A listing of the percentage of lessons that included each feature by the elapsing of time is included in appendix $F$. To create each histogram, each lesson was divided into 250 segments, each representing 0.4 percent of lesson time. The codes applied to each lesson at the start of each segment were tabulated, using weighted data, and

[^5]
## The lesson signature for Switzerland

## Purpose

Seventy-one percent of eighth-grade Swiss mathematics lessons began with a review segment, with a majority of lessons maintaining that focus through the first 20 percent of lesson time (figure 6.6 and table F.6, appendix F). The review of previously introduced content constituted an average of 34 percent of lesson time (figure 3.8). As shown in the lesson signature, a majority of Swiss lessons shifted to the introduction and practice of new content by about one-third of the way through the lesson, with an increasing percentage of lessons focused on the practice of new content as lesson time elapsed (figure 6.6 and table F.6, appendix F). This observation is generally consistent with an earlier analysis that showed Swiss lessons devoted an average of 63 percent of lesson time to introducing and practicing new content (figure 3.8). During the time when a majority of Swiss lessons focused on the review of previously learned content, a majority of lessons were conducted through whole-class, public discussion (figure 6.6 and table F.6, appendix F). This follows the observations of country experts who suggested that during the review phase the teacher takes a leading role but involves students by asking questions and engaging in "interactive instruction" (see the hypothesized Swiss country model, figure E.6, appendix E).

Classroom interaction
Although whole-class, public interaction was common throughout most of the first half and at the very end of Swiss eighth-grade mathematics lessons, a majority of Swiss lessons also devoted lesson time to individual student or small-group work for a period of time during the second half of the lesson (figure 6.6 and table F.6, appendix F). Indeed, as reported in an earlier analysis, eighth-grade Swiss mathematics lessons used 44 percent of lesson time, on average, for individual student and small-group work, surpassed only by Dutch lessons (table 3.6). During this period of lesson time, students spent an average of three-quarters of the private time working individually, with the remaining one-quarter working in pairs or small groups ( 74 and 26 percent per lesson, figure 3.10). The time during which eighth-grade Swiss students worked individually or in smallgroup work largely coincided with the time during which, in a majority of lessons, students were asked to work on sets of problems (concurrent problems; figure 6.6 and table F.6, appendix F). According to Swiss country experts, working privately on problems is a common activity when students are practicing new content in order to become more efficient in executing solution procedures (see the hypothesized Swiss country model, figure E.6, appendix E).

## Content activities

Although the content of some eighth-grade Swiss lessons was delivered by focusing on independent problems, the largest percentage of lessons utilized sets of problems (concurrent problems) as either whole-class or seatwork (figure 6.6 and table F.6, appendix F). Indeed, as revealed in earlier analyses, an average of 31 percent of lesson time in Switzerland was spent on independent problems and an average of 53 percent of lesson time was spent on sets of problems (concurrent problems; figure 3.4). According to Swiss country experts, two patterns of mathematics teaching were predicted to be observed in the videotaped lessons: one would focus on the introduction of new knowledge through a kind of Socratic dialogue between teacher and students (figure E.6, appendix E) and the second would focus largely on practicing content introduced in previous lessons (figure E.7, appendix E). Although analyses conducted for this study do not point to one or the other hypothesized model as being predominant in eighth-grade Swiss mathematics lessons, it
seems relatively clear that, in the majority of lessons, the introduction and practice of new mathematics content was conveyed to students through working on sets of problems rather than through working on independent, individual problems (figure 6.6 and table F.6, appendix F). This is a feature of eighth-grade Dutch mathematics lessons as well, as pointed out earlier. Finally, a majority of problems introduced per lesson in eighth-grade Swiss lessons exhibited a low level of procedural complexity, meaning that these problems could be solved using four or fewer steps (figure 4.1). An additional 22 percent of problems per lesson were found to be of moderate complexity, and 12 percent of high complexity (figure 4.1).

Looking across the indicators of mathematics teaching in Switzerland suggests a mixed picture. From the lesson signature and other data, it appears that on average eighth-grade Swiss mathematics lessons devoted some time to review but spent the bulk of lesson time on the introduction and practice of new content. To convey to students the new content, Swiss lessons employed a mix of independent problems and sets of problems (concurrent problems), though a majority of lessons utilized sets of problems during most of the latter half of the lesson, which coincided with an increasing focus on the practice of new content. Among the participating countries, Switzerland is unique in that it operates under three separate educational systems, depending on the Canton (state) and the dominant language spoken in the area (i.e., French, Italian, or German) (Clausen, Reusser, and Klieme forthcoming). Assuming that the operation of these three systems results in different decisions being made about content and how it is taught in the classroom, this makes summarizing across the various indicators of mathematics teaching to find a "country-level" pattern challenging.
FIGURE 6.6. Swiss eighth-grade mathematics lesson signature: 1999


The lesson signature for the United States
Purpose
Through most of the first half of the lesson time in the United States, the majority of eighthgrade mathematics lessons focused on reviewing previously learned content (figure 6.7 and table F.7, appendix F). Indeed, an earlier analysis showed that, of the 94 percent of lessons that engaged students in review during some portion of the lesson (table 3.4), an average of 53 percent of lesson time was spent reviewing previously learned material, among the highest percentages of all the countries (figure 3.8). Moreover, teachers in 28 percent of eighth-grade U.S. mathematics lessons were found to spend the entire lesson on reviewing previously learned content, also one of the highest percentages among the countries examined (figure 3.9). The relative emphasis on review was predicted by country experts who were fairly detailed in their description of review and practice segments in comparison to the description of introducing new content (see the hypothesized United States country model, figure E.8, appendix E). For example, the hypothesized U.S. model created by country experts included three different goals for review: to assess or evaluate, to re-teach, and to "warm-up" in preparation for the lesson.

Around half way through the lesson, a majority of eighth-grade U.S. mathematics lessons shifted focus to the introduction and practice of new content (figure 6.7 and table F.7, appendix F). Nonetheless, averaging across all the eighth-grade mathematics lessons, the United States was among the countries with the smallest percentage of lesson time devoted to introducing and practicing new content (48 percent, figure 3.8). Although in some of the countries there was a detectable difference in the emphasis placed within the lessons on either reviewing previously learned content or introducing and practicing new content, there was no such difference found in the United States (figure 3.8).

Classroom interaction
On average, 67 percent of eighth-grade mathematics lesson time in the United States was spent in whole-class, public interaction (table 3.6). This pattern was relatively prominent throughout most of the lesson, as seen in the lesson signature (figure 6.7 and table F.7, appendix F). The United States was one of the few countries in which some lessons began with students working on a set of problems as seatwork ( 21 percent, table F.7, appendix F). This may be consistent with what the country experts described in the hypothesized country model as the conducting of a "warm-up" activity, which is reportedly designed to secure and activate old knowledge (figure E.8, appendix E). One way in which the lesson might conclude, suggests the hypothesized model, is for students to practice new material while working on their own.

## Content activities

As with the other countries, the delivery of content in eighth-grade U.S. mathematics lessons was accomplished primarily by working through problems. As noted above, a majority of U.S. eighth-grade mathematics lessons focused on the introduction and practice of new content beginning in the second half of the lesson. Though U.S. eighth-grade mathematics teachers appeared to engage students with both independent and sets of problems, a majority of U.S. lessons utilized independent problems to convey mathematical content during the middle portion of the lesson (figure 6.7 and table F.7, appendix F). This is consistent with an earlier analysis that showed 51 percent of U.S. mathematics lesson time devoted to working on independent problems, on average (figure 3.4). Moreover, another analysis showed that, on average, an
eighth-grade U.S. mathematics lesson included 10 independent problems, among the highest frequency of all the countries (table 3.3). When eighth-grade students worked privately-as when they were assigned sets of problems (concurrent problems) to be completed in seatwork-students usually worked individually ( 80 percent of private interaction time per lesson, figure 3.10) and usually spent their time repeating procedures that were introduced earlier in the lesson ( 75 percent of private time per lesson; figure 5.13).

When taking into consideration all the problems presented in the U.S. lessons, 69 percent of the problems per lesson were found to be posed with the apparent intent of using proceduresproblems that are typically solved by applying a procedure or set of procedures-a higher percentage than problems that were posed with the apparent intent of making connections between ideas, facts, or procedures, or problems that were posed with the apparent intent of eliciting a mathematical convention or concept (stating concepts; figure 5.8). When the problems introduced in the lesson were examined a second time for processes made public while working through the problems, 91 percent of the problems per lesson in the United States were found to have been solved by giving results only without discussion of how the answer was obtained or by focusing on the procedures necessary to solve the problem (figure 5.9). Moreover, when the 17 percent of problems per lesson that were posed to make mathematical connections were followed through to see whether the connections were stated or discussed publicly, less than one percent per lesson were solved by explicitly and publicly making the connections (figure 5.12). Finally, an expert review of the mathematics problems introduced into U.S. lessons revealed that 67 percent of problems per lesson were deemed of low procedural complexity-based on the number of steps it took to solve a problem using common solution methods (figure 4.1). An additional 27 percent of problems per lesson were found to be of moderate procedural complexity, and 6 percent of high complexity.

All of these observations suggest that on average U.S. eighth-grade mathematics lessons were conducted largely through whole-class, public discussions that focused students' attention on both previously learned and new content by working on multiple independent problems, supplemented by practice on the occasional set of problems, with the goal of learning and using procedures.
FIGURE 6.7. U.S. eighth-grade mathematics lesson signature: 1999

NOTE: The graph represents both the frequency of occurrence of a feature and the elapsing of time throughout a lesson. For each feature listed along the left side of the graph, the histogram (or bar) represents the percentage of eighth-grade mathematics lessons that exhibited the feature-the thicker the histogram, the larger the percentage of lessons that exhibited the feature. From left to right, the moment during the lesson time, and disappears when fewer than 5 percent of lessons were marked (due to technological limitations). By following each histogram from left to right, one can get an idea of the percentage of lessons that included the feature as lesson time elapsed. A listing of the percentage of lessons that included each feature by the elapsing of time is included in appendix F. To create each histogram, each lesson was divided into 250 segments, each representing 0.4 percent of lesson time. The codes applied to each lesson at the start of each segment were tabulated, using weighted data, and

[^6]
## Summary

Information about eighth-grade mathematics teaching displayed in the lesson signatures and the accompanying descriptions complements the information presented in chapters 2 to 5 in several ways. First, the time during the lesson that particular lesson features were evident reveals the flow of lessons in ways that were not apparent from comparing the occurrence of individual features. Second, the co-occurrence of particular features during the lesson suggested interpretations about the nature of individual features. For example, non-problem segments that occurred at the beginning of the lesson along with a high percentage of public interaction and review suggested that these segments involved a whole-class discussion or brief presentation on material previously learned. Third, by invoking information from country experts and contained in the hypothesized country models, it was possible to speculate about the meaning of particular patterns found in the signatures. In general, the country models helped to put back together individual features separated for coding and analysis.

A summary of impressions about individual countries gained from viewing the lesson signatures includes the following. As noted earlier in this report (figure 3.8), the emphasis on introducing new material in Japan, on practicing new material in Hong Kong SAR, and on review in the Czech Republic and the United States all were reinforced in the lesson signatures. The emphasis in the Netherlands on private work also was supported.

Another set of impressions concerns the issue of convergence versus variability across mathematics lessons within countries. The height of the histograms, or the width of the bands, in the lesson signature graphs indicates the extent to which eighth-grade mathematics lessons for a particular country displayed the same profile across time. Lessons in Hong Kong SAR and Japan showed some convergence along the purpose dimensions, and lessons in the Netherlands showed particular convergence in a mid-lesson shift from public to private interaction. Other countries showed less convergence, as indicated by much overlap among variables within a dimension and few definitive peaks in the histograms.

Variability among lessons within countries might come from several sources. One source is individual differences among teachers. Different teachers might organize and implement lessons in different ways. Another source is the more systematic differences that could result from several different methods of teaching co-existing within a single country. One of the hypotheses arising from the TIMSS 1995 Video Study was that lessons within countries show considerable similarity compared with lessons across countries (Stigler and Hiebert 1999). These similarities might result from cultural "scripts" of teaching that become widely shared within a country.

Although the data presented in this report are, in general, consistent with this hypothesis, the variability within some countries suggests a more complicated picture. Switzerland is an instructive case. The lesson signature for Switzerland (figure 6.6) showed a lack of convergence along several dimensions. There appeared to be no clear consistency among lessons with regard to when during the lesson particular features were evident. Concurrent research in Switzerland suggests that this variability might be explained by the different language areas within the country (Clausen, Reusser, and Klieme forthcoming) and by educational reform activity, currently underway, that has yielded two different methods of teaching mathematics (Reusser et al. forthcoming). These results suggest it is useful to search the lesson signatures for indicators of variability within countries as well as for points of convergence.

## The Roles Played by Individual Lesson Features Within Different Teaching Systems

The fact that lessons within each country can accumulate to display a characteristic pattern means that the same basic ingredients of lessons can be arranged to yield recognizable systems of teaching (Stigler and Hiebert 1999). A system of teaching can be thought of as a recurring similarity in the way in which the basic lesson ingredients interact. These interactions change during a lesson as new ingredients are introduced, ingredients in-use disappear, and the emphasis placed on particular sets of ingredients waxes and wanes. A partial picture of systems, based on a small set of variables, is portrayed by the lesson signatures. As noted above, there are some conditions under which more than one system of teaching might operate within a given country.

From a system perspective, the meaning of each ingredient depends on its role in the system: when it occurs, the other features that co-occur, and the function it is serving at the time. The same ingredients can mean different things to the students (and the teacher) within different systems of teaching. The consequence is that individual ingredients or features can provide more information about teaching than is obtained through comparing them one at a time outside of the systems in which they are functioning. What is needed is a description of the system(s) of teaching in each country and then an analysis of the roles played by individual features within each system.

A full system analysis of eighth-grade mathematics teaching is beyond the scope of this study, in part because it would require a richer database than is available here (e.g., more than one lesson per teacher). But the lesson signatures presented earlier provide a preview, suggested by the data available, of country-based systems of teaching. This section extends the system analysis by considering two examples of the way in which similar ingredients can play quite different roles within different systems of teaching.

As a first example, consider the different roles that private interaction can play in eighth-grade mathematics lessons. Private interaction can occur at different points in the lesson (see the lesson signatures), and it can be used to engage students in different kinds of work. As stated earlier, eighth-grade students in Japan engaged in activities other than repeating procedures during a greater percentage of private work time than students in the other countries (figure 5.13). In addition, by viewing the co-occurrence of private interaction with lesson segments of different purposes, the lesson signatures suggest that some private work time might be used to introduce new content (Japan, the Netherlands, and Switzerland, figures 6.4, 6.5, and 6.6), to practice procedures introduced during the lesson (Australia, the Czech Republic, the Netherlands, and the United States; figures 6.1, 6.2, 6.5 and 6.7), and to review procedures and definitions already learned (the Czech Republic and the United States; figures 6.2 and 6.7). The different purposes indicate that, by definition, private work time can play different roles and provide different kinds of learning opportunities for students.

As a second example of the different meanings that can be associated with the same lesson ingredient, consider reviewing in the Czech Republic and the United States. Eighth-grade mathematics lessons in the Czech Republic devoted a greater percentage of time, on average, to review ( 58 percent), than any of the other countries except the United States (53 percent; figure 3.8). It appears, however, that it would be a mistake to assume that this shared emphasis on reviewing results in a similar experience for Czech and U.S. students. As described in the following paragraphs, the lesson signatures for each country signal one of the differences and hint at its meaning.

The review segments occurred near the beginning of lessons in both countries (figures 6.2, 6.7). But during the review segments of the Czech mathematics lessons, a mixed form of interaction could occur. When viewing the videotapes, and by studying the hypothesized Czech country model (see appendix E), it is apparent that this mixed form often occurred when one or two students were called to the front of the room to be publicly "graded." The teacher assigned a review problem for the student(s) to work on the chalkboard while the rest of the students attended to the dialogue between the teacher and the student(s) being graded or, sometimes, were given a choice to either watch and listen or to work on their own. After the student(s) finished the problem on the chalkboard, the teacher asked the student(s) questions about the work and then announced the grade.

Segments like the Czech grading were not seen in the United States lessons. Impressions from viewing the videotapes and information in the hypothesized country models (appendix E) indicated that reviewing in the United States usually occurred through a whole-class discussion, with the teacher answering questions and working through problems at the chalkboard requested by the students, or through students working individually on a set of "warm-up" problems. These different kinds of segments in the Czech Republic and the United States were all marked review, but it seems they could provide different learning experiences for the students.

The examples indicate that differences in teaching exist among the countries in this study, even along individual variables that might show similar frequencies of occurrence. To examine fully these more subtle differences, a combination of quantitative and qualitative analyses would be needed to identify patterns or systems of teaching and then to analyze the roles played within these systems by individual features of teaching.

## Conclusions

There are no simple or easy stories to tell about eighth-grade mathematics teaching from the TIMSS 1999 Video Study results. More than anything, the findings of this study expand the discussion of teaching by underscoring its complexity.

One thing is clear however: the countries that show high levels of achievement on TIMSS do not all use teaching methods that combine and emphasize features in the same way. Different methods of mathematics teaching can be associated with high scores on international achievement tests. Eighth-grade Japanese students often perform well in mathematics (Gonzales et al. 2000), and Japanese eighth-grade mathematics teaching contains a number of distinctive features. Nonetheless, it appears that these features are not a necessary condition for high achievement in other countries. Teachers in Hong Kong SAR, the other participating country with a TIMSS mathematics score as high as Japan, used methods of teaching that contained a number of features different from Japan, while teachers in the other high-achieving countries employed still different features.

The comparison between Japan and Hong Kong SAR is especially instructive because they were the two highest achieving countries in the study (table 6.1). In both countries, 76 percent of lesson time, on average, was spent working with new content and 24 percent of lesson time was spent reviewing previous content (figure 3.8). The new content introduced in mathematics lessons in these countries was worked with in different ways however. In Japanese lessons, more time
(than in all the other countries) was devoted to introducing the new content and in Hong Kong SAR more time (than in the Czech Republic, Japan, and Switzerland) was devoted to practicing the new content (figure 3.8). Consistent with this emphasis, a larger percentage of mathematics problems in Japanese eighth-grade mathematics lessons (than in all the other countries except the Netherlands) were presented with the apparent intent of asking students to make mathematical connections, and a larger percentage of mathematics problems in Hong Kong SAR lessons (than in all the other countries except the Czech Republic) were presented with the apparent intent of asking students to use procedures (figure 5.8). These different emphases are reinforced by recalling that a larger percentage of private work time in Hong Kong SAR lessons (along with those in the Czech Republic) was devoted to repeating procedures already learned than in Japanese (and Swiss) lessons (figure 5.13). Given that students in both Japan and Hong Kong SAR have performed well on international achievement tests such as TIMSS, it is interesting that their instructional practices (summarized in table 6.2) lie on the opposite ends of these dimensions.

## TABLE 6.2. Similarities and differences between eighth-grade mathematics lessons in Japan and Hong Kong

 SAR on selected variables: 1995 and 1999| Lesson variable | Japan ${ }^{1}$ | Hong Kong SAR |
| :---: | :---: | :---: |
| Reviewing ${ }^{2}$ (figure 3.8) | 24 percent of lesson time | 24 percent of lesson time |
| New content (figure 3.8) | 76 percent of lesson time | 76 percent of lesson time |
| Introducing new content ${ }^{3}$ | 60 percent of lesson time | 39 percent of lesson time |
| Practicing new content ${ }^{4}$ | 16 percent of lesson time | 37 percent of lesson time |
| Problems ${ }^{5,6}$ (as stated) (figure 5.8) | Making connections (54 percent of problems) | Using procedures (84 percent of problems) |
| Private work activity 7,8 (figure 5.13) | Something other than practicing procedures or mix (65 percent of work time) | Practicing procedures <br> (81 percent of work time) |

${ }^{1}$ Japanese mathematics data were collected in 1995.
${ }^{2}$ Reviewing: No differences detected.
${ }^{3}$ Introducing new content: JP>HK.
${ }^{4}$ Practicing new content: HK>JP.
${ }^{5}$ Making connections: JP>HK.
${ }^{6}$ Using procedures: HK>JP.
${ }^{7}$ Percent of private time devoted to something other than practicing procedures or mix: JP>HK.
${ }^{8}$ Percent of private time devoted to practicing procedures: HK>JP.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

If the learning goal for students is high performance on assessments of mathematics, the findings of this study suggest that there is no single method that mathematics teachers in relatively high-achieving countries use to achieve that goal. Different methods of mathematics teaching were found in different high-achieving countries. This conclusion suggests that informed choices of which teaching methods to use will require more detailed descriptions of learning goals than simply high performance on international tests. A particular country might have specific learning goals that are highly valued (see chapter 2, figure 2.1 and table 2.5) and for which particular methods of teaching may be better aligned than others. The results of this study make it clear that an international comparison of teaching, even among mostly high-achieving countries, cannot, by itself, yield a clear answer to the question of which method of mathematics teaching may be best to implement in a given country.

At the same time, the results of the study suggest that there are many similarities across countries, especially in the basic ingredients used to construct eighth-grade mathematics lessons. It is
likely that many teachers in each country are familiar with these ingredients. All participating countries, for example, devoted at least 80 percent of lesson time, on average, to solving mathematics problems (figure 3.3) and all countries devoted some lesson time, on average, to presenting new content (figure 3.8) However, mathematics teachers in the different countries used these ingredients with different emphases or arranged them in different ways.

Interpreting the results from this study requires a thoughtful and analytic approach, including follow-up analyses and research that can more precisely examine the possible effects that particular methods or approaches may have on student learning. Through these kinds of activities, the ultimate aim of a study such as this can be realized: a deeper understanding of classroom mathematics teaching and a deeper understanding of how teaching methods can be increasingly aligned with learning goals for students.

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## Results From the TIMSS 1999 Video Study

## APPENDIX A

Sampling, Questionnaires, Video Data
Coding Teams, and Statistical Analyses

## A1. Sampling

The sampling objective for the TIMSS 1999 Video Study was to obtain a representative sample of eighth-grade mathematics lessons in each participating country. ${ }^{1}$ Meeting this objective would enable inferences to be made about the national populations of lessons for the participating countries. In general, the sampling plan for the TIMSS 1999 Video Study followed the standards and procedures agreed to and implemented for the TIMSS 1999 assessments (see Martin, Gregory, and Stemler 2002). The school sample was required to be a Probability Proportionate to Size (PPS) sample. A PPS sample assigns probabilities of selection to each school proportional to the number of eligible students in the eighth-grade in schools countrywide. Then, one mathematics and/or one science eighth-grade class per school was sampled, depending on the subject(s) to be studied in each country.

Most of the participating countries drew separate samples for the Video Study and the assessments. ${ }^{2}$ For this and other reasons, the TIMSS 1999 assessment data cannot be directly linked to the video database. ${ }^{3}$

## A1.1. Sample Size

All of the TIMSS 1999 Video Study countries were required to include at least 100 schools in their initial selection of schools; however some countries chose to include more for various reasons. For example, Switzerland wished to analyze its data by language group, and therefore obtained a nationally representative sample that is also statistically reliable for the French-, Italian-, and German-language regions of that country. The Japanese data from the TIMSS 1995 Video Study included only 50 schools.

The TIMSS 1999 Video Study final sample included 638 eighth-grade mathematics lessons. Table 1 indicates the sample size and participation rate for each country.

[^7]| TABLE A.1. Sample size and participation rate for each country in the TIMSS 1999 Video Study |
| :--- | ---: | ---: | ---: |

${ }^{1}$ The participation rates including replacement schools are the percentage of all schools (i.e., original and replacements) that participated.
${ }^{2}$ Unweighted participation rates are computed using the actual numbers of schools and reflect the success of the operational aspects of the study (i.e., getting schools to participate).
${ }^{3}$ Weighted participation rates reflect the probability of being selected into the sample and describe the success of the study in terms of the population of schools to be represented.
${ }^{4}$ For Australia, the Czech Republic, and the Netherlands, these figures represent the participation rates for the combined mathematics and science samples.
${ }^{5}$ Japanese mathematics data were collected in 1995.
${ }^{6}$ The response rates after replacement for Japan differ from that reported previously (Stigler et al. 1999). This is because the procedure for calculating response rates after replacement has been revised to correspond with the method used in the TIMSS 1995 and TIMSS 1999 achievement studies.
${ }^{7}$ In the Netherlands, a mathematics lesson was filmed in 78 schools.
${ }^{8}$ In Switzerland, 74 schools participated from the German-language area ( 99 percent unweighted and weighted participation rate,), 39 schools participated from the French-language area ( 95 percent unweighted and weighted participation rate), and 27 schools participated from the Italian-language area (77 percent unweighted and weighted participation rate). SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

## A1.2. Sampling Within Each Country

Within the specified guidelines, the participating countries each developed their own strategy for obtaining a random sample of eighth-grade lessons to videotape for the TIMSS 1999 Video Study. For example, in two countries the video sample was a sub-sample of the TIMSS 1995 or TIMSS 1999 achievement study schools. ${ }^{4}$

The national research coordinators were responsible for selecting or reviewing the selection of schools and lessons in their country. ${ }^{5}$ Identical instructions for sample selection were provided to all of the national research coordinators. For each country, a sample of at least 100 eighthgrade mathematics classrooms was selected for videotaping. National random samples of schools were drawn following the same procedure used to select the sample for the TIMSS 1999 main study. In all cases, countries provided the relevant sampling variables to Westat, so that they could appropriately weight the school samples.

Complete details about the sampling process in each country can be found in the technical report (Jacobs et al. forthcoming).

[^8]
## A1.3. Videotaping Lessons

As noted in chapter 1, only one mathematics class was randomly selected within each school. No substitutions of teachers or class periods were allowed. The designated class was videotaped once, in its entirety, without regard to the particular mathematics topic being taught or type of activity taking place. The only exception was that teachers were not videotaped on days they planned to give a test for the entire class period.

Teachers were asked to do nothing special for the videotape session, and to conduct the class as they had planned. The scheduler and videographer in each country determined on which day the lesson would be filmed.

Most of the filming took place in 1999. In some countries filming began in 1998 and ended in 1999, and in others countries filming began in 1999 and ended in 2000. The goal was to sample lessons throughout a normal school year, while accommodating how academic years are organized in each country.

## A2. Questionnaires

To help understand and interpret the videotaped lessons, questionnaires were collected from the eighth-grade mathematics teachers of each lesson. The teacher questionnaire was designed to elicit information about the professional background of the teacher, the nature of the mathematics course in which the lesson was filmed, the context and goal of the filmed lesson, and the teacher's perceptions of its typicality. Teacher questionnaire response rates are shown in table A.2.

TABLE A.2. Teacher questionnaire response rates

|  | Teacher questionnaire response rate <br> (unweighted) |  |
| :--- | :---: | ---: |
| Country | Percent | Sample size |
|  | 100 | 87 |
|  | 100 | 100 |
| Czech Republic | 100 | 100 |
| Hong Kong SAR | 96 | 75 |
| Netherlands | 99 | 138 |
| Switzerland | 100 | 83 |
| United States |  |  |

NOTE: Japan did not collect a new mathematics video sample for the TIMSS 1999 Video Study.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

The questionnaire was developed in English and consisted of 27 open-ended questions and 32 closed-ended questions. Each country could modify the questionnaire items to make them culturally appropriate. In some cases, questions were deleted from the questionnaires for reasons of sensitivity or appropriateness. Country-specific versions of the questionnaire were reviewed for comparability and accuracy. Additional details regarding the development of the questionnaire, along with a copy of the U.S. version of the teacher questionnaire, can be found in the technical
report (Jacobs et al. forthcoming). Copies of the teacher questionnaires can also be found online at http://www.lessonlab.com.

The open-ended items in the teacher questionnaire required development of quantitative codes, a procedure for training coders, and a procedure for calculating inter-coder reliability. An 85 percent within-country inter-coder reliability criterion was used. The reliability procedures were similar to those used in the TIMSS 1995 assessment to code students' responses to the openended tasks (Mullis et al. 1996; Mullis and Martin 1998).

Short questionnaires also were distributed to the students in each videotaped lesson; however student data are not presented in this report. More information about the student questionnaire, and a copy of the U.S. version of the student questionnaire, can be found in the technical report (Jacobs et al. forthcoming).

## A3. Video Data Coding

This section provides information about the teams involved in developing and applying codes to the video data. More details about each of these groups and the codes they developed and applied can be found in the technical report (Jacobs et al. forthcoming).

## A3.1. The Mathematics Code Development Team

An international team was assembled to develop codes to apply to the TIMSS 1999 Video Study mathematics data. The team consisted of country associates (bilingual representatives from each country ${ }^{6}$ ) and was directed by a mathematics education researcher (see appendix B for team members). The mathematics code development team was responsible for creating and overseeing the coding process, and for managing the international video coding team. The team discussed coding ideas, created code definitions, wrote a coding manual, gathered examples and practice materials, designed a coder training program, trained coders and established reliability, organized quality control measures, consulted on difficult coding decisions, and managed the analyses and write-up of the data.

The mathematics code development team worked closely with two advisory groups: a group of national research coordinators representing each of the countries in the study, and a steering committee consisting of five North American mathematics education researchers (see appendix $B$ for advisory group members).

## A3.2. The International Video Coding Team

Members of the international video coding team represented all of the participating countries (see appendix B for team members). They were fluently bilingual so they could watch the lessons in their original language, and not rely heavily on the English-language transcripts. In almost all cases, coders were born and raised in the country whose lessons they coded.

[^9]Coders in the international video coding team applied 45 codes in seven coding passes to each of the videotaped lessons. They also created a lesson table for each video, which combined information from a number of codes. For example, the lesson tables noted when each mathematical problem began and ended, and included a description of the problem and the solution. These tables served a number of purposes: they acted as quick reference guides to each lesson, they were used in the development process for later codes, and they enabled problems to be further coded by specialist coding teams. ${ }^{7}$

## A3.3. Coding Reliability

As with any study that relies upon coding, it is important to establish clear reliability criteria. Based on procedures previously used and documented for the TIMSS 1995 Video Study and as described in the literature (Bakeman and Gottman 1997), percentage agreement was used to estimate inter-rater reliability and the reliability of codes within and across countries for all variables presented in the report. Percentage agreement allows for consideration of not only whether coders applied the same codes to a specific action or behavior, for example, but also allows for consideration of whether the coders applied the same codes within the same relative period of time during the lesson. That is, the reliability of coding in this study was judged based on two general factors: (1) that the same code was applied and (2) that it was applied during the same relative time segment in the lesson. Thus, it was not deemed appropriate to simply determine that the same codes were applied, but that they were applied to the same point in the lesson (here referred to as time segment) as well.

The calculation of percentage of agreement in this study is defined as the proportion of the number of agreements to the number of agreements and disagreements. Estimates of inter-rater and code reliability followed procedures described in Bakeman and Gottman (1997). Table A. 3 reports the reliability of applying codes to the video data at two points: at or very near the beginning of applying codes (initial reliability) and at the midpoint of applying codes to the video data (midpoint reliability). Coders established initial reliability on all codes in a coding pass prior to their implementation. After the coders finished coding approximately half of their assigned set of lessons (in most cases about 40 to 50 lessons), coders established midpoint reliability. The minimum acceptable reliability score for each code (averaging across coders) was 85 percent. Individual coders or coder pairs had to reach at least 80 percent reliability on each code. ${ }^{8}$

Initial reliability was computed as agreement between coders and a master document. A master document refers to a lesson or part of a lesson coded by consensus by the mathematics code development team. To create a master, the country associates independently coded the same lesson and then met to compare their coding and discuss disagreements until consensus was achieved. Masters were used to establish initial reliability. This method is considered a rigorous and cost-effective alternative to inter-coder reliability (Bakeman and Gottman 1997).

[^10]Midpoint reliability was computed as agreement between pairs of coders. By halfway through the coding process, coders were considered to be more expert in the code definitions and applications than the mathematics code development team. Therefore, in general, the most appropriate assessment of their reliability was deemed to be a comparison among coders rather than to a master document. Inter-rater agreement was also used to establish initial reliability in some of the later coding passes, but only for those codes for which coders helped to develop coding definitions.

A percentage agreement reliability statistic was computed for each coder by dividing the number of agreements by the sum of agreements and disagreements (Bakeman and Gottman 1997). Average reliability was then calculated across coders and across countries for each code. In cases where coders did not reach the established reliability standard, they were re-trained and re-tested using a new set of lessons. Codes were dropped from the study if 85 percent reliability could not be achieved (or if individual coders could not reach at least 80 percent reliability). As indicated in table A.3, all codes presented in the report met or exceeded the minimum acceptable reliability standard established for this study.

What counted as an agreement or disagreement depended on the specific nature of each code, and is explained in detail in Jacobs et al. (forthcoming). Some codes required coders to indicate a time. In these cases, coders' time markings had to fall within a predetermined margin of error. This margin of error varied depending on the nature of the code, ranging from 10 seconds to 2 minutes. Rationales for each code's margin of error are provided in Jacobs et al. (forthcoming).

Exact agreement was required for codes that had categorical coding options. In other words, if a code had four possible coding categories, coders had to select the same coding category as the master. In most cases, coders had to both mark a time (i.e., note the in- and/or out-point of a particular event) and designate a coding category. In these cases, it was first determined whether coders reliably marked the same or nearly the same in- and out-points, within the established margin of error. If reliability could not be established between coders based on marking the inand out-time of codes, then reliability for the actual coding category was not calculated. In these cases, as explained above, coders were re-trained and re-tested using a different set of lessons.

Percentage agreement was used to estimate inter-rater reliability and the reliability of the codes within and across countries for all the variables presented in this report. Percentage agreement allowed us to take into account the markings of both in- and out-points of the codes applied to the videotaped lessons when computing the reliability for a code. All three marks (i.e., in-point, out-point, and label) were included in the calculation. Percentage agreement was selected to calculate reliability for all codes because most codes included marking times as well as labels.

While initial and midpoint reliability rates are reported, coders were monitored throughout the coding process to avoid reliability decay. If a coder did not meet the minimum reliability standard, additional training was provided until acceptable reliability was achieved. The data reported in the report only include data from coders who were evaluated as reliable.

Table A. 3 lists the initial and midpoint reliability scores for each code, averaged across coders.

| TABLE A.3.Code | ability statistics | applied by the International Coding Team, |
| :---: | :---: | :---: |
|  | Initial reliability ${ }^{1}$ (percent) | Midpoint reliability ${ }^{2}$ (percent) |
| Lesson (LES) | 93 | 99 |
| Classroom interaction (CI) | 94 | 92 |
| Content activity (CC) | 90 | 87 |
| Concurrent problem (CP) | 94 | 90 |
| Assignment of homework (AH) | 99 | 93 |
| Goal statement (GS) | 99 | 89 |
| Outside interruption (OI) | 96 | 96 |
| Summary of lesson (SL) | 98 | 99 |
| Homework (H) | 99 | 98 |
| Real-life connection (RLC) | 98 | 100 |
| Graphs (GR) | 97 | 98 |
| Tables (TA) | 99 | 98 |
| Drawings/diagrams (DD) | 97 | 94 |
| Physical materials (PM) | 95 | 97 |
| Student choice of solution method (SC) | 90 | 93 |
| Proof/verification/derivation (PVD) | 99 | 97 |
| Number of target results (NTR) | 96 | 94 |
| Length of working on (LWO) | 95 | 94 |
| Facilitating exploration (FE) | 96 | 95 |
| Chalkboard (CH) | 96 | 100 |
| Projector (PRO) | 98 | 100 |
| Television or video (TV) | 100 | 100 |
| Textbook or worksheets (TXW) | 98 | 98 |
| Special mathematical materials (SMM) | 92 | 93 |
| Real-world objects (RWO) | 98 | 100 |
| Calculators (CALC) | 98 | 95 |
| Computers (COMP) | 100 | 98 |
| Multiple solution methods (MSM) | 99 | 98 |
| Problem summary (PSM) | 97 | 95 |
| Contextual information (CON) | 92 | 91 |
| Mathematical concept/theory/idea (CTI) | 92 | 94 |
| Activity (AC) | 97 | 97 |
| Announcing or clarifying homework or test (HT) | 95 | 98 |
| Private work assignment (PWA) | 93 | 98 |
| Organization of students (OS) | 96 | 96 |
| Public announcements (PA) | 86 | 86 |
| Purpose (P) | 87 | 94 |

[^11]A variety of additional quality control measures were put in place to ensure accurate coding. These measures included: (1) discussing difficulties in coding reliability lessons with the mathematics code development team and/or other coders, (2) checking the first two lessons coded by each coder, either by a code developer or by another coder, and (3) discussing hard-to-code lessons with code developers and/or other coders.

## A3.4. Specialist Coding Groups

The majority of codes for which analyses were conducted for in this report were applied to the video data by members of the international video coding team, who were cultural "insiders" and fluent in the language of the lessons they coded. However, not all of them were experts in mathematics or teaching. Therefore, several specialist coding teams with different areas of expertise were employed to create and apply special codes regarding the mathematical nature of the content, the pedagogy, and the discourse.

## A3.4.1. Mathematics Problem Analysis Group

The mathematics problem analysis group was comprised of individuals with expertise in mathematics and mathematics education (see appendix B for group members). They developed and applied a series of codes to all of the mathematical problems in the videotaped lessons, using lesson tables prepared by the international video coding team.

The mathematics problem analysis group constructed a comprehensive, detailed, and structured list of mathematical topics covered in eighth grade in all participating countries. Each problem marked in a lesson was connected to a topic on the list.

In addition to coding the mathematical topic of problems, the group also coded the procedural complexity of each problem, the relationship among problems, and identified application problems (see chapter 4 for definitions of procedural complexity and problem relationship, and chapter 5 for the definition of application problems).

The members of this group each established reliability with the director of the group by coding a randomly selected set of lessons from each country. They computed initial reliability as well as reliability after approximately two-thirds of the lessons had been coded. The percent agreement was above 85 percent for each code at both time points.

The director prepared a "master" for each lesson. Table A. 4 lists the other coders' percentage agreement with the director on each code, calculated as the number of agreements divided by the sum of agreements and disagreements.

## TABLE A.4. Initial and midpoint reliability statistics for each code applied by the Mathematics Problem

 Analysis Group, by code: 1999| Code | Initial reliability <br> (percent) | Midpoint reliability <br> (percent) |
| :--- | ---: | :--- |
| Topic | 89 | 90 |
| Procedural complexity | 87 | 90 |
| Relationship | 88 | 88 |

${ }^{1}$ Initial reliability refers to reliability established on a designated set of lessons before coders began work on their assigned lessons. 2Midpoint reliability refers to reliability established on a designated set of lessons after coders completed approximately two-thirds of their assigned lessons.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

## A3.4.2. Mathematics Quality Analysis Group

A second specialist group possessed special expertise in mathematics and teaching mathematics at the post-secondary level (see appendix B for group members). The same group previously was commissioned to develop and apply codes for the TIMSS 1995 Video Study. The mathematics quality analysis group reviewed a randomly selected subset of 120 lessons ( 20 lessons from each country except Japan). Japan was not included in this exercise because the group already had analyzed a sub-sample of the Japanese lessons as part of the 1995 Video Study.

Specially trained members of the international video coding team created expanded lesson tables for each lesson in this subset. The resulting 120 tables all followed the same format: they included details about the classroom interaction, the nature of the mathematical problems worked on during class time, descriptions of time periods during which problems were not worked on, mathematical generalizations, labels, links, goal statements, lesson summaries, and other information relevant to understanding the content covered during the lesson. Furthermore, the tables were "country-blind," with all indicators that might reveal the country removed. For example, "pesos" and "centavos" were substituted as units of currency, proper names were changed to those deemed neutral to Americans, and lessons were identified only by an arbitrarily assigned ID number. The mathematics quality analysis group worked solely from these written records, and had no access to either the full transcript or the video data.

The mathematics quality analysis group created and applied a coding scheme that focused on mathematical reasoning, mathematical coherence, the nature and level of mathematical content, and the overall quality of the mathematics in the lessons. The scheme was reviewed by mathematics experts in each country and then revised based on the feedback received. The group applied their coding scheme by studying the written records of the lessons and reaching consensus about each judgment. Due to the small sample size, only descriptive analyses of the group's coded data are included in this report (see appendix D).

## A3.4.3. Problem Implementation Analysis Team

The problem implementation analysis team analyzed a subset of mathematical problems and examined (1) the types of mathematical processes implied by the problem statement and (2) the types of mathematical processes that were publicly addressed when solving the problem (see appendix B for group members).

Using the video data, translated transcripts, and the same lesson tables provided to the mathematics problem analysis group, the problem implementation analysis team analyzed only those problems that were publicly completed during the videotaped lesson. Problems had to be publicly completed in order for the group to code for problem implementation. Furthermore, the group did not analyze data from Switzerland, since most of the Swiss transcripts were not translated into English.

Reliability was established by comparing a set of 10 lessons from each country coded by the director of the team with one outside coder. These lessons were randomly selected from those lessons that included at least one problem that was publicly completed during the lesson. Reliability of at least 85 percent was achieved for all countries.

Average inter-rater agreement for problem statements and implementations is shown in table A.5. Percentage agreement was calculated as the number of agreements divided by the sum of agreements and disagreements.

| TABLE A.5. | Reliability statistics for each code applied by the Problem Implementation Analysis Group, by <br> code: 1999 |
| :--- | :--- |
| Rode | Reliability <br> (percent) |
| Problem statement | 90 |
| Problem implementation | 90 |

SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

## A3.4.4. Text Analysis Group

The text analysis group used all portions of the mathematics lesson transcripts designated as public interaction to conduct various discourse analyses (see appendix B for group members). The group utilized specially designed computer software for these quantitative analyses of classroom talk.

Because of resource limitations, computer-assisted analyses were applied to English translations of lesson transcripts. ${ }^{9}$ In the case of the Czech Republic, Japan, and the Netherlands all lessons were translated from the respective native languages, and in the case of Hong Kong SAR, 34 percent of the lessons were conducted in English, so 66 percent were translated. English translations of Swiss lessons were not available.

[^12]
## A4. Statistical Analyses

Most of the analyses presented in this report are comparisons of means or distributions across seven countries for video data and across six countries for questionnaire data. The TIMSS 1999 Video Study was designed to provide information about and compare mathematics instruction in eighth-grade classrooms. For this reason, the lesson rather than the school, teacher, or student was the unit of analysis in all cases.

Analyses were conducted in two stages. First, means or distributions were compared across all available countries using either one-way ANOVA or Pearson Chi-square procedures. For some continuous data, additional dichotomous variables were created that identified either no occurrence of an event ( $\operatorname{code}=0$ ) or one or more occurrences of an event ( $\operatorname{code}=1$ ). Variables coded dichotomously were usually analyzed using ANOVA, with asymptotic approximations.

Next, for each analysis that was significant overall, pairwise comparisons were computed and significance determined by the Bonferroni adjustment. The Bonferroni adjustment was made assuming all combinations of pairwise comparisons. For continuous variables, Student's $t$ values were computed on each pairwise contrast. Student's $t$ was computed as the difference between the two sample means divided by the standard error of the difference. Determination that a pairwise contrast was statistically significant with $\mathrm{p}<.05$ was made by consulting the Bonferroni $t$ tables published by Bailey (1977). For categorical variables, the Bonferroni Chi-square tables published in Bailey (1977) were used.

The degrees of freedom were based on the number of replicate weights, which was 50 for each country. Thus, in any comparison between two countries there were 100 replicate weights, which were used as the degrees of freedom.

A significance level criterion of .05 was used for all analyses. All differences discussed in this report met at least this level of significance, unless otherwise stated. Terms such as "less," "more," "greater," "higher," or "lower," for example, are applied only to statistically significant comparisons. The inability to find statistical significance is noted as "no differences detected." In some cases, large apparent differences in data are not significant due to large standard errors, small sample sizes, or both.

All tests were two-tailed. Statistical tests were conducted using unrounded estimates and standard errors, which also were computed for each estimate. Standard errors for estimates shown in figures in the report are provided in appendix C.

The analyses reported here were conducted using data weighted with survey weights, which were calculated specifically for the classrooms in the TIMSS 1999 Video Study (see Jacobs et al. forthcoming for a more detailed description of weighting procedures).

## APPENDIX B Participants in the TIMSS 1999 Video Study of Mathematics Teaching

## Director of TIMSS 1999 Video Study of Mathematics Teaching

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| Douglas Grouws |  |
| Carolyn Kieran |  |
| Magdalene Lampert |  |
| Glenda Lappan | Eugene Owen |
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Arthur Lee

## APPENDIX C Standard Errors for Estimates Shown in Figures and Tables

| Standard errors for estimates shown in figures and tables, by country ${ }^{1}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Table/Figure | Category | AU | CZ | HK | JP2 | NL | sw | US |
| Table 2.1 | Mathematics | 4.1 | 1.3 | 3.0 | - | 1.9 | 3.6 | 6.1 |
|  | Science | 6.2 | 4.9 | 5.2 | - | 6.2 | 4.2 | 7.3 |
|  | Education | 6.6 | 4.0 | 2.7 | - | 3.6 | 4.8 | 6.7 |
|  | Other | 5.3 | 4.6 | 4.9 | - | 5.9 | 4.1 | 5.8 |
| Table 2.2 | Certified to teach one or more areas | 0.0 | 1.0 | 0.0 | - | 2.2 | 0.6 | 2.4 |
|  | Certified to teach mathematics at grade 8 | 6.3 | 4.0 | 4.1 | - | 4.1 | 4.1 | 5.5 |
|  | Certified to teach mathematics at another or unspecified grade | 4.7 | 3.7 | 3.2 | - | 3.4 | $\ddagger$ | $\ddagger$ |
|  | Certified to teach science at grade 8 | 6.2 | 4.4 | 5.0 | - | 6.4 | 3.6 | 5.3 |
|  | Certified to teach science at another or unspecified grade | 4.5 | 2.9 | 3.4 | - | 2.8 | 2.0 | 5.1 |
|  | Certified to teach education at grade 8 | $\ddagger$ | 3.8 | $\ddagger$ | - | $\ddagger$ | 4.2 | 5.5 |
|  | Certified to teach education at another or unspecified grade | 4.0 | $\ddagger$ | $\ddagger$ | - | $\ddagger$ | $\ddagger$ | 3.8 |
|  | Certified to teach another subject at grade 8 | 4.2 | 4.4 | 5.4 | - | 4.5 | 4.9 | 4.5 |
|  | Certified to teach another subject at another or unspecified grade | 4.3 | 2.1 | 3.2 | - | $\ddagger$ | $\ddagger$ | 2.8 |
| Table 2.3 | Years teaching | 1.1 | 1.1 | 0.8 | - | 1.1 | 1.0 | 1.2 |
|  | Years teaching mathematics | 1.1 | 1.5 | 0.8 | - | 1.0 | 1.1 | 1.3 |
| Table 2.4 | Hours per week teaching mathematics | 0.7 | 0.5 | 0.5 | - | 0.8 | 0.4 | 1.2 |
|  | Hours per week teaching other classes | 0.7 | 0.5 | 0.6 | - | 0.7 | 0.5 | 0.9 |
|  | Hours per week meeting with other teachers | 0.2 | 0.2 | 0.1 | - | 0.2 | 0.1 | 0.3 |
|  | Hours per week doing mathematics-related work at school | 0.7 | 0.4 | 0.6 | - | 0.4 | 0.2 | 0.4 |
|  | Hours per week doing mathematics-related work at home | 0.6 | 0.3 | 0.5 | - | 0.5 | 0.3 | 0.5 |
|  | Hours per week doing other school-related activities | 0.8 | 1.0 | 0.8 | - | 0.6 | 0.7 | 0.7 |
|  | Hours per week teaching and doing other school-related activities | 1.1 | 1.2 | 1.4 | - | 1.3 | 0.7 | 1.4 |
| Figure 2.1 | Perspective goals | 5.9 | 2.1 | $\ddagger$ | - | 3.4 | 4.4 | 3.7 |
|  | Process goals | 5.7 | 2.6 | 2.3 | - | 1.1 | 2.3 | 2.1 |
|  | Content goals | 6.3 | 2.4 | 2.7 | - | 3.4 | 3.7 | 4.9 |


| Standard errors for estimates shown in figures and tables, by country ${ }^{1}$-Continued |  |  |  |  |  |  |  |  |
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| Table/Figure | Category | AU | CZ | HK | JP2 | NL | SW | US |
| Table 2.5 | Using routine operations | 6.4 | 5.1 | 5.0 | - | 7.1 | 4.4 | 5.1 |
|  | Reasoning mathematically | 2.5 | 3.1 | 3.7 | - | 5.5 | 2.1 | 2.7 |
|  | Applying mathematics to real world problems | 3.7 | 4.0 | 2.7 | - | 4.7 | 2.6 | 5.3 |
|  | Knowing mathematical content | 5.1 | 3.3 | 2.9 | - | 4.2 | 3.3 | 3.6 |
|  | No process goals identified | 3.6 | 2.4 | 1.8 | - | 3.8 | 3.4 | 4.2 |
| Table 2.6 | Curriculum guidelines | 3.9 | 2.1 | 5.4 | - | 7.5 | 4.9 | 6.7 |
|  | External exams or tests | - | $\ddagger$ | 5.1 | - | 4.0 | 5.5 | 6.3 |
|  | Mandated textbook | 6.0 | 4.8 | 5.3 | - | 2.8 | 4.6 | 6.9 |
|  | Teacher's comfort with or interest in the topic | 6.5 | 4.8 | 5.1 | - | 4.4 | 5.1 | 6.1 |
|  | Teacher's assessment of students' interests or needs | 6.1 | 5.8 | 5.1 | - | 4.1 | 4.7 | 5.7 |
|  | Cooperative work with other teachers | 5.2 | $\ddagger$ | 3.9 | - | 5.6 | 3.4 | 3.6 |
| Figure 2.2 | Disagree | 4.0 | 3.3 | 3.4 | - | $\ddagger$ | 3.0 | 2.9 |
|  | No opinion | 5.3 | 4.7 | 4.7 | - | 4.3 | 4.7 | 4.5 |
|  | Agree | 6.3 | 4.5 | 3.7 | - | 4.9 | 5.4 | 5.5 |
| Figure 2.3 | Not at all | $\ddagger$ | 3.5 | 5.1 | - | 3.7 | 2.6 | $\ddagger$ |
|  | A little | 5.6 | 5.4 | 3.6 | - | 6.0 | 4.6 | 4.1 |
|  | A fair amount or a lot | 6.3 | 5.3 | 4.9 | - | 6.4 | 4.9 | 4.2 |
| Figure 2.4 | Sometimes or seldom | 6.0 | $\ddagger$ | 4.6 | - | $\ddagger$ | 4.1 | 5.1 |
|  | Often | 7.2 | 4.9 | 5.0 | - | 7.0 | 6.0 | 5.6 |
|  | Almost always | 5.4 | 4.5 | 4.7 | - | 6.9 | 4.8 | 5.8 |
| Figure 2.5 | Worse than usual | 2.5 | 5.7 | 4.2 | - | 5.9 | 2.1 | 3.0 |
|  | About the same | 5.1 | 5.8 | 5.9 | - | 6.2 | 4.4 | 5.8 |
|  | Better than usual | 5.2 | 3.7 | 5.2 | - | 2.9 | 4.3 | 5.4 |
| Figure 2.6 | Less difficult | 4.8 | 2.0 | 3.6 | - | $\ddagger$ | 1.3 | 5.2 |
|  | About the same | 4.7 | 3.8 | 4.5 | - | 3.4 | 2.8 | 6.1 |
|  | More difficult | 3.0 | 3.2 | 2.8 | - | $\ddagger$ | 2.5 | 2.8 |
| Figure 2.7 | Worse than usual | 3.6 | 5.2 | 5.0 | - | 3.0 | 2.2 | 2.0 |
|  | About the same | 4.4 | 4.7 | 4.9 | - | 3.6 | 2.4 | 3.0 |
|  | Better than usual | 2.3 | 4.6 | $\ddagger$ | - | $\ddagger$ | 2.0 | 2.5 |
| Figure 2.8 | Time spent planning similar lessons | 2.1 | 1.6 | 1.7 | - | 0.8 | 2.5 | 3.1 |
|  | Time spent planning the videotaped lesson | 5.1 | 3.9 | 7.0 | - | 2.1 | 2.3 | 5.4 |


| Standard errors for estimates shown in figures and tables, by country ${ }^{1}$-Continued |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Table/Figure | Category | $A \cup$ | CZ | HK | JP2 | NL | SW | US |
| Table 2.7 | Number of lessons in the sequence | 0.6 | 0.8 | 0.5 | - | 0.9 | 1.1 | 0.6 |
|  | Placement in the sequence | 0.5 | 0.7 | 0.4 | - | 0.6 | 0.9 | 0.5 |
| Table 3.1 | Lesson duration - average | 1.4 | 0.1 | 1.4 | 0.3 | 0.8 | 0.4 | 1.9 |
|  | Lesson duration - standard deviation | 1.0 | 0.2 | 1.5 | 0.3 | 2.0 | 3.1 | 2.3 |
| Figure 3.2 | Mathematical work | 0.6 | 0.2 | 0.5 | 0.4 | 0.7 | 0.3 | 0.5 |
|  | Mathematical organization | 0.6 | 0.2 | 0.4 | 0.3 | 0.4 | 0.2 | 0.4 |
|  | Non-mathematical work | 0.3 | 0.1 | 0.2 | 0.2 | 0.4 | 0.2 | 0.3 |
| Figure 3.3 | Problem segments | 2.3 | 1.3 | 1.3 | 2.4 | 0.9 | 1.7 | 1.9 |
|  | Non-problem segments | 2.1 | 1.2 | 1.3 | 2.4 | 0.6 | 1.6 | 1.7 |
| Table 3.3 | Independent problems - number | 1.9 | 1.0 | 0.5 | 0.1 | 1.1 | 0.5 | 1.0 |
|  | Answered only problems - number | 1.0 | 0.1 | 0.1 | $\ddagger$ | 0.6 | 1.5 | 1.4 |
| Figure 3.4 | Independent problems - percent of lesson time | 3.2 | 2.4 | 2.8 | 5.3 | 3.0 | 2.8 | 3.1 |
|  | Concurrent problems - percent of lesson time | 3.4 | 2.6 | 2.6 | 5.1 | 3.2 | 2.9 | 3.1 |
|  | Answered only problems - percent of lesson time | 0.2 | 0.1 | 0.1 | $\ddagger$ | 0.3 | 0.4 | 0.7 |
| Figure 3.5 | Time per independent problem | 0.5 | 0.3 | 0.4 | 1.5 | 0.3 | 0.6 | 1.1 |
| Figure 3.7 | Independent and concurrent problems longer than 45 seconds | 4.7 | 2.9 | 2.7 | 1.3 | 4.0 | 3.3 | 3.5 |
| Figure 3.8 | Reviewing - percent of lesson time | 4.3 | 2.9 | 3.1 | 3.9 | 5.1 | 3.0 | 4.6 |
|  | Introducing new content - percent of lesson time | 3.1 | 2.0 | 2.9 | 4.2 | 4.4 | 2.8 | 2.7 |
|  | Practicing new content - percent of lesson time | 3.4 | 1.9 | 2.8 | 2.6 | 3.7 | 2.2 | 3.5 |
| Table 3.4 | Reviewing - at least one segment | 3.2 | 0.0 | 4.2 | 7.5 | 5.9 | 3.6 | 2.7 |
|  | Introducing new content - at least one segment | 6.5 | 4.2 | 2.9 | 2.9 | 6.0 | 3.6 | 5.7 |
|  | Practicing new content - at least one segment | 6.6 | 4.8 | 4.5 | 5.3 | 6.4 | 4.3 | 6.4 |
| Figure 3.9 | Entirely review | 6.5 | 4.2 | 2.9 | 2.9 | 5.8 | 3.6 | 5.7 |
| Table 3.5 | Shifts in purpose | 0.2 | 0.1 | 0.2 | 0.1 | 0.1 | 0.1 | 0.2 |
| Table 3.6 | Public interaction | 2.5 | 1.9 | 1.6 | 2.5 | 2.6 | 2.1 | 2.5 |
|  | Private interaction | 2.5 | 1.3 | 1.6 | 2.2 | 2.8 | 1.9 | 2.6 |
|  | Optional, student presents information | 0.1 | 2.0 | 0.7 | 0.8 | 0.2 | 0.4 | 0.4 |
| Figure 3.10 | Worked individually | 5.8 | 3.0 | 1.5 | 3.4 | 4.0 | 3.7 | 4.4 |
|  | Worked in pairs or groups | 5.8 | 3.0 | 1.5 | 3.4 | 4.0 | 3.8 | 4.5 |


| Standard errors for estimates shown in figures and tables, by country ${ }^{1}$-Continued |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Table/Figure | Category | AU | CZ | HK | JP2 | NL | SW | US |
| Table 3.7 | Shifts in classroom interaction | 0.3 | 0.4 | 0.3 | 0.4 | 0.3 | 0.3 | 0.4 |
| Figure 3.11 | Homework assigned | 6.4 | 4.3 | 4.1 | 5.1 | 5.4 | 4.2 | 6.4 |
| Table 3.8 | Future homework problems - number | 1.4 | 0.2 | 0.5 | 0.2 | 1.5 | 0.6 | 1.0 |
|  | Future homework problems - time | 0.9 | 0.4 | 0.7 | 0.6 | 1.4 | 1.2 | 0.8 |
| Table 3.9 | Previous homework problems - number | 1.7 | 0.1 | 0.1 | 0.2 | 1.8 | 1.9 | 1.9 |
|  | Previous homework problems - time | 0.4 | 0.1 | 0.3 | 0.4 | 3.4 | 1.8 | 2.6 |
| Figure 3.12 | Goal statement | 4.9 | 3.2 | 4.9 | 8.9 | 5.5 | 5.0 | 6.1 |
| Figure 3.13 | Summary statement | 3.0 | 4.6 | 4.1 | 7.8 | $\ddagger$ | 0.8 | 2.8 |
| Figure 3.14 | Outside interruption | 6.1 | 3.1 | 4.6 | 5.3 | 6.6 | 2.6 | 6.4 |
| Figure 3.15 | Non-mathematical segment within the mathematics portion of the lesson | 2.9 | 1.8 | 3.1 | $\ddagger$ | 5.6 | 2.7 | 4.4 |
| Figure 3.16 | Public announcement unrelated to the current assignment | 4.6 | 2.9 | 3.3 | 6.9 | 6.0 | 4.1 | 4.8 |
| Table 4.1 | Number | 6.0 | 4.0 | 3.4 | $\ddagger$ | 3.2 | 3.6 | 4.5 |
|  | Whole numbers, fractions, decimals | 5.2 | 2.6 | 2.5 | $\ddagger$ | 1.6 | 4.0 | 4.2 |
|  | Ratio, proportion, percent | 4.7 | 1.4 | 1.9 | $\ddagger$ | 1.8 | 4.2 | 2.1 |
|  | Integers | 1.2 | 2.3 | 1.3 | $\ddagger$ | 1.5 | 1.3 | 2.1 |
|  | Geometry | 5.3 | 4.0 | 3.8 | 3.3 | 5.7 | 3.8 | 5.8 |
|  | Measurement | 2.8 | 2.1 | 1.5 | 5.6 | 2.9 | 2.7 | 4.7 |
|  | Two-dimensional geometry | 4.2 | 3.3 | 3.5 | 6.9 | 4.3 | 3.4 | 1.9 |
|  | Three-dimensional geometry | 2.2 | 1.7 | 2.0 | $\ddagger$ | 3.1 | 1.6 | 2.0 |
|  | Statistics | 2.6 | 1.7 | 1.3 | $\ddagger$ | 3.0 | 0.9 | 2.1 |
|  | Algebra | 4.8 | 4.7 | 4.7 | 4.6 | 6.2 | 3.7 | 5.3 |
|  | Linear expressions | 2.9 | 2.9 | 2.3 | $\ddagger$ | 2.5 | 1.5 | 1.6 |
|  | Solutions and graphs of linear equations and inequalities | 3.9 | 4.0 | 3.9 | 4.6 | 5.3 | 2.9 | 5.0 |
|  | Higher-order functions | $\ddagger$ | 2.1 | 2.3 | $\ddagger$ | 1.5 | 1.2 | 2.5 |
|  | Trigonometry | $\ddagger$ | $\ddagger$ | 3.2 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ |
|  | Other | $\ddagger$ | 0.3 | $\ddagger$ | $\ddagger$ | $\ddagger$ | 0.5 | 0.8 |
| Figure 4.1 | Low complexity problems | 3.7 | 2.9 | 3.8 | 3.3 | 3.9 | 3.1 | 5.0 |
|  | Moderate complexity problems | 2.7 | 2.5 | 3.3 | 6.3 | 3.1 | 2.5 | 3.8 |
|  | High complexity problems | 2.6 | 2.4 | 1.8 | 7.2 | 2.6 | 2.1 | 2.4 |

Standard errors for estimates shown in figures and tables, by country ${ }^{1}$-Continued

| Table/Figure | Category | $A U$ | CZ | HK | JP2 | NL | SW | US |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Figure 4.2 | Low complexity 2-D geometry problems | 9.4 | 7.2 | 8.2 | 4.2 | 9.7 | 6.7 | 20.4 |
|  | Moderate complexity 2-D geometry problems | 8.9 | 6.9 | 8.0 | 4.9 | 7.8 | 6.1 | 23.4 |
|  | High complexity 2-D geometry problems | 6.0 | 8.0 | 2.8 | 6.0 | 7.9 | 4.9 | 13.2 |
| Figure 4.3 | Proof - percent of problems | $\ddagger$ | 0.4 | 0.5 | 6.6 | $\ddagger$ | 1.2 | $\ddagger$ |
| Figure 4.4 | Proof - at least one | $\ddagger$ | 1.7 | 3.2 | 7.1 | $\ddagger$ | 2.8 | $\ddagger$ |
| Figure 4.5 | Proof - percent of 2-D geometry problems | $\ddagger$ | 1.9 | 1.6 | 8.7 | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Figure 4.6 | Unrelated problems | 1.5 | 1.0 | 0.8 | $\ddagger$ | 0.5 | 0.7 | 2.6 |
|  | Repetition problems | 2.9 | 1.9 | 2.7 | 4.9 | 2.5 | 2.3 | 3.0 |
|  | Thematically related problems | 1.8 | 1.5 | 1.0 | 4.3 | 1.9 | 0.8 | 1.1 |
|  | Mathematically related problems | 1.8 | 1.4 | 2.3 | 5.4 | 2.3 | 2.0 | 2.4 |
| Table 4.2 | Unrelated - number of problems | 0.1 | 0.1 | 0.1 | $\ddagger$ | 0.1 | 0.1 | 0.3 |
| Figure 4.7 | Unrelated 2-D geometry problems | 8.8 | 5.5 | 4.0 | $\ddagger$ | 1.4 | 4.2 | 27.0 |
|  | Repetition 2-D geometry problems | 1.1 | 5.2 | 6.9 | 3.8 | 8.3 | 5.1 | 17.2 |
|  | Thematically related 2-D geometry problems | 7.4 | 5.1 | 4.1 | 5.8 | 9.7 | 2.1 | 15.7 |
|  | Mathematically related 2-D geometry problems | 4.4 | 4.1 | 5.2 | 6.3 | 3.2 | 4.9 | 4.2 |
| Figure 4.8 | Single topic | 5.7 | 4.9 | 4.7 | 4.6 | 6.0 | 5.2 | 4.1 |
| Figure 5.1 | Set up used mathematical language or symbols only | 3.2 | 2.6 | 2.5 | 2.1 | 4.7 | 3.8 | 4.9 |
|  | Set up contained a real life connection | 3.2 | 2.4 | 2.3 | 2.5 | 4.6 | 3.7 | 4.8 |
| Figure 5.2 | Graph | 2.3 | 1.4 | 2.1 | 3.7 | 3.0 | 1.7 | 3.1 |
|  | Table | 5.3 | 1.4 | 2.8 | 1.7 | 2.2 | 3.1 | 3.3 |
|  | Drawing or diagram | 5.1 | 3.1 | 4.0 | 5.9 | 4.4 | 4.0 | 5.2 |
| Figure 5.3 | Physical materials - problems | 3.6 | 2.4 | 1.1 | 6.1 | 1.1 | 2.6 | 2.3 |
| Figure 5.4 | Physical materials - 2-D geometry problems | 4.9 | 8.4 | 3.3 | 6.8 | 4.0 | 9.2 | $\ddagger$ |
| Figure 5.6 | Applications | 4.8 | 3.5 | 4.1 | 4.9 | 4.8 | 4.9 | 4.6 |
| Figure 5.7 | ```Target result presented publicly - concurrent problems Target result presented publicly - independent problems``` | 5.6 2.1 | 4.1 1.5 | 4.5 1.4 | 10.4 4.3 | 3.7 2.0 | 3.7 2.6 | 6.0 2.8 |


| Standard errors for estimates shown in figures and tables, by country ${ }^{1}$ - Continued |  |  |  |  |  |  |  |  |
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| Table/Figure | Category | AU | CZ | HK | JP2 | NL | sw | US |
| Table 5.1 | More than one solution method presented problems | 0.5 | 0.8 | 0.9 | 4.1 | 2.5 | 1.1 | 1.8 |
|  | More than one solution method presented at least one | 5.7 | 4.1 | 4.8 | 8.3 | 6.6 | 3.7 | 5.8 |
| Table 5.2 | Students had a choice of solution methods problems | 1.9 | 1.3 | 1.0 | 4.4 | $\ddagger$ | 1.7 | 2.7 |
|  | Students had a choice of solution methods at least one | 4.4 | 3.8 | 4.1 | 7.5 | $\ddagger$ | 4.0 | 0.1 |
| Table 5.3 | Examining methods - problems | 0.5 | 0.1 | 0.4 | 2.3 | $\ddagger$ | 1.2 | 0.5 |
|  | Examining methods - at least one | 0.3 | 0.2 | 0.4 | 0.6 | $\ddagger$ | 0.4 | 0.4 |
| Table 5.4 | Problem summary | 1.8 | 1.5 | 1.6 | 3.9 | 1.3 | 1.5 | 1.2 |
| Figure 5.8 | Using procedures problem statement | 5.3 | 3.1 | 2.7 | 8.3 | 5.7 | - | 3.3 |
|  | Stating concepts problem statement | 4.1 | 1.8 | 0.8 | 1.6 | 3.7 | - | 2.1 |
|  | Making connections problem statement | 2.7 | 2.5 | 2.6 | 8.3 | 5.1 | - | 2.9 |
| Figure 5.9 | Giving results only implementation | 5.6 | 2.4 | 2.3 | 1.3 | 2.0 | - | 3.1 |
|  | Using procedures implementation | 5.0 | 2.2 | 2.9 | 3.9 | 4.2 | - | 3.1 |
|  | Stating concepts implementation | 3.1 | 1.9 | 1.8 | 4.8 | 4.0 | - | 1.1 |
|  | Making connections implementation | 0.8 | 1.4 | 2.0 | 3.4 | 4.6 | - | 0.5 |
| Figure 5.10 | Using procedures problem statement and giving results only implementation | 7.3 | 2.6 | 2.5 | 2.0 | 2.7 | - | 4.0 |
|  | Using procedures problem statement and using procedures implementation | 6.8 | 2.5 | 2.9 | 7.2 | 4.5 | - | 3.8 |
|  | Using procedures problem statement and stating concepts implementation | 2.0 | 1.9 | 1.9 | 7.4 | 3.5 | - | 1.0 |
|  | Using procedures problem statement and making connections implementation | 0.7 | 1.3 | 1.7 | 4.1 | 4.5 | - | $\ddagger$ |
| Figure 5.11 | Stating concepts problem statement and giving results only implementation | 6.2 | 9.2 | 6.8 | $\ddagger$ | 6.2 | - | 7.8 |
|  | Stating concepts problem statement and stating concepts implementation | 6.0 | 7.2 | 6.8 | 0.0 | 6.9 | - | 5.9 |
|  | Stating concepts problem statement and making connections implementation | 1.1 | 4.2 | $\ddagger$ | $\ddagger$ | 4.8 | - | 8.6 |

Standard errors for estimates shown in figures and tables, by country ${ }^{1}$-Continued

| Standard errors for estimates shown in figures and tables, by country ${ }^{1}$-Continued |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Table/Figure | Category | AU | CZ | HK | JP2 | NL | SW | US |
| Figure 5.12 | Making connections problem statement and giving results only implementation | 6.3 | 3.1 | 2.1 | 1.9 | 2.5 | - | 6.4 |
|  | Making connections problem statement and using procedures implementation | 7.9 | 5.2 | 5.7 | 5.3 | 5.4 | - | 6.4 |
|  | Making connections problem statement and stating concepts implementation | 6.5 | 4.4 | 5.4 | 4.1 | 7.9 | - | 4.8 |
|  | Making connections problem statement and making connections implementation | 4.9 | 5.6 | 6.2 | 4.2 | 7.7 | - | $\ddagger$ |
| Figure 5.13 | Repeating procedures assignment | 5.3 | 3.8 | 3.4 | 3.7 | 5.2 | 3.8 | 4.1 |
|  | Other than repeating procedures assignment or mix | 4.2 | 3.8 | 3.2 | 2.4 | 3.5 | 3.4 | 2.6 |
| Table 5.5 | Mathematical information | 4.4 | 2.8 | 3.6 | 2.1 | 7.7 | 4.1 | 4.2 |
|  | Contextual information | 5.7 | 2.6 | 3.8 | 2.6 | 7.0 | 3.8 | 4.7 |
|  | Mathematical activity | 2.8 | 1.7 | 1.0 | 3.0 | 1.0 | 0.9 | 4.1 |
|  | Announcements | 4.1 | 2.2 | 2.8 | 3.0 | 6.9 | 3.6 | 4.2 |
| Figure 5.14 | Student words per 50 minutes of public interaction | 55.4 | 35.4 | 59.1 | 89.9 | 85.9 | - | 71.7 |
|  | Teacher words per 50 minutes of public interaction | 143.2 | 98.9 | 143.2 | 99.4 | 166.0 | - | 166.4 |
| Figure 5.15 | Teacher words to every one student word | 0.8 | 0.6 | 1.4 | 2.1 | 1.7 | - | 0.6 |
| Figure 5.16 | Teacher utterances that were $25+$ words | 1.5 | 1.3 | 1.2 | 2.3 | 1.5 | - | 1.6 |
|  | Teacher utterances that were 5+ words | 1.0 | 0.7 | 0.7 | 2.1 | 1.2 | - | 0.8 |
|  | Teacher utterances that were 1-4 words | 0.9 | 0.7 | 0.7 | 2.1 | 1.1 | - | 0.8 |
| Figure 5.17 | Student utterances that were 10+ words | 0.6 | 0.6 | 0.5 | 0.7 | 0.7 | - | 0.6 |
|  | Student utterances that were 5+ words | 1.3 | 1.2 | 1.1 | 1.5 | 1.1 | - | 1.0 |
|  | Student utterances that were 1-4 words | 1.3 | 1.2 | 1.1 | 1.5 | 1.1 | - | 1.0 |
| Table 5.6 | Chalkboard | 1.8 | 0.0 | 1.8 | 1.9 | 2.0 | 3.8 | 5.7 |
|  | Projector | 5.0 | 4.5 | 3.2 | 2.7 | 1.8 | 4.1 | 6.3 |
|  | Textbook or worksheet | 3.0 | 0.0 | 1.4 | 3.2 | 0.0 | 2.3 | 1.5 |
|  | Special mathematics materials | 5.9 | 5.1 | 4.6 | 2.8 | 4.5 | 4.0 | 5.2 |
|  | Real world objects | 4.8 | 3.4 | 1.8 | 6.3 | 3.1 | 4.0 | 3.8 |
| Figure 5.18 | Computational calculators | 7.0 | 4.7 | 5.0 | $\ddagger$ | 2.8 | 3.4 | 5.5 |


| Standard errors for estimates shown in figures and tables, by country ${ }^{1}$ - Continued |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Table/Figure | Category | AU | CZ | HK | JP2 | NL | SW | US |
| Figure D. 1 | Elementary | 6.9 | 0.0 | 0.0 | - | 6.9 | 8.2 | 8.2 |
|  | Elementary/moderate | 11.4 | 8.2 | 5.0 | - | 9.2 | 8.2 | 9.9 |
|  | Moderate | 10.5 | 9.2 | 11.2 | - | 11.2 | 10.5 | 11.2 |
|  | Moderate/advanced | 8.2 | 11.4 | 10.9 | - | 10.5 | 10.9 | 9.2 |
|  | Advanced | 0.0 | 9.2 | 9.2 | - | 0.0 | 5.0 | 0.0 |
| Table D. 1 | Conceptual | 11.4 | 11.2 | 11.5 | - | 11.2 | 9.9 | 11.4 |
|  | Procedural | 0.0 | 0.0 | 0.0 | - | 0.0 | 0.0 | 5.0 |
|  | Notational | 10.9 | 11.2 | 11.4 | - | 11.4 | 11.4 | 11.4 |
| Figure D. 2 | Deductive reasoning | 0.0 | 5.0 | 8.2 | - | 5.0 | 6.9 | 6.9 |
| Figure D. 3 | Development of a rationale | 9.9 | 6.9 | 9.2 | - | 6.9 | 9.9 | 0.0 |
| Figure D. 4 | Generalizations | 6.9 | 6.9 | 6.9 | - | 9.2 | 9.2 | 0.0 |
| Figure D. 5 | Fragmented | 0.0 | 5.0 | 0.0 | - | 5.0 | 0.0 | 0.0 |
|  | Moderately fragmented | 8.2 | 8.2 | 0.0 | - | 6.9 | 6.9 | 10.9 |
|  | Mixed | 6.9 | 10.5 | 0.0 | - | 9.2 | 9.2 | 8.2 |
|  | Moderately thematic | 8.2 | 9.2 | 6.9 | - | 6.9 | 5.0 | 9.2 |
|  | Thematic | 11.2 | 10.5 | 6.9 | - | 11.4 | 10.9 | 10.5 |
| Figure D. 6 | Undeveloped | 6.9 | 0.0 | 0.0 | - | 6.9 | 8.2 | 11.2 |
|  | Partially developed | 9.2 | 9.2 | 6.9 | - | 10.9 | 5.0 | 9.2 |
|  | Moderately developed | 10.5 | 11.2 | 8.2 | - | 10.5 | 9.2 | 6.9 |
|  | Substantially developed | 11.2 | 10.5 | 11.4 | - | 9.2 | 11.4 | 9.9 |
|  | Fully developed | 0.0 | 6.9 | 9.2 | - | 5.0 | 8.2 | 5.0 |
| Figure D. 7 | Very unlikely | 0.0 | 5.0 | 0.0 | - | 6.9 | 6.9 | 8.2 |
|  | Doubtful | 10.5 | 6.9 | 5.0 | - | 10.5 | 6.9 | 11.4 |
|  | Possible | 10.5 | 9.2 | 10.5 | - | 10.5 | 10.5 | 9.9 |
|  | Probable | 10.5 | 11.4 | 10.5 | - | 9.2 | 11.2 | 8.2 |
|  | Very likely | 6.9 | 6.9 | 10.9 | - | 6.9 | 6.9 | 0.0 |
| Figure D. 8 | Low | 8.2 | 5.0 | 0.0 | - | 9.9 | 6.9 | 11.2 |
|  | Moderately low | 9.2 | 9.2 | 6.9 | - | 8.2 | 8.2 | 8.2 |
|  | Moderate | 10.5 | 9.2 | 8.2 | - | 10.9 | 9.9 | 9.2 |
|  | Moderately high | 10.5 | 11.2 | 11.4 | - | 9.2 | 10.9 | 9.9 |
|  | High | 5.0 | 8.2 | 10.5 | - | 5.0 | 8.2 | 0.0 |

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| Standard errors for estimates shown in figures and tables, by country ${ }^{1}$-Continued |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Table/Figure | Category | AU | CZ | HK | JP2 | NL | SW | US |
| Figure D. 9 | Coherence | 0.3 | 0.3 | 0.1 | - | 0.3 | 0.3 | 0.3 |
|  | Presentation | 0.2 | 0.2 | 0.2 | - | 0.2 | 0.3 | 0.3 |
|  | Student engagement | 0.2 | 0.2 | 0.2 | - | 0.3 | 0.3 | 0.2 |
|  | Overall quality | 0.3 | 0.3 | 0.2 | - | 0.3 | 0.3 | 0.3 |

[^13]
## APPENDIX D Results From the Mathematics Quality Analysis Group

\|udgments about the mathematics content of the lessons were made by the mathematics quality analysis group, a team of mathematicians and teachers of post-secondary mathematics (see appendix A for a description of this group). The mathematics quality analysis group developed a coding scheme that focused on the content of the lessons and then applied the scheme by reaching consensus on each judgment. The members of this group examined country-blind written records of 20 lessons selected randomly from each country's sample (see appendix A). Japan was not included because the same group already had analyzed Japanese lessons as part of the TIMSS 1995 Video Study which meant, among other things, that potential country bias could not be adequately reduced (see Stigler et al. [1999] and Manaster [1998] for a report of the group's findings in the 1995 study).

The findings of the mathematics quality analysis group are based on a relatively small, randomly selected sub-sample of lessons and, consequently, are considered preliminary. Because the results are based on a sub-sample, the results are raw percentages rather than weighted percentages. The percentages and ratings shown in the figures are descriptive only; no statistical comparisons were made. Readers are urged to be cautious in their interpretations of these results because the subsample, due to its relatively small size, might not be representative of the entire sample or of eighth-grade mathematics lessons in each country.

## Curricular Level of the Content

The data shown in chapter 4, table 4.1, display the relative emphasis given to different topics, on average, across the full sample in each country. These percentages provide one estimate of the level of content. Another estimate can be obtained by asking experts in the field to review the lessons for the curricular level of the content.

One of the codes developed by the mathematics quality analysis group placed each lesson in the sub-sample into one of five curricular levels, from elementary (1) to advanced (5). The moderate or mid level (3) was defined to include content that usually is encountered by students just prior to the standard topics of a beginning algebra course that often is taught in the eighth grade. One rating was assigned to each lesson based on the rating that best described the content of the lesson, taken as a whole.

Figure D. 1 shows the percentage of eighth-grade mathematics lessons assigned to each rating. Because these analyses were limited to a subset of the total sample of lessons, the percentages were not compared statistically and the results should be interpreted with caution. This same cautionary note applies to all of the findings of the mathematics quality analysis group and is noted on each of the figures that present the findings of this group. In figure D.1, as in all figures in this special section, percentages indicate the number of lessons in the sub-sample that contain a particular feature. In other words, 100 percent and 0 percent are indications that all or none of the sub-sample lessons contained a feature, and are not meant to imply that all or no lessons in the country contain the feature.

FIGURE D.1. Percentage of eighth-grade mathematics lessons in sub-sample at each content level, by country: 1999

${ }^{1}$ AU=Australia; CZ=Czech Republic; HK=Hong Kong SAR; NL=Netherlands; SW=Switzerland; and US=United States. NOTE: Lessons included here are a random sub-sample of lessons in each country. Results should be interpreted with caution because they might not be representative of the entire sample. The number in the parentheses is the ranking number for that category. A moderate ranking was defined to include content that usually is encountered by students just prior to the standard topics of a beginning algebra course that is often taught in the eighth grade.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

Averaging the content level ratings of each country's sub-sample of lessons gives a summary rating for each country. Additional caution is needed when interpreting the summary ratings because, for example, an elementary lesson and an advanced lesson are unlikely to average to the same experience for students as two moderate lessons. With this caveat in mind, the ratings for countries with the most advanced (5) to the most elementary (1) content in the sub-sample of lessons, were the Czech Republic and Hong Kong SAR (3.7), Switzerland (3.0), the Netherlands (2.9), the United States (2.7), and Australia (2.5) (see also figure D.9).

## Nature of the Content

The distinction between different kinds of mathematical knowledge has been used frequently by researchers to describe different kinds of mathematics learning and to describe the outcomes of different kinds of learning environments (Hiebert 1986). Common distinctions separate knowledge of concepts, procedures, and written notation or definitions.

The mathematics quality analysis group characterized the mathematics presented in the subsample of lessons as conceptual, procedural, or notational. Conceptual mathematics was defined as the development of mathematical ideas or procedures. Segments of conceptual mathematics might include examples and explanations for why things work like they do. Procedural mathematics was defined as the presentation of mathematical procedures without much explanation, or the practice of procedures that appeared to be known already by the students.

Often, the development and first application of a solution procedure was coded as conceptual whereas subsequent applications of the method were coded as procedural. The notational code was used when the presentation or discussion centered on mathematical definitions or notational conventions.

Table D. 1 shows the percentage of eighth-grade mathematics lessons that contained segments of conceptual, procedural, and notational mathematics. Because these analyses were limited to a subset of the total sample of lessons, the percentages were not compared statistically and the results should be interpreted with caution.

TABLE D.1. Percentage of eighth-grade mathematics lessons in sub-sample that contained segments of conceptual, procedural, and notational mathematics, by country: 1999

|  | Percentage of sub-sampled lessons that contained segments <br> of the following types of mathematics: |  |
| :--- | :---: | :---: |
| Country | Conceptual | Procedural |
| Australia | 55 | 100 |
| Czech Republic | 60 | 100 |
| Hong Kong SAR | 50 | 100 |
| Netherlands | 40 | 100 |
| Switzerland | 75 | 100 |
| United States | 45 | 95 |

NOTE: Lessons included here are a random sub-sample of lessons in each country. Results should be interpreted with caution because they might not be representative of the entire sample.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

Almost all lessons in the sub-samples for every country contained segments that were coded procedural. Between 35 percent and 55 percent of the lessons in the sub-samples contained segments of notational mathematics and between 40 percent and 75 percent of the lessons in the sub-samples contained segments of conceptual mathematics.

## Mathematical Reasoning

As noted in chapter 4, one hallmark of doing mathematics is engaging in special forms of reasoning, such as deduction (deriving conclusions from stated assumptions using a logical chain of inferences). Other forms of mathematical reasoning include generalization (recognizing that several examples share more general properties) and using counter-examples (finding one example that does not work to prove that a mathematical conjecture cannot be true). These special reasoning processes provide one way to distinguish mathematics from other disciplines (National Research Council 2001a; Whitehead 1948).

Because the findings from the TIMSS 1995 Video Study indicated that not all countries provide eighth-graders opportunities to engage in deductive reasoning (Stigler et al. 1999; Manaster 1998), the mathematics quality analysis group expanded the mathematical reasoning coding scheme it had used for the 1995 Video Study in an attempt to identify special reasoning forms that might be present in eighth-grade mathematics lessons.

Figure D. 2 shows the results of applying the group's definition of deductive reasoning to the subsample of eighth-grade mathematics lessons. Such reasoning could occur as part of problem or non-problem segments. The reasoning did not need to include a formal proof, only a logical chain of inferences with some explanation.

The percentage of eighth-grade mathematics lessons in the sub-sample that contained deductive reasoning by the teacher or students is shown in figure D.2. Because these analyses were limited to a subset of the total sample of lessons, the percentages were not compared statistically and the results should be interpreted with caution.

FIGURE D.2. Percentage of eighth-grade mathematics lessons in sub-sample that contained deductive reasoning, by country: 1999


[^14]A maximum of 15 percent of the sub-samples of lessons in any country contained instances of deductive reasoning. As noted earlier, Japanese lessons were not included in these analyses because the mathematics quality analysis group had examined a sub-sample of Japanese lessons for the TIMSS 1995 Video Study.

Deductive reasoning is not the only special form of mathematical reasoning. The mathematics quality analysis group coded the previously described sub-sample of 20 lessons in each country (except Japan) for other special kinds of mathematical reasoning in which eighth-graders seem to be capable of engaging (National Research Council 2001a). "Developing a rationale" was defined by the mathematics quality analysis group as explaining or motivating, in broad mathematical terms, a mathematical assertion or procedure. For example, teachers might show that the rules for adding and subtracting integers are logical extensions of those for adding and subtracting whole numbers, and that these more general rules work for all numbers. When such explanations took a systematic logical form, they were coded as deductive reasoning (see figure D.2); when they took a less systematic or precise form, they were coded as developing a rationale.

Figure D. 3 shows that a maximum of 25 percent of the eighth-grade mathematics lessons in any country's sub-sample included instances of developing a rationale. As before, these analyses were limited to a subset of the total sample of lessons so the percentages were not compared statistically and the results should be interpreted with caution.

FIGURE D.3. Percentage of eighth-grade mathematics lessons in sub-sample that contained the development of a rationale, by country: 1999

${ }^{1}$ AU=Australia; CZ=Czech Republic; HK=Hong Kong SAR; NL= Netherlands; SW=Switzerland; and US=United States. NOTE: Lessons included here are a random sub-sample of lessons in each country. Results should be interpreted with caution because they might not be representative of the entire sample.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

The mathematics quality analysis group also examined the sub-sample of lessons to determine the occurrence of two other forms of reasoning: generalization and counter-example. Generalization might involve, for example, graphing several linear equations such as $y=2 x+3$,
$2 y=x-2$, and $y=-4 x$, and making an assertion about the role played by the numbers in these equations in determining the position and slope of the associated lines. Generalization, then, involves inducing general properties or principles from several examples.

As shown in figure D.4, generalizations occurred in a maximum of 20 percent of the eighthgrade mathematics lessons in any country. Because these analyses were limited to a subset of the total sample of lessons, the percentages were not compared statistically and the results should be interpreted with caution.

FIGURE D.4. Percentage of eighth-grade mathematics lessons in sub-sample that contained generalizations, by country: 1999

${ }^{1}$ AU=Australia; CZ=Czech Republic; HK=Hong Kong SAR; NL= Netherlands; SW=Switzerland; and US=United States. NOTE: Lessons included here are a random sub-sample of lessons in each country. Results should be interpreted with caution because they might not be representative of the entire sample.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

A final kind of special mathematical reasoning-using a counter-example-involves finding an example to show that an assertion cannot be true. For instance, suppose someone claims that the area of a rectangle gets larger whenever the perimeter gets larger. A counter-example would be a rectangle whose perimeter becomes larger but the area does not become larger.

The mathematics quality analysis group found that, in the sub-sample of eighth-grade mathematics lessons, demonstrating that a conjecture cannot be true by showing a counter-example occurred in 10 percent of the lessons in Australia and 5 percent of the lessons in Hong Kong SAR. The other countries showed no evidence of counter-example use.

## Overall Judgments of Mathematical Quality

The mathematics quality analysis group judged the overall quality of the mathematics in the sub-sample of lessons along several dimensions: coherence, presentation, student engagement, and overall quality. Each lesson was rated from 1 (low) to 5 (high) on each dimension. Whereas
most of the group's codes reported to this point marked the occurrence of particular features of a lesson, the group's overall judgments of quality considered each lesson as a whole. As stated earlier, country-identifying marks had been removed from the written records to mask the country from which the lessons came. Recall also that Japanese lessons were not included in the group's sub-sample.

Coherence was defined by the group as the (implicit and explicit) interrelation of all mathematical components of the lesson. A rating of 1 indicated a lesson with multiple unrelated themes or topics and a rating of 5 indicated a lesson with a central theme that progressed saliently through the whole lesson.

Figure D. 5 shows the percentage of eighth-grade mathematics lessons in the sub-sample assigned to each level of coherence. Averaging across all the lessons within each country's sub-sample yields the following general ratings of countries based on lesson coherence: Hong Kong SAR (4.9), Switzerland (4.3), Australia (4.2), the Netherlands (4.0), the Czech Republic (3.6), and the United States (3.5). Because these analyses were limited to a subset of the total sample of lessons, the ratings were not compared statistically and the results should be interpreted with caution.

FIGURE D.5. Percentage of eighth-grade mathematics lessons in sub-sample rated at each level of coherence, by country: 1999

${ }^{1}$ AU=Australia; CZ=Czech Republic; HK=Hong Kong SAR; NL=Netherlands; SW=Switzerland; and US=United States.
NOTE: Lessons included here are a random sub-sample of lessons in each country. Results should be interpreted with caution because they might not be representative of the entire sample. The number in the parentheses is the ranking number for that category. For Hong Kong SAR, no lessons in the sub-sample were found to be mixed, moderately fragmented, or fragmented.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

It is worth pausing here to make an observation about the results related to lesson coherence. The Czech Republic makes an interesting case. The full sample of eighth-grade mathematics lessons from the Czech Republic contained, on average, a higher percentage of unrelated problems per lesson than Hong Kong SAR, the Netherlands, and Switzerland (see figure 4.6 in chapter 4), and the mathematics quality analysis group judged that 50 percent of their sub-sample of Czech
lessons contained at least moderately fragmented portions (figure D.5). But recall the findings in chapter 3, which showed that the lessons in the Czech Republic displayed relatively high profiles of pedagogical coherence (e.g., lesson goal and summary statements, and interruptions to lessons), compared with some of the other countries (figures 3.12, 3.13, 3.15, and 3.16). This suggests that there are several dimensions of lesson coherence and that they are not necessarily interdependent.

Another characteristic of overall quality defined by the mathematics quality analysis group was presentation-the extent to which the lesson included some development of the mathematical concepts or procedures. Development required that mathematical reasons or justifications were given for the mathematical results presented or used. This might be done, for example, by the teacher drawing clear connections between what was known and familiar to the students and what was unknown. Presentation ratings took into account the quality of mathematical arguments. Higher ratings meant that sound mathematical reasons were provided by the teacher (or students) for concepts and procedures. Mathematical errors made by the teacher reduced the ratings. A rating of 1 indicated a lesson that was descriptive or routinely algorithmic with little mathematical justification provided by the teacher or students for why things work like they do. A rating of 5 indicated a lesson in which the concepts and procedures were mathematically motivated, supported, and justified by the teacher or students.

Figure D. 6 shows the percentage of eighth-grade mathematics lessons in the sub-sample assigned to each level of presentation. Averaging the ratings of all the lessons within each country's subsample yields the following general ratings of countries based on presentation: Hong Kong SAR (3.9), Switzerland (3.4), the Czech Republic (3.3), Australia (3.0), the Netherlands (2.8), and the United States (2.4). Because these analyses were limited to a subset of the total sample of lessons, the ratings were not compared statistically and the results should be interpreted with caution.

## FIGURE D.6. Percentage of eighth-grade mathematics lessons in sub-sample rated at each level of presenta-

 tion, by country: 1999
${ }^{1}$ AU=Australia; CZ=Czech Republic; HK=Hong Kong SAR; NL=Netherlands; SW=Switzerland; and US=United States. NOTE: Lessons included here are a random sub-sample of lessons in each country. Results should be interpreted with caution because they might not be representative of the entire sample. The number in the parentheses is the ranking number for that category. SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

Student engagement was defined by the mathematics quality analysis group as the likelihood that students would be actively engaged in meaningful mathematics during the lesson. A rating of very unlikely (1) indicated a lesson in which students were asked to work on few of the problems in the lesson and those problems did not appear to stimulate reflection on mathematical concepts or procedures. In contrast, a rating of very likely (5) indicated a lesson in which students were expected to work actively on, and make progress solving, problems that appeared to raise interesting mathematical questions for them and then to discuss their solutions with the class.

Figure D. 7 shows the percentage of eighth-grade mathematics lessons in the sub-sample assigned to each level of student engagement. Averaging across all the lessons within each country's subsample yields the following general ratings of countries based on student engagement: Hong Kong SAR (4.0), the Czech Republic (3.6), Switzerland (3.3), Australia (3.2), the Netherlands (2.9), and the United States (2.4). Because these analyses were limited to a subset of the total sample of lessons, the ratings were not compared statistically and the results should be interpreted with caution.

FIGURE D.7. Percentage of eighth-grade mathematics lessons in sub-sample rated at each level of student engagement, by country: 1999

${ }^{1}$ AU=Australia; CZ=Czech Republic; HK=Hong Kong SAR; NL=Netherlands; SW=Switzerland; and US=United States. NOTE: Lessons included here are a random sub-sample of lessons in each country. Results should be interpreted with caution because they might not be representative of the entire sample. The number in the parentheses is the ranking number for that category. SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

The mathematics quality analysis group made a final overall judgment about the quality of mathematics in each lesson in their sub-sample. This overall quality judgment took into account all previous codes and was defined as the opportunities that the lesson provided for students to construct important mathematical understandings. Ratings ranged from 1 for low to 5 for high.

Figure D. 8 shows the percentage of eighth-grade mathematics lessons in the sub-sample assigned to each level of overall quality. Averaging across all the lessons within each country's sub-sample yields the following general rating of countries based on overall quality of the mathematics presented: Hong Kong SAR (4.0), the Czech Republic (3.4), Switzerland (3.3), Australia (2.9), the Netherlands (2.7), and the United States (2.3). Because these analyses were limited to a subset of the total sample of lessons, the ratings were not compared statistically and the results should be interpreted with caution.

FIGURE D.8. Percentage of eighth-grade mathematics lessons in sub-sample rated at each level of overall quality, by country: 1999

${ }^{1}$ AU=Australia; CZ=Czech Republic; HK=Hong Kong SAR; NL=Netherlands; SW=Switzerland; and US=United States.
NOTE: Lessons included here are a random sub-sample of lessons in each country. Results should be interpreted with caution because they might not be representative of the entire sample. The number in the parentheses is the ranking number for that category. SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

A summary display of the overall judgments of the mathematics quality analysis group is found in figure D.9. The general ratings reported above for the sub-sample of lessons of each country for coherence, presentation, student engagement, and overall mathematics quality are plotted on the same figure. As the figure shows, the relative standing of Hong Kong SAR was consistently high and the relative standing of the United States was consistently low. The other four countries received general ratings that fell in between and that varied depending on the dimension examined. Again, these ratings were based on a sub-sample of lessons and, therefore, might not be representative of the entire sample and of eighth-grade mathematics lessons in each country.
 sion of content quality, by country: 1999


NOTE: Lessons included here are a random sub-sample of lessons in each country. Results should be interpreted with caution as they may not be representative of the entire sample. Rating along each dimension based on scale of 1 to 5 , with 1 being the lowest possible rating and 5 being the highest.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

## Summary

It is difficult to draw conclusions about similarities and differences among countries from the findings just presented because of their descriptive and exploratory character. Two points are worth noting, however.

A first point is that where there is overlap between the variables defined by the mathematics quality analysis group and those described in the chapters of this report, the findings are not inconsistent with each other. For example, in chapter 4 it was reported that a relatively small percentage of mathematics problems (and lessons) in countries other than Japan involved proofs. The mathematics quality analysis group found similarly infrequent instances of special mathematical reasoning, including deductive reasoning.

A second point is that the findings reported in this appendix could be considered hypotheses worthy of further examination. Because the quality of mathematical content is theoretically an important contributor to the learning opportunities for students (National Research Council 2001a), and because the mathematics quality analysis group developed a series of high-inference codes for evaluating the quality of content, it is likely that an application of the codes to the full sample of TIMSS 1999 Video Study lessons would add to the findings presented in chapter 4. In addition, the results presented in this appendix identify constructs of mathematical content that would benefit from development and further application in other studies that aim to describe the quality of content in mathematics lessons.

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## APPENDIX E Hypothesized Country Models

Hypotheses were developed about specific instructional patterns that might be found in eighthgrade mathematics classrooms in each country. ${ }^{1}$ The process began by considering the four or five field test videos collected in each country-eighth-grade mathematics lessons that provided an initial opportunity to observe teaching in the different countries in the sample. ${ }^{2}$ An international group of representatives (i.e., the field test team) met together for an entire summer, viewing and reflecting on these tapes. They followed a structured protocol to generate hypotheses that could later be tested by quantitative analyses of the full data set. First, the country representatives closely examined field test lessons from their own country, and nominated the one that was "most typical." Then, the entire group viewed and discussed each typical lesson at length, noting in particular the similarities and differences among countries. These discussions provided consensus that six dimensions framed mathematics classroom practice and were of interest across countries and lessons: Purpose, Classroom Routine, Actions of Participants, Content, Classroom Talk, and Climate. These dimensions were then used to create hypothesized country models-holistic representations of a "typical" mathematics lesson in each country.

The hypothesized country models were presented to National Research Coordinators, the Mathematics Steering Committee, and other colleagues in each country including eighth-grade mathematics teachers and educators, and refined over a period of several months. The goal was to retain an "insider perspective," and faithfully represent in the coding system the critical features of eighth-grade mathematics teaching in each country. The hypothesized country models served two purposes toward this end. First, the models provided a basis on which to identify key, universal variables for quantitative coding. Second, they described a larger context that might be useful in interpreting the coding results. The hypothesized country models are presented in this appendix.

TABLE E.1. Key to symbols and acronyms used in hypothesized models

| Symbol/acronym | Meaning |
| :--- | ---: |
| T | Teacher |
| S | Student |
| SS | Students |
| HW | Homework |
| BB | Blackboard |
| $\\|:\\|$ | Segment |
| $\\|:$ | may repeat |

[^15]FIGURE E.1. Hypothesized country model for Australia

| Purpose | Review | Introduction of | Assignment of task | Practice/application and re-instruction |  |  | Conclusion |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Reinforce knowledge; check/ correct/review homework; reinstruct | Acquisition of knowledge | Assignment of task | Practice/ application | Reassignment of task | Practice/ application | Reinforce knowledge |
|  |  |  |  | Application of knowledge | Assignment of task | Application of knowledge |  |
| Classroom routine | review of relevant material previously worked on | presentation of new material | assignment of task | completion of task | assignment of task | completion of task | summary of new material; assignment of homework |
| Actions of participants | T- [at front] ask Ss questions; elicit/embellish responses; demonstrate examples on BB | T-[at front] provides information asking some Ss questions and using examples on BB | ```T - [at front] describes textbook/work- sheet task``` | T-[roams room] provides assistance to Ss as needed and observes Ss progress on set task | $\begin{aligned} & \mathrm{T}-\text { [at front] } \\ & \text { re-explains } \\ & \text { textbook/work- } \\ & \text { sheet task } \end{aligned}$ | T-[roams room] provides assistance to Ss as needed and observes Ss progress on set task | T- [at front] provides information and asks Ss questions |
|  | Ss - [in seats] respond to and ask T questions; listen to T explanations, watch demonstrations | Ss - [in seats] listen to T explanations and respond to T questions | Ss - [in seats] listen to T descriptions | Ss - [in seats] work individually or in pairs on task | Ss - [in seats] listen to T descriptions | Ss - [in seats] work individually or in pairs on task | Ss - [in seats] listen to T descriptions; respond to and ask T questions |
| Content | related to previous lesson | definitions/examples building on ideas previously worked on | description of task; focus on textbook/worksheet problems | textbook/worksheet problems | description of task; focus on textbook/ worksheet problems | textbook/worksheet problems | textbook/worksheet problems; homework problems |
| Classroom talk | T talks most; Ss one-word responses | Mix of T/S talk although discussion clearly T directed | T provides direct instructions | mix of $T / S$ and $S / S$ talk - including explanations and questions | T provides direct instructions | mix of $T / S$ and $S / S$ talk - including explanations and questions | mix of $T$ and $T / S s$ talk - including explanations and questions |
| Climate | somewhat informal - relaxed yet focused |  |  |  |  |  |  |

SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

| FIGURE E.2. Hypothesized country model for the Czech Republic |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Purpose | Review |  |  | Constructing new knowledge |  |  | Practice |  |
|  | Evaluating | Securing old knowledge | Re-instruction | Activating old knowledge | Constructing new topics | Formulating the new information | Practice "work-ing-through" | Using knowledge in different problems |
| Classroom routine | oral exam test homework | set of problems; homework | dialogue | Experiment, solving problems, demonstration, dialogue | dialogue |  | dialogue solving problems | solving problems |
| Actions of participants | T-giving grade | T-gives individual help | T-explaining procedure |  |  | T-writing notes at the board |  |  |
|  | Ss - solving problems at the board | Ss - at the board |  | Ss - answering questions, solving problems at the board |  |  | Ss - solving problem; more then one student solving one problem |  |
| Classroom talk | answering questions; fast pace |  | T talk most | T-S dialogue | teacher talks most of the time; slow pace | teacher talks most of the time |  | more mathematically open questions |
| Content | content probably from unit |  |  | special problems prepared in special order, solutions are very visible, strong connection with new topics | step-by-step solving problem, solutions very visible | mathematical <br> statements and <br> definitions; <br> something new <br> that students <br> don't know |  | stronger connection with real life |
| Climate | few mistakes allowed students very quiet serious atmosphere | mistakes are not graded but not expected, students talk loudly |  |  |  |  | more mistakes allowed |  |

SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.
FIGURE E.3. Hypothesized country model for Hong Kong SAR

| Purpose | Review | Instruction | Consolidation |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | To review material learned in the past <br> To prepare for the present lesson | To introduce and explain new concepts and/or skills | To practice the skills learned |  |  |
| Classroom routine | T-goes over relevant material learned in the past, sometimes through asking Ss questions | T-introduces a new topic <br> T - explains the new concepts/skills <br> T-shows one or more worked examples | Seatwork <br> T - assigns seatwork <br> Ss - work on seatwork <br> T - helps individual Ss | Evaluation <br> T - asks some Ss to work on the board T-discusses the work on the board with Ss | Homework <br> T-assigns homework <br> Ss - start doing homework |
| Actions of participants | T-talks at the blackboard Ss - listen in their seats Ss - answer questions from their seats | T - explains at the blackboard <br> T - works examples on the blackboard <br> Ss - listen and/or copy notes at their seats | T-talks at blackboard Ss - listen and then work in their seats T-walks around the class | Some Ss work on the board T-discusses Ss' work on the board Ss - listen in their seats | T-talks at blackboard <br> Ss - listen in their seats |
| Classroom talk | T- talks most of the time Pace relatively fast Convergent questions by T Conversation evaluation | T- talks most of the time Pace relatively slow Mostly convergent questions and some divergent questions Less evaluative | Some informal S talk (with each other) Pace relatively slow |  |  |
| Content | Usually low demand of the cognitive processes | Higher demand in the cognitive processes <br> Definitions/proofs/examples Heavy reliance on textbook | Medium demand on the cognitive processes Select exercises <br> Focus on procedures or skills |  |  |
| Climate | Serious <br> Relatively quiet <br> Mistakes less acceptable | Serious <br> Relatively quiet <br> Mistakes more acceptable | Less serious <br> Less quiet <br> Mistakes more acceptable |  |  |

SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.
FIGURE E.4. Hypothesized country model for the Netherlands

| Purpose | Re-instruction |  | Instruction |  | Assignment of task | Students attempt problems |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Classroom routine | Going over old assignment Nakijken |  | Presenting new material |  | Assignment of task | Student problem solving continued work on old assignment and/or initial efforts on new assignment |
| Actions of participants | Option 1 Completion of each problem as a class | Option 2 <br> T gives hints for selected problems | Option 1 T verbalizes | Option 2 Complete reliance on text | T-writes assignment on BB (or may give verbally) | If "Re-instruction" follows Option 2, Ss first work on the old assignment, then work on the new T - available to answer S-initiated questions <br> T - gives mostly procedural assistance <br> T - generally provides answers freely; doesn't require much S input <br> T - may give semi-public assistance (at front of room) or private assistance (at Ss desks) |
|  | T-goes through assignment, problem by problem at the front of the class, with or without use of the BB; Emphasis is on procedures | T - provides partial assistance (e.g., hints) on selected problems at the front of the class; T - provides answers on paper (e.g., answer sheet, access to T manual); Emphasis is on procedures | T-verbalizes text presentation and/or points to selected features of the text presentation | None |  |  |
|  | Ss - follow along at their desks, respond to T questions, ask clarifying questions | Ss - follow along at their desks; Very low S involvement | Ss - listen to T at their seats | Ss - read about new topic(s) from the text, at their desks | Ss - write assignment into their agendas | Ss - work in pairs at their desks and ask T for assistance when necessary, either at their desks or at the front of the room |
| Content | Small number of multi-part problems from the text (~5); Assignment given yesterday and worked on as HW; Generally one solution method provided |  | Heavy reliance on text; new material presented within the context of a task/problem |  | Small number of multi-part problems from the text (~5) to be continued tonight as HW; Ss only need to find one solution method (any one solution is okay) |  |
|  | Problems are in a real-world context (might be considered "application"), situations vary across tasks, T rarely solicits errors |  |  |  |  |  |
| Classroom talk | Option 1 | Option 2 | Option 1 | Option 2 | Direct instruction; T verbalizes the assignment as written on the BB | S-S talk regarding assignment; 1-on-1 (or 2- to 3-on-1) private, S -T conversations initiated by S , but then dominated by T |
|  | T asks Ss questions and rephrases Ss' responses | T briefly gives partial information on selected problems; Ss rarely ask questions Less S talk than in Option 1 | Direct instruction | None |  |  |
|  | Low level of evaluation/low concern for assessment |  |  |  |  |  |
| Climate |  | High level of $S$ freedom and responsibility |  | High level of S freedom and responsibility |  | Moderate level of noise is accepted by T |
|  | High error tolerance by the T T-S relationship is relaxed |  |  |  |  |  |

SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.
Hypothesized country models for Switzerland

| Purpose | Opening | Construction of new cognitive structure (Aufbau, Begriffsbildung) | Working-through (operatives Ueben) | Practice (automatization, rehearsal) Ueben |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Classroom routine | Collecting homework, informal talk | Interactive instruction T-presentation 'real action' T - modeling problem solving | Interactive instruction Problem solving | Ss writing or reading at their desks Interactive instruction |  |
| Actions of participants |  | T - asks questions and explains, demonstrates procedure, or states a problem... |  |  |  |
|  |  | Ss - answer questions, observe T, imitate, act, solve problems; work as a whole class | (See Notes) <br> Ss - work as a whole class | Ss - individual, group, or pair work |  |
| Content |  | New concept is introduced in a step-by-step fashion, starting from Ss previous knowledge and/or their everyday experience <br> Goal: Ss understand the concept (on their level of knowledge); <br> Usefulness of concept (for further learning, and as a tool for everyday practice) emphasized; <br> Visualization (Anschauung) is important; <br> New information is reinforced (presented at board or textbook in a standardized fashion) | Sequence of carefully selected tasks related to new topic <br> ("operatives Ueben") | Collection of tasks related to new topic |  |
| Content (relationship between tasks) |  | Relationship between tasks: no set (often: problem-like situation) | Relationship between tasks: Set 2 | Relationship between tasks: Set 1 | Relationship between tasks: Set 1, 2, ... |
| Classroom talk |  | Lehrgespraech (Interactive instruction; long waittime, Ss expected to actively participate in construction process) | Interactive instruction | T-S-dialogue |  |
| Climate |  | See Notes |  |  |  |

SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.
FIGURE E.6. Hypothesized classroom patterns of Swiss mathematics lessons without introduction of new knowledge

| Purpose | Opening | Working-through or practice goal: understanding and/or proficiency | Practice goal: understanding and/or proficiency | Re-instruction, sharing | Practice goal: understanding and/or proficiency | Re-instruction, sharing | Using knowledge in different situations/to solve different problems Anwenden |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Classroom routine | Collecting homework, informal talk | Interactive instruction | Ss solve tasks | Sharing and checking Ss' solutions (Besprechung) <br> - interactive instruction <br> - S presentation <br> - discussion | Ss solve tasks | Sharing and checking Ss' solutions (Besprechung) <br> - interactive instruction <br> - S presentation <br> - discussion | problem solving interactive instruction |
| Actions of participants |  | Classwork | Ss - individual, group, or pair work | Classwork | Ss - individual, group, or pair work | Classwork | Ss - individual, group, or pair work |
| Content |  | Topic: introduced in a previous lesson <br> T may start with short review of topic, and solving some examples of tasks |  |  | Progression to more demanding tasks, finally: to demanding application problems (possibly not in the same lesson, but later) |  | Character of tasks: Given new situations but connection to mathematical concepts is not obvious |
| Content (relationship between tasks) |  | Relationships between tasks: no set, or Set 1 or Set 2 | Relationships between tasks: Set 1 or Set 2 | Relationships between tasks: Set 1 or Set 2 | Relationships between tasks: Set 1 or Set 2 | Relationships between tasks: Set 1 or Set 2 | Relationships between tasks: Set 2 or no set |
| Classroom talk |  | Interactive instruction | T-S-dialogue, and/or S-S-conversation | Interactive instruction/discussion | T-S-dialogue, and/or S-S-conversation | Interactive instruction/discussion | T-S-dialogue, S-Sconversation, discussion... |
| Climate |  |  |  |  |  |  |  |

SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

Notes to hypothesized classroom patterns of Swiss mathematics lessons with introduction of new knowledge

Most frequently a new topic (concept) might be co-constructed by means of interactive instruction (Lehrgespraech). The means of guidance are primarily teacher questions and hints. The procedure is oriented toward the Socratic dialogue. The teacher questions serve two main purposes: (1) to guide and initiate students' thinking (e.g., propose a certain point of view, or perspective on a problem), and (2) to diagnose students' actual understanding. An important feature of quality of a Lehrgespraech is the need for sufficient wait-time after the teacher's questions.

The introduction phase may include some further actions that may be embedded in the interactive instruction, such as teacher presentation, or modeling or "real actions."

Reform 1:
In reform-oriented classrooms another pattern of introduction lessons might be expected: (1) student independent problem solving in pairs, groups, or individually (inventing procedures for solving new, open problems, discovering principles, regularities, and so on); (2) discussion of the different approaches and negotiating an accepted approach. This approach (influenced by scholars of mathematics didactics in Germany and the Netherlands) is presently recommended in teacher education and professional development. (It is unclear if this is observable at the eighthgrade level.)

Notes to hypothesized classroom patterns of Swiss mathematics lessons without introduction of new knowledge

As a general pattern an alternation between students solving tasks on their own and of sharing/checking/re-instruction based on students' work in a classwork sequence may be expected, but the duration of and total amount of the phases is not predictable.

The sequence of activity units varies, and does not always start with a classwork phase.
The first unit may provide some special kinds of tasks (warm-up, or a motivating starting task).
In most cases, the teacher will vary the social structure (e.g., classwork - individual work - classwork - pair work - and so on).

There is a progression from easier to more demanding tasks over the entire learning phase; usually the progression leads to application problems (most often, applied story problems).

Not all students always solve the same tasks (individualization of instruction).
Reform 2:
In some reform classrooms there will be no or almost no classwork phase and each student may be proceeding through a weekly assigned collection of learning tasks (arranged in collaboration with the teacher; individualized instruction). As with Reform 1, it is not clear if and how many teachers are in fact practicing this reform model of instruction (which is recommended in teacher development) at the eighth-grade level.
FIGURE E.7. Hypothesized country model for the United States

| FIGURE E.7. | Hypothesized country model for the United States |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Purpose | Review of previously learned material A |  |  | Acquisition of knowledge B | Practice and re-instruction C |  |
|  | Assess/evaluate | Assess/evaluate, re-instruct, secure knowledge | Secure knowledge, activate knowledge |  |  |  |
| Classroom routine | Quiz <br> A1 | Checking homework A2 | Warm-up/ brief review A3 | Presenting new material $B$ | Solving problems (not for homewor C1 | for homework) C2 |
| Actions of participants | T-tells or solicits answers <br> T - at the front | T-tells or solicits answers <br> T - may work through difficult problems T - at the front | T-tells or solicits answers <br> T-may work through problems T - at the front | Information provided mostly by T <br> T - tells students when, why, and how to <br> use certain procedures <br> T - asks short-answer questions <br> T - may do an example problem <br> T - at the front | T-Ss through example problems T - at the front | T-walks around the room T - provides assistance to Ss who raise their hands |
|  | Ss - Students take quiz <br> Ss - provide or check their answers Ss - at their seats | Ss - provide or check their answers Ss - at their seats Ss - may put their answers on the board | Ss - complete problem(s) Ss - provide or check their answers Ss - at their seats | Ss - listen and answer T's questions Ss - may work on an activity, as explicitly instructed by the T <br> Ss - at their seats | Ss - help the teacher do the problems Ss - at their seats | Ss - work individually or in small groups at their seats Ss - may state their answers as a class |
| Classroom talk | Known-answer questions, relatively quick pace, more student turns, T evaluates, recitation? | Known-answer questions, relatively quick pace, more student turns, T evaluates | Known-answer questions, relatively quick pace, more student turns, T evaluates | Fewer student turns, direction instruction? lecture? | Recitation, more student turns, direct instruction? | T-S dialogue, S-S dialogue (private talk) |
| Content | Content related to previous lesson | Content related to previous lesson | Content may or may not be closely related to the new topic | Simple rules or definitions stated by T, focus is mostly on procedure (little reflection on concepts) | More problems very similar to what the T has just shown | More problems very similar to what the T has just shown |
| Climate | T wants correct answers |  |  |  |  | Friendly atmosphere |

SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

## U.S. Notes

Recitation = A series of short, known-answer questions posed by the teacher, to solicit correct answers from students. Consists mainly of Initiation-Response-Evaluation sequences.

An alternative U.S. classroom pattern occasionally exists that does not resemble this model. These are considered "reform" mathematics lessons. They typically consist of an open-ended problem posed by the teacher, a long period of seatwork during which the students work on the problem, and then a period of "sharing" when the students provide their answers and the teacher summarizes the key points.

Teaching Mathematics in Seven Countries

## Results From the TIMSS 1999 Video Study

## APPENDIX F <br> Numeric Values for the Lesson Signatures

TABLE F.1. Percentage of Australian lessons marked at each 10 percent interval of the lessons: 1999

|  | Percent interval (time) of the lessons |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Beginning |  |  |  | Midpoint |  |  |  |  | End |  |
|  | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Review | 87 | 77 | 52 | 37 | 32 | 27 | 23 | 23 | 22 | 23 | 23 |
| Introduction of new content | 12 | 23 | 33 | 40 | 36 | 31 | 33 | 33 | 29 | 23 | 23 |
| Practice of new content | $\ddagger$ | $\ddagger$ | 8 | 17 | 26 | 35 | 36 | 36 | 40 | 47 | 47 |
| Public interaction | 99 | 73 | 70 | 64 | 48 | 49 | 33 | 32 | 37 | 42 | 92 |
| Private interaction | $\ddagger$ | 36 | 37 | 40 | 52 | 54 | 67 | 72 | 66 | 59 | 9 |
| Optional, student presents information | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | 4 | $\ddagger$ | $\ddagger$ |
| Mathematical organization | 32 | 6 | 5 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | 5 | 47 |
| Non-problem | 23 | 24 | 17 | 18 | 15 | 10 | 6 | $\ddagger$ | 7 | 10 | 36 |
| Concurrent problem classwork | $\ddagger$ | 14 | 15 | 10 | 7 | 8 | 5 | 6 | 14 | 17 | $\ddagger$ |
| Concurrent problem seatwork | $\ddagger$ | 31 | 26 | 30 | 40 | 43 | 54 | 62 | 62 | 56 | 6 |
| Answered-only problems | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Independent problem 1 | 6 | 18 | 9 | 6 | 7 | 6 | $\ddagger$ | 5 | 4 | $\ddagger$ | $\ddagger$ |
| Independent problem 2-5 | $\ddagger$ | 31 | 17 | 17 | 16 | 16 | 10 | 10 | $\ddagger$ | 5 | $\ddagger$ |
| Independent problem 6-10 | $\ddagger$ | 21 | 23 | 6 | 10 | 7 | 10 | 10 | 5 | $\ddagger$ | $\ddagger$ |
| Independent problems 11+ | $\ddagger$ | $\ddagger$ | $\ddagger$ | 8 | 8 | 9 | 8 | 10 | 5 | 6 | $\ddagger$ |

[^16]TABLE F.2. Percentage of Czech lessons marked at each 10 percent interval of the lessons: 1999

| TABLE F. 2. | ed | 10 | int | he | : 199 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Percent interval (time) of the lessons |  |  |  |  |  |  |  |  |  |  |
|  | Beginning |  |  |  | Midpoint |  |  |  |  | End |  |
|  | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Review | 99 | 96 | 92 | 79 | 62 | 50 | 40 | 35 | 30 | 30 | 32 |
| Introduction of new content | $\ddagger$ | 4 | 6 | 21 | 36 | 44 | 43 | 34 | 22 | 15 | 10 |
| Practice of new content | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | 7 | 18 | 32 | 47 | 56 | 58 |
| Public interaction | 98 | 59 | 62 | 67 | 70 | 59 | 67 | 60 | 56 | 41 | 98 |
| Private interaction | $\ddagger$ | 30 | 28 | 20 | 8 | 19 | 13 | 20 | 23 | 41 | $\ddagger$ |
| Optional, student presents information | $\ddagger$ | 12 | 18 | 18 | 23 | 24 | 23 | 23 | 21 | 20 | $\ddagger$ |
| Mathematical organization | 17 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | 13 |
| Non-problem | 45 | 17 | 13 | 18 | 14 | 13 | 10 | 13 | 10 | 6 | 68 |
| Concurrent problem classwork | $\ddagger$ | $\ddagger$ | 11 | 12 | 14 | 11 | 9 | 9 | 6 | 5 | 5 |
| Concurrent problem seatwork | $\ddagger$ | 24 | 30 | 24 | 10 | 15 | 10 | 15 | 22 | 29 | 4 |
| Answered-only problems | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ |  | $\ddagger$ |
| Independent problem 1 | 7 | 26 | 18 | 11 | 10 | 5 | 6 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Independent problem 2-5 | $\ddagger$ | 21 | 12 | 16 | 24 | 28 | 26 | 19 | 17 | 17 | $\ddagger$ |
| Independent problem 6-10 | $\ddagger$ | 20 | 10 | 4 | 9 | 18 | 22 | 20 | 22 | 21 | 4 |
| Independent problems 11+ | $\ddagger$ | 19 | 26 | 13 | 20 | 15 | 22 | 28 | 19 | 19 | 5 |

[^17]TABLE F.3. Percentage of Hong Kong SAR lessons marked at each 10 percent interval of the lessons: 1999

|  | Percent interval (time) of the lessons |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Beginning |  | 20 | 30 | Midpoint |  |  | 70 | 80 | 90 | End100 |
|  | 0 | 10 |  |  | 40 | 50 | 60 |  |  |  |  |
| Review | 77 | 58 | 33 | 26 | 22 | 14 | 13 | 12 | 12 | 10 | 8 |
| Introduction of new content | 23 | 40 | 58 | 55 | 47 | 40 | 40 | 40 | 29 | 23 | 23 |
| Practice of new content | $\ddagger$ | $\ddagger$ | 11 | 20 | 34 | 46 | 47 | 50 | 61 | 67 | 67 |
| Public interaction | 100 | 86 | 88 | 81 | 82 | 78 | 68 | 63 | 64 | 63 | 97 |
| Private interaction | $\ddagger$ | 11 | 10 | 19 | 19 | 20 | 26 | 31 | 28 | 31 | 4 |
| Optional, student presents information | $\ddagger$ | 4 | 3 | $\ddagger$ | 4 | 5 | 6 | 8 | 11 | 11 | $\ddagger$ |
| Mathematical organization | 16 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | 3 | 15 |
| Non-problem | 43 | 29 | 23 | 12 | 11 | 8 | 9 | 4 | $\ddagger$ | 5 | 50 |
| Concurrent problem classwork | $\ddagger$ | $\ddagger$ | $\ddagger$ | 7 | 9 | 13 | 17 | 13 | 19 | 24 | 13 |
| Concurrent problem seatwork | $\ddagger$ | 6 | 10 | 15 | 21 | 22 | 23 | 30 | 32 | 32 | 3 |
| Answered-only problems | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Independent problem 1 | 20 | 38 | 27 | 10 | 7 | 6 | 6 | 4 | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Independent problem 2-5 | $\ddagger$ | 27 | 36 | 44 | 42 | 34 | 23 | 21 | 17 | 16 | $\ddagger$ |
| Independent problem 6-10 | $\ddagger$ | $\ddagger$ | $\ddagger$ | 8 | 11 | 14 | 15 | 21 | 14 | 11 | $\ddagger$ |
| Independent problems 11+ | $\ddagger$ | $\ddagger$ | 4 | $\ddagger$ | 5 | 8 | 8 | 10 | 7 | $\ddagger$ | 4 |

[^18]TABLE F.4. Percentage of Japanese lessons marked at each 10 percent interval of the lessons: 1995

|  | Percent interval (time) of the lessons |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Beginning |  |  |  | Midpoint |  |  |  |  | End |  |
|  | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Review | 73 | 66 | 38 | 31 | 25 | 13 | 9 | 10 | 10 | 6 | $\ddagger$ |
| Introduction of new content | 27 | 38 | 62 | 72 | 73 | 75 | 71 | 62 | 58 | 55 | 55 |
| Practice of new content | $\ddagger$ | $\ddagger$ | $\ddagger$ | \# | $\ddagger$ | 12 | 20 | 28 | 33 | 39 | 40 |
| Public interaction | 98 | 64 | 53 | 67 | 67 | 59 | 65 | 35 | 57 | 62 | 98 |
| Private interaction | $\ddagger$ | 37 | 42 | 33 | 32 | 40 | 35 | 63 | 46 | 34 | $\ddagger$ |
| Optional, student presents information | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | \# | $\ddagger$ | 8 | $\ddagger$ | 10 | $\ddagger$ |
| Mathematical organization | $\ddagger$ | $\ddagger$ | $\ddagger$ | \# | \# | \# | $\ddagger$ | \# | $\ddagger$ | $\ddagger$ | 16 |
| Non-problem | 31 | 32 | 21 | 17 | 15 | 12 | 17 | $\ddagger$ | $\ddagger$ | 15 | 55 |
| Concurrent problem classwork | $\ddagger$ | $\ddagger$ | $\ddagger$ | 9 | $\ddagger$ | 8 | 8 | $\ddagger$ | $\ddagger$ | 10 | 7 |
| Concurrent problem seatwork | $\ddagger$ | 9 | 10 | \# | \# | \# | 10 | 13 | 18 | 17 | \# |
| Answered-only problems | $\ddagger$ | $\ddagger$ | $\ddagger$ | \# | $\ddagger$ | \# | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | \# |
| Independent problem 1 | 17 | 53 | 51 | 44 | 34 | 22 | 19 | 19 | 17 | $\ddagger$ | $\ddagger$ |
| Independent problem 2-5 | $\ddagger$ | $\ddagger$ | 19 | 31 | 38 | 57 | 49 | 54 | 61 | 52 | 23 |
| Independent problem 6-10 | $\ddagger$ | $\ddagger$ | $\ddagger$ | \# | $\ddagger$ | $\ddagger$ | \# | \# | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Independent problems 11+ | $\ddagger$ | $\ddagger$ | $\ddagger$ | \# | \# | $\ddagger$ | \# | \# | $\ddagger$ | $\ddagger$ | \# |

ents representing 0.4 percent of total lesson time in a 50 -minute lesson this
$\ddagger$ Reporting standards not met. Too few cases to be reported.
NOTE: The percentage of lessons coded for a feature at any equates to segments of approximately 12 seconds each. Within each segment, the codes applied to the lessons are tabulated to derive the percentage of lessons exhibiting the feature. While many of the features listed above are mutually exclusive within each of the three dimensions (e.g., reviewing, introducing new content, and practicing new content within the purpose dimension), the percentages may not sum to 100 within a dimension due to the possibility of (a) a shift in codes within a segment in which case both codes would have been counted, (b) a segment being coded as "unable to make a judgment," (c) categories not reported, (d) momentary overlaps between the end of one feature and the start of another in which case both would be counted, and (e) rounding.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999,
TABLE F.5. Percentage of Dutch lessons marked at each 10 percent interval of the lessons: 1999

|  | Percent interval (time) of the lessons |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Beginning |  |  |  | Midpoint |  |  | 70 | 80 |  | End <br> 100 |
|  | 0 | 10 | 20 | 30 | 40 | 50 | 60 |  |  | 90 |  |
| Review | 64 | 61 | 53 | 46 | 36 | 30 | 26 | 25 | 25 | 24 | 24 |
| Introduction of new content | 29 | 31 | 36 | 37 | 37 | 34 | 34 | 29 | 28 | 27 | 27 |
| Practice of new content | $\ddagger$ | $\ddagger$ | 4 | 11 | 22 | 29 | 35 | 40 | 41 | 43 | 43 |
| Public interaction | 99 | 75 | 64 | 65 | 51 | 41 | 34 | 16 | 16 | 9 | 74 |
| Private interaction | 6 | 24 | 35 | 34 | 46 | 56 | 69 | 82 | 84 | 91 | 38 |
| Optional, student presents information | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Mathematical organization | 23 | 6 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | 13 |
| Non-problem | 9 | 11 | 5 | $\ddagger$ | 6 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | 15 |
| Concurrent problem classwork | $\ddagger$ | $\ddagger$ | $\ddagger$ | 6 | 9 | 8 | 10 | 8 | 8 | $\ddagger$ | 4 |
| Concurrent problem seatwork | $\ddagger$ | 23 | 33 | 35 | 43 | 56 | 67 | 81 | 84 | 91 | 47 |
| Answered-only problems | $\ddagger$ | 7 | 8 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Independent problem 1 | 16 | 24 | 9 | 7 | $\ddagger$ | 5 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Independent problem 2-5 | $\ddagger$ | 42 | 42 | 23 | 15 | 7 | 5 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Independent problem 6-10 | $\ddagger$ | 19 | 13 | 32 | 21 | 12 | 7 | 6 | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Independent problems 11+ | $\ddagger$ | $\ddagger$ | 8 | 9 | 10 | 14 | 10 | $\ddagger$ | 12 | 8 | $\ddagger$ |

[^19]TABLE F.6. Percentage of Swiss lessons marked at each 10 percent interval of the lessons: 1999

|  | Percent interval (time) of the lessons |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Beginning |  |  |  | Midpoint |  |  |  | 80 | 90 | End |
|  | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 |  |  | 100 |
| Review | 71 | 62 | 53 | 39 | 30 | 27 | 22 | 19 | 19 | 19 | 19 |
| Introduction of new content | 27 | 34 | 41 | 46 | 50 | 49 | 47 | 40 | 36 | 31 | 27 |
| Practice of new content | $\ddagger$ | $\ddagger$ | 7 | 14 | 19 | 20 | 28 | 37 | 42 | 48 | 51 |
| Public interaction | 98 | 73 | 67 | 57 | 51 | 48 | 44 | 44 | 41 | 45 | 93 |
| Private interaction | 3 | 29 | 33 | 39 | 46 | 49 | 58 | 55 | 58 | 61 | 13 |
| Optional, student presents information | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Mathematical organization | 21 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | 24 |
| Non-problem | 28 | 22 | 17 | 10 | 11 | 10 | 6 | 9 | 5 | 7 | 34 |
| Concurrent problem classwork | $\ddagger$ | $\ddagger$ | 9 | 11 | 8 | 10 | 12 | 14 | 14 | 18 | 5 |
| Concurrent problem seatwork | 3 | 23 | 27 | 32 | 36 | 41 | 49 | 48 | 52 | 55 | 13 |
| Answered-only problems | 2 | 6 | 3 | $\ddagger$ | \# | 3 | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ | $\ddagger$ |
| Independent problem 1 | 15 | 30 | 20 | 12 | 10 | 6 | 10 | 5 | 5 | 6 | 4 |
| Independent problem 2-5 | $\ddagger$ | 18 | 23 | 21 | 21 | 20 | 14 | 12 | 11 | 13 | 3 |
| Independent problem 6-10 | $\ddagger$ | キ | 13 | 5 | 5 | 4 | 4 | 4 | 5 | $\ddagger$ | $\ddagger$ |
| Independent problems 11+ | $\ddagger$ | $\ddagger$ | 5 | 4 | 3 | 4 | 4 | 7 | 8 | 4 | $\ddagger$ |

${ }^{\ddagger}$ Reporting standards not met. Too few cases to be reported. NOTE: The percentage of lessons coded for a feature at any equates to segments of approximately 12 seconds each. Within each segment, the codes applied to the lessons are tabulated to derive the percentage of lessons exhibiting the feature. While many of the features listed above are mutually exclusive within each of the three dimensions (e.g., reviewing, introducing new content, and practicing new content within the purpose dimension), the percentages may not sum to 100 within a dimension due to the possibility of (a) a shift in codes within a segment in which case both codes would have been counted, (b) a segment being coded as "unable to make a judgment," (c) categories not reported, (d) momentary overlaps between the end of one feature and the start of another in which case both would be counted, and (e) rounding.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.


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[^0]:    ${ }^{1}$ This does not take into account differences between participating countries based on teachers' reports of years teaching, academic preparation, lesson goals, and the like, as reported in chapter 2.

[^1]:    ${ }^{2}$ The analysis used to develop the lesson signatures divided each lesson into 250 segments, each representing 0.4 percent of the total lesson length. For example, the analysis accounts for how a 50 -minute lesson was coded approximately every 12 seconds.
    ${ }^{3}$ Pixel is short for "picture element." A pixel is the smallest unit of visual information that can be used to build an image. In the case of the printed page, pixels are the little dots or squares that can be seen when a graphics image is enlarged or viewed up close.

[^2]:    reported as the percentage of lessons exhibiting each feature at particular moments in time.
    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third Intern

    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

[^3]:    ${ }^{4}$ Even when taking into consideration that the Japanese sample included a large percentage of lessons on two-dimensional geometry, the percentage of problems per lesson of each level of procedural complexity remained relatively consistent (see figure 4.2).

[^4]:    reported as the percentage of lessons exhibiting each feature at particular moments in time.
    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third Intern

    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

[^5]:    reported as the percentage of lessons exhibiting each feature at particular moments in time.
    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third Interna
    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

[^6]:    reported as the percentage of lessons exhibiting each feature at particular moments in time.
    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third Interna
    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

[^7]:    ${ }^{1}$ Australia, the Czech Republic, Japan, the Netherlands, and the United States also collected data on eighth-grade science lessons.
    ${ }^{2}$ For the German-speaking area of Switzerland, the video sample was a sub-sample of the TIMSS 1995 assessment schools. For Hong Kong SAR most, but not all, of the video sample was a sub-sample of the TIMSS 1999 assessment schools.
    ${ }^{3}$ Australia and Switzerland conducted separate studies that involved testing the mathematics achievement of the videotaped students.

[^8]:    ${ }^{4}$ For the German-language area of Switzerland, the video sample was a sub-sample of the TIMSS assessment schools. For Hong Kong SAR most, but not all, of the video sample was a sub-sample of the TIMSS 1999 assessment schools.
    ${ }^{5}$ Since it was based on the TIMSS 1999 assessment sample, the Hong Kong SAR school sample was selected and checked by Statistics Canada. In the United States, Westat selected the school sample and LessonLab selected the classroom sample.

[^9]:    ${ }^{6}$ The mathematics team did not include a representative from Japan because Japanese mathematics lessons were not filmed as part of the TIMSS 1999 video data collection.

[^10]:    ${ }^{7}$ A subset of these lesson tables, from all countries except Japan, were expanded and then coded by the mathematics quality analysis group, described below.
    ${ }^{8}$ The minimum acceptable reliability score for all codes (across coders and countries) was 85 percent. For coders and countries, the minimum acceptable reliability score was 80 percent. That is, the reliability of an individual coder or the average of all coders within a particular country was occasionally between 80 and 85 percent. In these cases clarification was provided as necessary, but re-testing for reliability was not deemed appropriate.

[^11]:    ${ }^{1}$ Initial reliability refers to reliability established on a designated set of lessons before coders began work on their assigned lessons.
    ${ }^{2}$ Midpoint reliability refers to reliability established on a designated set of lessons after coders completed approximately half of their total assigned lessons.
    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

[^12]:    ${ }^{9}$ Transcriber/translators were fluent in both English and their native language, educated at least through eighth grade in the country whose lessons they translated, and had completed two-weeks training in the procedures detailed in the TIMSS 1999 Video Study Transcription and Translation Manual (available in Jacobs et al. forthcoming). A glossary of terms was developed to help standardize translation within each country.

[^13]:    ${ }^{1} \mathrm{AU}=$ Australia; CZ=Czech Republic; HK=Hong Kong SAR; NL=Netherlands; SW=Switzerland; and US=United States.
    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study, Video Study, 1999

[^14]:    ${ }^{1}$ AU=Australia; CZ=Czech Republic; HK=Hong Kong SAR; NL=Netherlands; SW=Switzerland; and US=United States. NOTE: Lessons included here are a random sub-sample of lessons in each country. Results should be interpreted with caution because they might not be representative of the entire sample.
    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

[^15]:    ${ }^{1}$ The process of creating a hypothesized country model was not completed for Japan.
    ${ }^{2}$ Field test lessons were not collected in Hong Kong SAR because a final decision about participation in the study had not yet been made.

[^16]:    NOTE: The percentage of lessons coded for a feature at any point in time was calculated by dividing each lesson into 250 segments representing 0.4 percent of total lesson time. In a 50 -minute lesson, this equates to segments of approximately 12 seconds each. Within each segment, the codes applied to the lessons are tabulated to derive the percentage of lessons exhibiting the feature. While many of the features listed above are mutually exclusive within each of the three dimensions (e.g., reviewing, introducing new content, and practicing new content within the purpose dimension), the percentages may not sum to 100 within a dimension due to the possibility of (a) a shift in codes within a segment in which case both codes would have been counted, (b) a segment being coded
    ment," (c) categories not reported, (d) momentary overlaps between the end of one feature and the start of another in which case both would be counted, and (e) rounding. SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

[^17]:    ${ }^{\ddagger}$ Reporting standards not met. Too few cases to be reported.
    ${ }^{\ddagger}$ Reporting standards not met. Too few cases to be reported.
    NOTE: The percentage of lessons coded for a feature at any
    equates to segments of approximately 12 seconds each. Within each segment, the codes applied to the lessons are tabulated to derive the percentage of lessons exhibiting the feature. While many of the features listed above are mutually exclusive within each of the three dimensions (e.g., reviewing, introducing new content, and practicing new content within the purpose dimension), the percentages may
    not sum to 100 within a dimension due to the possibility of (a) a shift in codes within a segment in which case both codes would have been counted, (b) a segment being coded as "unable to make a judgnot sum to 100 within a dimension due to the possibility of (a) a shift in codes within a segment in which case both codes would have been counted, (b) a segment being coded as "unable to make a judgment," (c) categories not reported, (d) momentary overlaps between the end of one feature and the start of another in which case both would be counted, and (e) rounding.
    SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999. SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

[^18]:    NOTE: The percentage of lessons coded for a feature at any point in time was calculated by dividing each lesson into 250 segments representing 0.4 percent of total lesson time. In a 50 -minute lesson, this equates to segments of approximately 12 seconds each. Within each segment, the codes applied to the lessons are tabulated to derive the percentage of lessons exhibiting the feature. While many of the features listed above are mutually exclusive within each of the three dimensions (e.g., reviewing, introducing new content, and practicing new content within the purpose dimension), the percentages may not sum to 100 within a dimension due to the possibility of (a) a shift in codes within a segment in which case both codes would have been counted, (b) a segment being coded
    ment," (c) categories not reported, (d) momentary overlaps between the end of one feature and the start of another in which case both would be counted, and (e) rounding. SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

[^19]:    NOTE: The percentage of lessons coded for a feature at any point in time was calculated by dividing each lesson into 250 segments representing 0.4 percent of total lesson time. In a $50-\mathrm{minute}$ lesson, this equates to segments of approximately 12 seconds each. Within each segment, the codes applied to the lessons are tabulated to derive the percentage of lessons exhibiting the feature. While many of the features listed above are mutually exclusive within each of the three dimensions (e.g., reviewing, introducing new content, and practicing new content within the purpose dimension), the percentages may not sum to 100 within a dimension due to the possibility of (a) a shift in codes within a segment in which case both codes would have been counted, (b) a segment being coded
    ment," (c) categories not reported, (d) momentary overlaps between the end of one feature and the start of another in which case both would be counted, and (e) rounding. SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

[^20]:    ${ }^{\ddagger}$ Reporting standards not met. Too few cases to be reported. ${ }^{\ddagger}$ Repore: The percentage of lessons coded for a feature at any

    NOTE: The percentage of lessons coded for a feature at any point in time was calculated by dividing each lesson into 250 segments representing 0.4 percent of total lesson time. In a 50 -minute lesson, this
    equates to segments of approximately 12 seconds each. Within each segment, the codes applied to the lessons are tabulated to derive the percentage of lessons exhibiting the feature. While many of the features listed above are mutually exclusive within each of the three dimensions (e.g., reviewing, introducing new content, and practicing new content within the purpose dimension), the percentages may not sum to 100 within a dimension due to the possibility of (a) a shift in codes within a segment in which case both codes would have been counted, (b) a segment being coded as "unable to make a judgment," (c) categories not reported, (d) momentary overlaps between the end of one feature and the start of another in which case both would be counted, and (e) rounding. SOURCE: U.S. Department of Education, National Center for Education Statistics, Third International Mathematics and Science Study (TIMSS), Video Study, 1999.

