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Hybrid Codes: Past, Present and Future

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Abstract. Hybrid codes, in which the ions are treated kinetically and the electrons are assumed to be a massless fluid, have been widely used in space physics over the past two decades. These codes are used to model phenomena that occur on ion inertia and gyroradius scales, which fall between longer scales obtained by magnetohydrodynamic simulations and shorter scales attainable by full particle simulations. In this tutorial, the assumptions and equations of the hybrid model are discussed along with some most commonly used numerical implementations. Examples of results of two-dimensional hybrid simulations are used to illustrate the method, to indicate some of the tradeoffs that need to be addressed in a realistic calculation, and to demonstrate the utility of the technique for problems of contemporary interest. Some speculation about the future direction of space physics research using hybrid codes is also provided.

1 Hybrid Codes: Past

Generally, the term ”hybrid code” in plasma physics can refer to any simulation model in which one or more of the plasma species are treated as a single or multiple fluids, while the remaining species are treated kinetically as particles. The plasma can be coupled to the electromagnetic fields in a variety of ways: full Maxwell equations, low-frequency Darwin model, electrostatic only, etc. In this tutorial, we shall concentrate only on the most common type of hybrid code used in space plasmas: where all the ions are treated kinetically, the electrons are assumed to be an inertia-less and quasi-neutral fluid, and the electromagnetic fields are treated in the low-frequency approximation.

Because this tutorial is being presented in the context of the International School for Space Simulation (ISSS), we will mostly restrict the discussion of ”past” uses of hybrid methods in space physics to the articles published in the previous schools. Those articles give appropriate and timely references to research that was carried out at that time with hybrid codes that were then available. One-dimensional hybrid algorithms are discussed in some detail in the articles from the first (Winske and Leroy, 1984) and second (Winske, 1985) schools. Those article emphasized applications to collisionless shocks and low frequency waves in the ion foreshock driven by ions reflected at the bow shock. At the third school, Quest (1989) discussed hybrid codes more generally, comparing and contrasting several multi- dimensional algorithms. As we will see, the field has not advanced much beyond what was described in that article. Winske and Omidi (1993) gave a tutorial on hybrid codes at the fourth school that once again emphasized, for pedagogical reasons, one dimensional codes. In this case, ion beam instabilities were used to illustrate the main features of hybrid simulation methods. Given that rather complete treatment, the emphasis in this tutorial will be on two-dimensional implementations. As we expect to find numerous applications of hybrid codes and techniques at this meeting, we have restricted the discussion here to problems that we ourselves have worked on over the last few years. Students and young researchers who are not familiar with hybrid techniques are urged to consult the ISSS-4 article as a reference point to the following discussion.

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2 Hybrid vCodes: Present

Hybrid codes arise from the need to model phenomena that occur on shorter time and distance scales than can be treated by magnetohydrodynamics and yet do not resolve processes that occur on electron scales (e.g., electron gyroradius and electron Debye length scales, inverse electron gyrofrequency and electron plasma frequency time scales). The relevant scales are then the ion gyroradius and ion inertial spatial scales, and inverse ion gyrofrequency time scale. In space, these length scales typically are on the order of 10’s to 100’s of km and times on the order of seconds; these scales are readily resolved by satellite instrumentation. To model phenomena on these scales with a hybrid model, as contrasted with a Hall-MHD (two-fluid) code, implies assumptions about the descriptions of the plasma ions and electrons as well as the electromagnetic fields.

To be consistent with the hybrid model, the ions are treated kinetically, i.e., using standard particle-in-cell methods. Each simulation ion (charge q_i, mass m_i) is subject to the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$, which have values given on a spatial grid and are interpolated to the particle location. The updated particle information is collected at the grid points to determine the ion number density (n_i), charge density ($q_i n_i$), flow velocity $\boldsymbol{V}_i$ and current $\boldsymbol{J}_i = q_i n_i \boldsymbol{V}_i$. In order to eliminate kinetic electron effects, the electrons are treated as an inertia-less fluid ($m_e = 0$). The electron momentum equation is thus:

$$\frac{d}{dt} n_e m_e \boldsymbol{V}_e = 0 = -e n_e (\boldsymbol{E} + \frac{\boldsymbol{V}_e \times \boldsymbol{B}}{c}) - \boldsymbol{\nabla} \cdot \mathbf{P}_e \quad (1)$$

where $\boldsymbol{V}_e$ is the electron fluid velocity and $\mathbf{P}_e$ is the electron pressure tensor. Ignoring effects on the electron Debye length scale further implies that the plasma is quasi-neutral, so that the electron and ion charge densities are equal. $\mathbf{P}_e$ is almost always taken as a scalar, $\mathbf{P}_e = p_e \mathbf{I}$. Typically, an isothermal or adiabatic relation between the pressure and temperature is assumed. (For simplicity, we have also left off resistive coupling between the electrons and ions; this adds a term $e n_i \eta \cdot \boldsymbol{J}$ to the right-hand side of (1), where $\boldsymbol{J}$ is the total current.) The electromagnetic fields are treated in the low frequency approximation using Ampere's law and Faraday's law. As is done in two-fluid codes, Ampere's law is used to eliminate $\boldsymbol{V}_e$ and Faraday's is used to advance the magnetic field in time. Because $m_e = 0$, Eq. (1) can be solved for the electric field directly, so that no time advance of $\boldsymbol{E}$ is needed. The other Maxwell's equations, e.g., Poisson's equation is satisfied by virtue of the quasi-neutral approximation and boundary conditions; likewise $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$ is also satisfied.

The numerical implementation of the hybrid model in a simulation code is relatively straightforward. To advance the fields, one uses Faraday's law and the electric field at time step N to advance the magnetic field to time level N+1/2. From the particle push and moment collection, we have the ion current and density (either can collect directly at time level N + 1/2 or use the average between values at N and N+1), and thus all the information needed to evaluate the electric field at time level N+1/2. With this information, again Faraday's law can be used to push $\boldsymbol{B}$ to time level N+1. However, the advance of $\boldsymbol{E}$ from time level N+1/2 to N+1 is not so straightforward: $\boldsymbol{B}^{N+1}$ and ${n_i}^{N+1}$ are known, but not ${\boldsymbol{V}_i}^{N+1}$. The problem of implementing a good algorithm for hybrid codes then reduces to how best to calculate $\boldsymbol{E}^{N+1}$.

Historically the first hybrid algorithm, which continues to be widely used, is a predictor- corrector technique (Harned, 1982; Winske and Quest, 1986). The basic idea is to: (i) make a prediction of the fields at N+1; (ii) advance the particles in the predicted fields in order to compute the ion source terms at time level N+3/2; (iii) use the predicted current (and charge density) to compute predicted fields at N+3/2; and (iv) use the average of the electric field at N+1/2 and the predicted field at N+3/2 to get $\boldsymbol{E}^{N+1}$. In principle, the process could be repeated to improve the accuracy, but in practice this is never done. This method is still often used because it gives very good energy conservation and is rather robust. As we will show, however, there can be significant amount of short wavelength whistler noise generated by the application of this technique, which will require additional measures to remove it. It is evident that this technique will be somewhat slow, since one has to move the particles twice each time step.

The second type of hybrid algorithm involves the advance of the electric field to time level N+1 by an extrapolation of the ion flow velocity (or equivalently the ion current density) from time level N+1/2 to N+1 (Fujimoto, 1990; Thomas et al., 1990). Since the other quantities are known at N+1 already, with an extrapolated ${\boldsymbol{V}_i}^{N+1}$, $\boldsymbol{E}^{N+1}$ can be evaluated, and the time-stepping process can proceed to the next cycle. Intuitively this method is not quite as accurate, as we will show quantitatively when we discuss some simple examples, but for many problems, it is more than adequate and is more often used. The extrapolation of the ion velocity can be done in several ways. First, by merely saving the values of ${\boldsymbol{V}_i}^{N-1/2}$ and ${\boldsymbol{V}_i}^{N+1/2}$, one has can do a 4th order Bashford-Adams extrapolation. Alternatively, one can follow the philosophy of implicit plasma methods and advance a moment equation to give a better estimate of the ion current (Quest, 1989). However, this method requires the accumulation of the ion pressure tensor and the evaluation of an advective derivative, which would seem to negate main advantages of using a hybrid code that has particles to calculate the effects of $\mathbf{P}_i$ and the advection already. In the CAM-CL method (Matthews, 1994), which has become popular in recent years, the ion current is calculated by doing an extra half time step push using a mixed level evaluation of the electric field. Note that in all of these extrapolation methods only one push of the ions each time step is required and so the method is inherently faster.

We illustrate the use of hybrid codes and show differences that result from various algorithms using several test problems. One problems involves the evolution of an ion cyclotron instability driven by an ion temperature anisotropy, such as found in the magnetosheath and investigated in detail to develop a scaling law involving observationally convenient variables (Gary and Winske, 1993; Gary et al., 1997).

As a specific example, we show some results comparing predictor-corrector and velocity extrapolation algorithms for an initial ion temperature anisotropy, $T_{i\perp}/T_{i||} = 2.5$, with $\beta_{i||} = 8\pi T_{i||}/B_o^2 = 1$ in a system of length 80 c/ω_i along ($\hat{x}$) and 40 c/ω_i transverse ($\hat{y}$) to the magnetic field. In these calculations there are 128×64 cells with 100 particles per cell initially. The time step is $\Omega_i \Delta t = 0.05$ (Ω_i is the ion gyrofrequency), and a very small resistivity (resistive length is 10^{-4} of the cell size) is included. The predictor-corrector simulation smooths the source term during both the predic-

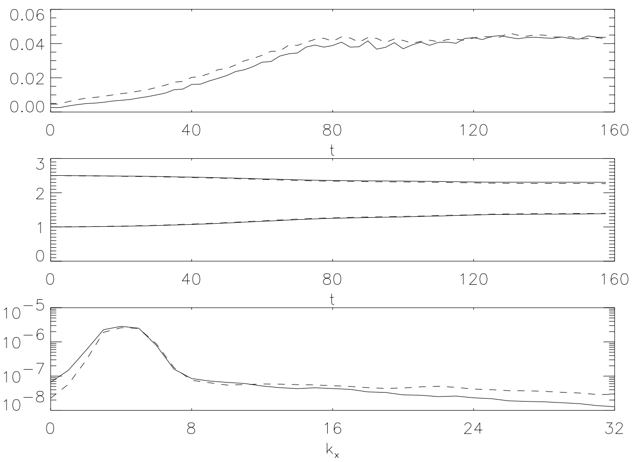

Fig. 1. Fig. 1. Comparison of velocity extrapolation (solid curves) and predictor-corrector (dashed curves) simulations of an ion temperature anisotropy instability. Top panel shows the time evolution of the magnetic fluctuations, the middle panel shows the time evolution of the ion temperature perpendicular and parallel to the magnetic field, and the bottom panel shows the power spectrum of the first 32 Fourier modes along the magnetic field at one time.

tor and corrector cycles; the velocity extrapolation simulation smooths the source terms twice each time step. Figure 1 shows a comparison of the two calculations (velocity extrapolation results are given as the solid curves, predictor-corrector results given as the dashed curves). The top panel shows the growth of the magnetic fluctuations, normalized by the ambient magnetic field, ($\delta B^2/B_o^2$). One sees growth and eventual saturation of the waves. The velocity extrapolation code gives a slightly smaller fluctuation level during the growth phase, with a similar linear growth rate, but comparable level of fluctuations in the nonlinear phase. The middle panel shows the time history of the parallel and perpendicular ion temperatures, averaged over the entire simulation domain. The temperatures are normalized to the initial parallel ion temperature. As expected, the perpendicular temperature decreases as the instability develops, while the parallel temperature increases. The results of the two calculations are practically indistinguishable. The bottom panel shows the power spectrum of the first 32 Fourier modes along the magnetic field direction (the direction of the most unstable growth) at $\Omega_i t = 150$. The dominant modes have similar magnitudes in the two simulations. The predictor-corrector simulation tends to have higher frequency components, whereas the velocity extrapolation code has more wave power at the longest modes. For these simulations, energy conservation is very good: over the simulation, the predictor-corrector code loses about 0.4% energy, while the velocity extrapolation code gains a small amount of energy, 1.2%.

A second problem concerns the excitation of oblique Alven/ion cyclotron waves driven by cold, relatively slow ion beams that are found in the plasma sheet boundary layer and the solar wind (Winske and Omidi, 1992; Daughton et al., 1999). Again for this problem, results of two-dimensional simulations in a doubly periodic system using both predictor-corrector and velocity extrapolation schemes will be shown in order to compare plasma and wave quantities and the effect of varying numerical parameters.

We will also compare results of 2-D hybrid simulations with those from Hall-MHD calculations. In both models we include the full electron pressure tensor in the generalized Ohm's law in order to initiate magnetic reconnection in a collisionless plasma (Hesse and Winske, 1994; Yin et al., 2001a). We will show how the two-fluid model lacks an important ion kinetic effect that is naturally included in the hybrid simulations. This effect can be included back in the Hall-MHD model in a predictor corrector manner, using test ions to calculate the off-diagonal terms of the ion pressure tensor in the predictor step that in turn modify the ion fluid velocity in the corrector step (Yin et al., 2001b,c).

3 Hybrid Codes: Future

In this tutorial, we also speculate on the future development and use of hybrid codes for space physics applications. We can see significant progress occurring in five general areas (Winske and Omidi, 1993; Dawson, 1999): (1) larger and more complex simulations, (2) inclusion of more physics, (3) improvements in diagnostics for better physical insight and comparison with data, (4) algorithm development for massively parallel computers, and (5) linking hybrid and fluid codes together. Some examples of these may include:

(1) The availability of faster CPUs, more memory, etc. will lead to larger scale simulations. Such calculations will include larger regions of space, e.g., the magnetopause or the magnetotail, three-dimensional effects, and/or more complex multi-species problems, such as the solar wind interaction with comets, unmagnetized planets, small moons, etc., beyond that which is presently available (Brecht et al., 1993; Nakamura and Scholer, 2000).

(2) More complex physics models may include, for example, semi-collisional plasmas, such as occur in the polar region, where the outflowing plasma is collisional near the Earth and becomes less so as it flows outward (Miller et al., 1993). We have already discussed another possibility: namely, the use of hybrid codes to understand new kinetic effects that occur near the reconnection site, which can be modeled in Hall-MHD and MHD codes.

(3) Improvements in diagnostics are likely to come through the use of commercial products, like IDL or EnSight, as the development of major visualization tools, especially in 3-D, is far to expensive for any particular research group. Hybrid codes offer unique possibilities for development of diagnostics that examine ion distributions, short wavelength fluctuations, etc., which can be expressed in a form convenient for comparing with data. This will be particularly useful for understanding spatial and temporal correlations between data from several different satellites (Cluster II).

(4) Computers that consist of 1000's of linked processors

seems to be the most economical future for large scale computing. Again, these can be very expensive machines that only the largest institutions can afford or they can be a group of inexpensive pc's that are ganged together. Understanding how to write algorithms that take advance the particular system's unique architecture can be time-consuming, but can pay off in the long run. The challenge for hybrid (and other PIC) codes on massively parallel architectures is to balance the load between processors for pushing the particles and to break up the computational domain in a convenient manner Liewer and Decyk (1989).

(5) Finally, there is the issue of including kinetic physics in large-scale fluid calculations for developing realistic space weather codes. As we have discussed earlier, kinetic effects found from hybrid simulations can be modeled in Hall- MHD code codes. In turn, a Hall-MHD code can be linked via adaptive-mesh-refinement methods to a global MHD code, thus providing an efficient way to include kinetic effects in a large-scale fluid code. Embedding an actual hybrid simulation in an MHD code would seem to be much more complex, given the disparate time and spatial scales between an ion kinetic model and an MHD model. In principle, this might be done be running the calculations on separate machines and exchanging appropriate boundary information to initialize the hybrid calculation or to update the MHD simulation each time step. It certainly provides the ultimate "grand-challenge" problem for graduates of ISSS-6!

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References

Brecht, S. H., Ferrante, J. R., and Luhmann, J. G., Three-dimensional simulations of the solar wind interaction with Mars, J. Geophys. Res., 98, 1345, 1993.

Daughton, W., Gary, S. P., and Winske, D., Electromagnetic proton/proton instabilities in the solar wind: Simulations, J. Geophys. Res., 104, 4657, 1999.

Dawson, J. M., Role of computer modeling of plasmas in the 21st century, Phys. Plasmas, 6, 4436, 1999.

Fujimoto, M., PhD Thesis, U. Tokyo, 1990.

Gary, S. P., and Winske, D., Simulations of ion cyclotron anisotropy instabilities in the terrestrial magnetosheath, J. Geophys. Res., 98, 9171, 1993.

Gary, S. P., Wang, J., Winske, D., and Fuselier, S. A., Proton temperature anisotropy upper bound, J. Geophys. Res., 102, 27159, 1997.

Harned, D. S., Quasineutral hybrid simulation of macroscopic plasma phenomena, J. Comput. Phys., 47, 452, 1982.

Hesse, M. and Winske, D., Hybrid simulations of collisionless reconnection in current sheets, J. Geophys. Res., 99, 11177, 1994.

Liewer, P. C. and Decyk, V. K., A general concurrent algorithm for plasma particle-in- cell simulation codes, J. Comput. Phys., 85, 302, 1989.

Matthews, A. P., Current advance method and cyclic leapfrog for 2-D multispecies hybrid plasma simulations, J. Comput. Phys., 112, 102, 1994.

Miller, R. H., Rasmussen, C. E., Gombosi, T. I., Khazanov, G. V., and Winske, D., Kinetic simulation of plasma flows in the inner magnetosphere, J. Geophys. Res., 98, 19301, 1993.

Nakamura, M. and Scholer, M., Structure of the magnetopause reconnection layer and of flux transfer events: Ion kinetic effects, J. Geophys. Res., 105, 23179, 2000.

Quest, K. B., Hybrid simulation, in Tutorial Courses: Third International School for Space Simulation, edited by B. Lembege and J. W. Eastwood, p. 177, Cepadues Ed., Toulouse, France, 1989.

Thomas, V. A., Winske, D., and Omidi, N., Reforming supercritical quasi-parallel shocks: 1. One- and two-dimensional simulations, J. Geophys. Res., 95, 18809, 1990.

Winske, D., Hybrid simulation codes with application to shocks and upstream waves, Space Sci. Rev., 42, 53, 1985.

Winske, D. and Leroy, M. M., Hybrid simulation techniques applied to the Earth's bow shock, in Computer Simulations of Space Plasmas, edited by H. Matsumoto and T. Sato, p. 255, D. Reidel/Terra Sci., Hingham MA, 1984.

Winske, D. and Omidi, N., Electromagnetic ion/ion cyclotron instability: Theory and simulations, J. Geophys. Res., 97, 14779, 1992.

Winske, D. and Omidi, N., Hybrid codes: Methods and applications, in Computer Space Plasma Physics: Simulations and Software, edited by H. Matsumoto and Y. Omura, p. 103 Terra, Tokyo, 1993.

Winske, D. and Quest, K. B., Electromagnetic ion beam instabilities: Comparison of one- and two-dimensional simulations, J. Geophys. Res., 91, 8789, 1986.

Yin, L., Winske, D., Gary, S. P., and Birn, J., Hybrid and Hall-MHD simulations of collisionless reconnection: Dynamics of the electron pressure tensor, J. Geophys. Res., in press, 2001a.

Yin, L., Winske, D., Gary, S. P., and Birn, J., Particle Hall-MHD simulation of collisionless reconnection: Finite ion gyro-orbit correction, Geophys. Res. Lett., in press, 2001b.

Yin, L., Winske, D., Gary, S. P., and J. Birn, J., Hybrid and Hall-MHD simulations of collisionless reconnection: Effects of the plasma pressure tensor, in Proceedings ISSS-6, 2001c.