False Alarms, True Alarms, and Statistics: Correct Usage of Decision Level and Minimum Detectable Amount

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Abstract

We frequently detect activity that is less than the minimum detectable activity! This occurs because the "minimum detectable activity" is misnamed, not because we are doing something impossible or nonsensical. The random nature of radioactive decay leads to erratic count rates when the total number of radioactive transitions detected is not very large (<100), a situation complicated by the need to subtract some value of "background" from the observation of the sample. Both classical and Bayesian statistical methods address this problem. Pioneering scientists grappled with the terminology for characterizing counting system performance and interpretation of data. Unfortunately, the choices made for terms in the 1960s and later have caused decades of confusion. Had Currie's 1968 term "critical level" (a.k.a. "decision level") been named the "false alarm level" and expressed only in terms of directly observed quantities (e.g., counts or count rates), it may have avoided the confusion so prevalent today. Had Currie's "detection level" (a.k.a. "lower limit of detection," "minimum detectable activity") been named "advertizing level" and expressed only in terms of the ultimate quantity of interest (e.g., Bq, uCi, Bq/kg, or pCi/g) we might not be in a situation in which the quantity it represents is so widely misused. "Never compare a measurement result with an advertizing level (MDA); compare measurement results with a false alarm level (DL)."

1 Suggested Readings

None of these references is perfect. Neither is this handout. But these, in Dan Strom's opinion, are the best.

For basic statistics, a truly superb reference for its insight, clarity, examples, and problems is Chapter 11 and Appendix E in Jim Turner's second edition of *Atoms, Radiation, and Radiation Protection* (Turner 1995).

For slightly more applied concepts, a current consensus standard with well-thought-out statistics and examples is *Performance Criteria for Radiobioassay* (Health Physics Society (HPS) 1996). Another with an applied bent is *Air Sampling in the Workplace* (Hickey et al. 1993). Recent publications in the environmental restoration area with statistics discussions include Multi-Agency Radiation Survey and Site Investigation Manual (MARSSIM) (http://www.epa.gov/radiation/marssim/) (U.S.Nuclear Regulatory Commission (NRC) et al. 1997) and *Minimum Detectable Concentrations with Typical Radiation Survey Instruments for Various Contaminants and Field Conditions* (U.S.Nuclear Regulatory Commission (NRC)

1998).

Two classic papers, which are still very much worth reading, are the 1963 and 1968 papers by pioneers in the field (Altshuler and Pasternack 1963; Currie 1968).

A paper with great insight, but which has been largely ignored, is Wes Nicholson's classic (Nicholson 1966) and references therein. Nicholson takes the conceptually clear approach of using underlying count *rates*, rather than framing the problem in terms of counts, as has been done by most others.

An especially interesting paper with many profound ideas dates from the late 1940s (Rainwater and Wu 1947). In particular, these authors explicitly introduce the Bayesian concept of inferring a distribution of observed results from a population parameter (e.g., a count rate), and then showing that the inverse relationship is symmetric: starting with an observation, one infers a distribution of population parameters that may have given rise to the observation.

For Bayesian statistics, be sure to read the definitive work by Little (Little 1982). Thomas Bayes's original 1763 treatise is still available (Bayes 1958) but is merely of historical interest. For elementary insight into Bayesian thinking, I recommend Don Berry's introductory text (Berry 1996). For recent Bayesian applications in radiation protection, visit http://www.pnl.gov/bayesian for introductory materials and important links, especially to the ground-breaking work at Los Alamos National Laboratory by Guthrie Miller, Bill Inkret, Harry Martz, and Mario Schillaci.

2 Introduction

The problem of detecting extremely small amounts of radioactive material is a problem of picking a small signal out of a significant noise. There are many natural examples when the signal-to-noise ratio is too small to detect something. Examples include trying to hear a whisper at a rock concert, listening from another room for the cough of a baby with the croup, straining to hear a single voice in a crowded stadium, trying to hear a dripping faucet next to a waterfall, looking to pick out the first star of the evening, trying to perceive the weight of a feather with a gloved hand, trying to catch a whiff of a scent that sets a dog barking. Often, natural detectors aren't up to it: we can't see the Cerenkov radiation from cosmic rays with our naked eyes, we can't sense low-LET ionizing particles with any of our senses. However, dark-adapted astronauts do report seeing flashes from high-Z, high-energy (HZE) particles in space, so the threshold of detection for, say, alpha particles in the eye, is not too much lower than our current sensitivity.

How can you separate two random processes that occur at the same time and give rise to indistinguishable electrical pulses from a detector? Specifically, one process, background, is presumed to be constant (How constant is it? The chi-square test for "expected" amount of variability can be used to determine how constant background is). The other process, the signal from the analyte in the unknown (How unknown is it? Bayes's theorem can be used to incorporate what you already know), differs from one sample to the next and is variable.

The *decision* one usually must make is whether, given a set of observations N_b, t_b, N_g, and t_g, there

is any analyte present in the unknown. This decision is obvious if $R_n >> R_b$. However, we often make this decision when the consequences of a wrong decision are significant. The question is particularly acute when assaying small quantities of transuranic (TRU) elements, such as plutonium, in waste or in environmental or bioassay samples. The decision about whether there is analyte present may also be the decision that "this waste drum is not suitable for near-surface disposal under 10 CFR 61 and must be shipped to a high-level waste repository;" or "this land is still contaminated and millions of dollars must be spent to clean it up;" or "this worker had an unexpected intake of plutonium that may have resulted in an $H_{E,50}$ greater than 5 rems." Thus it is important that we use the proper statistics for such decisions.

Classical or long-term frequentist statistical methods have been used for years to distinguish signal from noise, although they didn't mature until the 1960s. More recently, statistical inference methods based on Bayes' theorem (Bayes 1958) have begun to be used for this problem (Little 1982).

Innovative work employing Bayes' methods has been undertaken by one DOE site, the Los Alamos National Laboratory (LANL) (Miller et al. 1993; Miller et al. 1995; Miller et al. 1997; Miller et al. 1998). The Bayesian formalism is attractive because is incorporates prior knowledge in addition to the results of a given measurement, and it results in a distribution of likely outcomes rather than merely a point estimate with an uncertainty. However, the method has been criticized as being too subjective. At present, the DOE has taken no official position on the use of Bayesian methods; the method has been in use at the LANL for several years (Inkret and Miller 1995). In November of 1997 a workshop was held on Bayesian methods; proceedings are in preparation and a web site containing much of the material presented is available (http://www.pnl.gov/bayesian/; be sure to follow the link to LANL).

3 Classical Statistical Methods

3.1 Precision (Reproducibility), Bias, and Accuracy

The *precision* of a measure-ment result is an expression of the degree to which it can be repro-duced or repeated, that is, a measure of the agreement among individual measurements. Lack of precision is caused by random errors (indeterminate errors).

The *bias* is the amount that a measurement or average measurement differs from the "conventionally true value." Bias is caused by *systematic* errors (determinate errors).

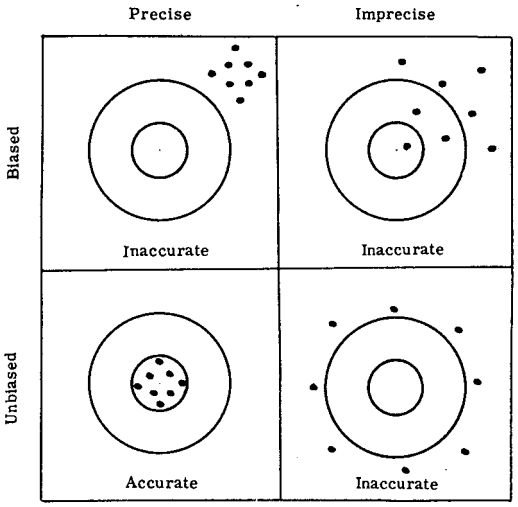

Figure 1. Diagram illustrating bias, precision, and accuracy (adapted from *Remington's Pharmaceutical Sciences*)

Bias and precision are generally independent quantities. A set of measurements of the same sample with a large standard deviation are not very precise (not very reproducible). A set of

measurements of the same sample with a small standard deviation are precise (reproducible). The bias of imprecise measurements may be high or low; similarly, the bias of precise measurements may be high or low.

The *accuracy* of a measurement result is the degree to which it is accurate (equals a "conventionally true value" or standard value for the quantity being measured) and precise (repeated measurements are close together). Unfortunately, accuracy is sometimes used as a synonym for bias, so be careful.

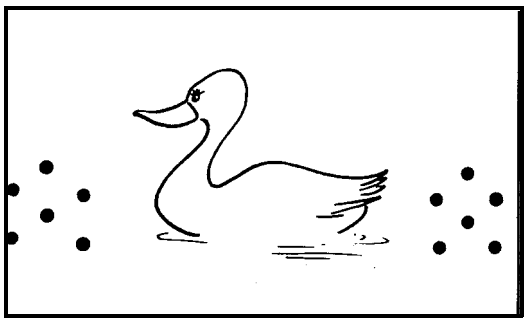

Figure 2. On the average, the duck was dead (adapted from *Remington's Pharmaceutical Sciences*)

3.2 Elementary Concepts

If N counts or events are observed in a time t, then the count rate, $dN/dt = R = N/t$. The variance of a Poisson (counting) variable, $\mathrm{Var}(N) = N$. The standard deviation, $s = \sqrt{N}$.

Counts due to background radiation must be subtracted for low-level measurements. For an observed number of background counts N_b during a background counting time t_b, the background count rate is

$$R_b = N_b / t_b . \qquad (1)$$

Similarly, for an observed number of gross counts N_g during a gross counting time t_g, the gross count rate, R_g, is

$$R_g = N_g / t_g . \qquad (2)$$

The net count rate, R_n, is

$$R_n \text{ in cps (or cpm)} = R_g - R_b = \frac{N_g}{t_g} - \frac{N_b}{t_b} \qquad (3)$$

The standard deviation of net count rate, $s(R_n)$, is

$$s(R_n) = \sqrt{R_g/t_g + R_b/t_b} = \sqrt{N_g/t_g^2 + N_b/t_b^2} . \qquad (4)$$

The standard deviation is a measure of precision or repeatability, and therefore is a valid, distribution-free measure of uncertainty due to random statistical fluctuations during counting. The coefficient of variation, also called the "relative standard deviation," expresses the standard deviation as a fraction of the net count rate:

$$\text{Coefficient of Variation} = C.V. = \frac{s_n}{R_n} \qquad (5)$$

The *bias* of a measurement result is the degree to which it equals a "conventionally true value" or standard value for the quantity being measured. Standard deviation is a measure of *precision* or repeatability, and therefore is a valid, distribution-free measure of uncertainty.

3.2.1 Relating Count Rate to Activity: Counting Yield

The counting yield K (also known as counting efficiency) is the expected number of counts observed per radioactive transition, usually a number less than 1 that is intrinsically dimensionless. Equally well, K relates count rate to activity as [counts per second] / [transitions per second] or cps/Bq. Other common expressions for K are in units of cpm/dpm, cpm/nCi, cpm/μCi, etc. The activity A is

$$A = \frac{R_n}{K}. \qquad (6)$$

K is determined experimentally for a given detector, geometry, absorption, radionuclide, etc.

$$s(A) = \frac{s_n}{K} = A(C.V.). \qquad (7)$$

If the counting yield K is wrong, the measurement result will be inaccurate or biased, regardless of how precise it is.

One should **report** the amount of activity in this sample as $\boldsymbol{A \pm s(A)}$ **Bq (1 s.d.)** (i.e., 68% confidence limits). Multiples of the standard deviation contain no more information thatn the standard deviation.

3.3 Qualitative Notions of Decision Level and Minimum Detectable Amount

Two very important statistical concepts, the decision level (*DL*; a.k.a. critical level [L_C]) and the minimum detectable amount (*MDA*; a.k.a. detection level [L_D], lower limit of detection [*LLD*], ...) are based on the standard deviation of the net count rate when an appropriate blank is being counted. *DL* and *MDA* are covered mathematically later. Suffice it to say at this point that one can determine, in advance of receiving a sample, how small an amount of radioactive material is likely to be distinguishable from background with a giving counting system and choice of counting times: this amount is the *MDA*. The *MDA* is the value that one can legitimately advertise that one can measure with reasonable assurance. Once one has made a measurement on a sample, one may wish to decide whether there is indeed any activity above background in the sample. This is done by comparing the counting result to the *DL*, a value typically about half the *MDA*. Yes, it is possible and not even infrequent to be sure one has detected activity less than the *MDA* but more than the *DL*. It should be remembered that the smaller the amount of activity in the sample is

Table 1. Comparison of decision level and minimum detectable amount

	"DL"	***"MDA"***
Name	decision level	minimum detectable amount
Former Name	critical level, L_C	detection level, L_D
Former Name		lower limit of detection, *LLD*; also, unfortunately, "lower level discriminator"
What?	the lowest useable action level	NOT an action level!
Use:	compare measurements to *DL*	use in statement of work for a contractor: how much will you charge to provide counting services with this *MDA*? use in planning. use in advertizing
When?	*a posteriori*: *after* the measurement is made	*a priori*: *before* the measurement is made
Defined in	HPS/ANSI N13.30	HPS/ANSI N13.30
Turner's name	"minimum significant measured activity"	"minimum detectable true activity"
Strom's name	"false alarm level"	"advertizing level"

compared to the *MDA*, the less likely it is to result in a number of counts above the *DL*.

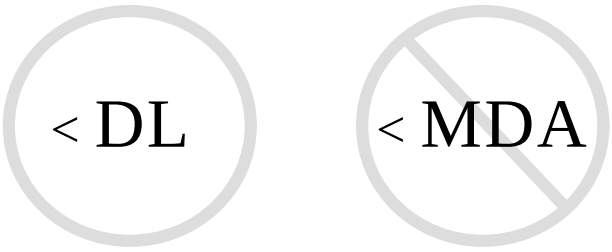

Figure 3. Never Compare a Measurement Result with the *MDA*; Compare It with the *DL*

Never compare a sample result to the *MDA*. Sample results should only be compared to a decision level. A statement that a result was "less than *MDA*" is statistical nonsense that originates with the poor choice of name for the *MDA*. The *MDA* is really the "if-it's-in-the-sample-you're-likely-to-detect-it" level, while the *DL* is the "if-you-got-a-result-above-this-it's-probably-real" level. A result above the *DL* is probably not a "false alarm."

One may want to evaluate the detection capability of a radioactivity measurement program. Toward this end, one may want to establish "action levels." An action level is a value of count rate (e.g., cps), concentration (e.g., $Bq \cdot m^{-3}$), or concentration × time (e.g., DAC-h) at or above which one chooses to take some action and below which that action is not deemed to be necessary. The simplest "action" one can take is to state, "Activity was (or was not) detected above background."

For operational purposes, the statistical concept of "decision level" is the lowest useable action level. Results of individual or pooled measurements are compared with the decision level. The decision level is a value chosen so that results above it are unlikely to be false alarms. Thus, the one chooses the decision level to be far enough above zero so that there is an acceptably low rate of false alarms due to random statistical fluctuations in the counting process (known to statisticians as "false positives").

The two correct decisions and the two principal errors one can make in statistical inference are shown in Table 2.

Table 2. Decisions based on measuring ("counting") whether radioactivity is present, in the presence of background.

		Is anything there? (Is any activity present [above blank]?)	
		Yes	No
Did I detect anything? (Was the result above the decision level?)	Yes	• I made the correct decision (no error)	• "false alarm" • false positive • I've committed a Type I error
	No	• the alarm should have sounded, but it didn't • false negative • I've committed a Type II error	• I made the correct decision (no error)

A Type I error is falsely concluding there's activity present when no activity is present.
A Type II error is falsely concluding there's no activity present when activity is present.

The probability of a Type I error is called α.
The probability of a Type II error is called β.

The number of standard deviations one must be above zero on the standard normal distribution to have a probability of α or β of being higher is known as the "standard normal deviate," k_α or k_β.

For $\alpha = 0.05$ (that is, a 5% chance of making a Type I error), $k_\alpha = 1.645$.
For $\beta = 0.05$ (that is, a 5% chance of making a Type II error), $k_\beta = 1.645$.

In many situations, the decision level is well below a level at which any action (other than recording the result) is taken.

Figure 8 shows the standard normal distribution with mean (μ) = 0 and standard deviation σ = 1. k_α is the value of the standard normal deviate *z* for which the corresponding *cumulative* standard normal distribution has a value of (1 - α). In the old days, one would look up values of k_α in a table; now, in a Microsoft Excel spreadsheet, putting =NORMSINV(1 - 0.05) returns 1.645.

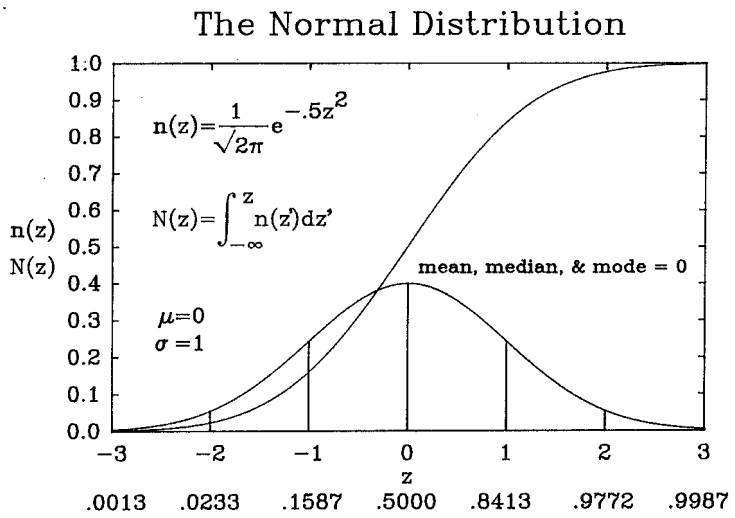

Figure 4. The Standard Normal Distribution, and the cumulative standard normal distribution. The standard normal deviate, *z*, is "the number of standard deviations from the mean."

Another concept one needs is that of "minimum detectable activity" or "minimum detectable concentration." Unlike the decision level or action levels with which individual or pooled results are compared, the minimum detectable quantities are performance gauges of a radioactivity measurement program that can be compared with a performance goal.

Whether or not one decides that activity is present, one must convert the observed count rate and its standard deviation into a result for the customer, such as activity, concentration, etc. One may also quote to the customer my degree of belief that the result is not background. A result with a net count rate equal to the *DL* has only a 5% chance of being due to background alone.

3.4 Sensitivity and Specificity

The terms below are defined for a binary condition, e.g., one has the measles or one doesn't.

Sensitivity: the proportion of correctly classified positives (the ability of a test to correctly classify a subject as positive).

Specificity: the proportion of correctly classified negatives.

Positive Predictive Value: the proportion of classified positives who really had the positive condition.

Negative Predictive Value: the proportion of those classified as negative who actually were negative.

Table 3. Sensitivity, specificity, positive predictive value, and negative predictive value

		Actual State	
		+	–
Classified State	+	*A*	*B*
	–	*C*	*D*

Sensitivity = $A/(A+C)$

Specificity = $D/(B+D)$

$PPV = A/(A+B)$

$NPV = D/(C+D)$

For interpretation of radiobioassay results, one can easily extrapolate the sensitivity and specificity to continuous variables which are a function of the underlying background and sample count rates, their respective counting times, and the value of the decision level. If one fixes the background count rate and the counting times, then pairs of sensitivity and specificity curves for each value of decision level can be plotted as a function of the true sample count rate.

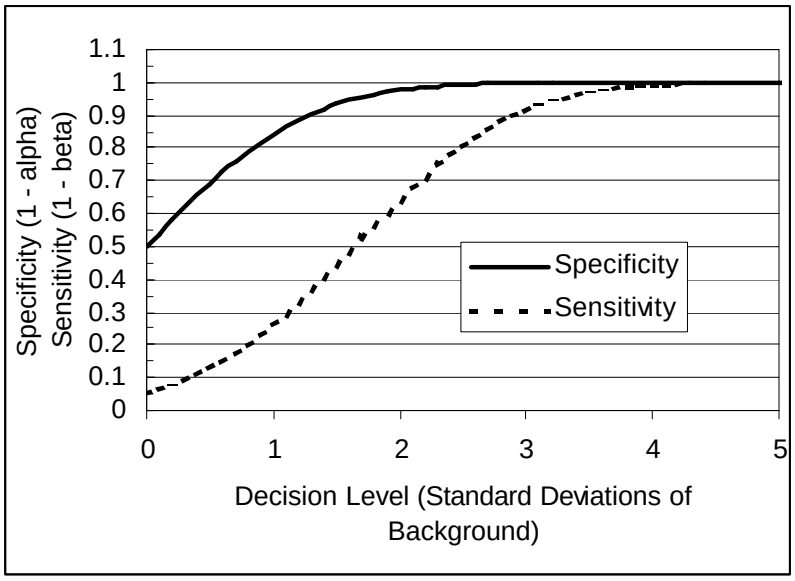

Figure 5. Specificity and Sensitivity as a function of *DL*

For radiobioassay, the proportions of values in each cell in the table depend on the nature of the population being tested. For routine bioassay, typically $(B + D) >> (A + C)$, that is, most persons are truly unexposed. The specificity is $(1 - \alpha)$ when everyone is unexposed. The minimum detectable amount *(MDA)* or L_D is that value of underlying true count rate that gives a sensitivity of $(1 - \beta)$ when the decision level is *(DL)* or L_C (Figure 5).

3.5 Detection Capability: Quantitative Treatment

3.5.1 Deciding Whether a Sample Is Above Background: The Decision Level

Any net count rate greater than the decision level represents the presence of activity in the sample. For significant number of background counts, the decision level for the net count rate is approximately (Strom and Stansbury 1992)(Lochamy 1976; Health Physics Society (HPS) 1996):

$$DL(R_n) \text{ in cps (or cpm)} = 1.645 \sqrt{R_b(1/t_b + 1/t_g)}, \quad (8)$$

where the 1.645 value corresponds to a 5% false alarm rate (i.e., 1 sample in 20 that has no activity present will exceed this count rate simply due to random statistical fluctuations).

One may assume that no activity is present in sample if the net count rate is less than the decision level; however, it is a good practice to record all counting results, whether above the decision level or not.

Equations are available in NUREG-1400 (Hickey et al. 1993) for cases where radioactive decay during sampling and counting may affect results. Equations are also provided in the appendix to that work that may improve precision and detection capability.

3.5.2 Measuring Detection Capability for a Counting System: Minimum Detectable Activity

A counting system may be characterized by a minimum detectable activity (*MDA*) for a specified choice of parameters such as counting times. Once a decision level has been specified by the choice of count times and the false alarm rate (here we adopt a 5% false alarm rate), it is possible to determine a value of activity that would yield a count rate less than the decision level a certain fraction of the time. This value of activity is called the *MDA*. The fraction of the time that an activity equal to the *MDA* would actually result in a count rate *less* than the decision level is called the *false negative rate*. Here we adopt a 5% false negative rate, i.e., 1 time in 20 a sample with an activity equal to the minimum detectable activity would actually result in a count rate less than the decision level. Under these assumptions, the *MDA* for the activity on the filter becomes

$$MDA \text{ in Bq (or } \mu\text{Ci)} = \frac{3 + 3.29\sqrt{R_b t_g (1 + t_g/t_b)}}{K t_g} \qquad (9)$$

where the terms are defined above (Currie 1968, 1984; Brodsky 1984; NCRP 1985). The *MDA* is a performance indicator for a counting system. Normally the *MDA* is compared with a performance goal rather than with the result of a measurement. The *MDA* is an amount of activity that yields a result above the decision level most of the time (95% of the time for this document). To contrast the decision level and the minimum detectable activity, consider the following: the decision level represents a count rate large enough that it is unlikely to be a "false alarm," but the minimum detectable activity represents an activity large enough that it is unlikely not to "set off the alarm," that is, an activity at or above the minimum detectable activity is likely to result in a count above the decision level (likely to "set off the alarm"). Note that it is quite possible that an activity less than the minimum detectable activity will "set off the alarm" or result in a count rate above the decision level.

For example, suppose that one has determined that 4 DAC-h are expected to result in an activity of 1.5 Bq (4×10^{-5} μCi) on the filter of an air sampler run for 8 hours. Would the counting system described in the examples (deleted to save space) have adequate detection capability to detect a 4 DAC-h exposure? The *MDA* becomes

$$MDA = \frac{3 + 3.29\sqrt{0.11 \times 100\ (1 + 100/1000)}}{0.33 \times 100} = 0.438\ \text{Bq} \qquad (10)$$

This is below the desired performance of 1.5 Bq (4E-5 µCi), so one can conclude that the counting system is adequate. If the minimum detectable activity had been greater than 1.5 Bq (4E-5 µCi), then one could have chosen to count the sample longer, used a more efficient counter, or chosen a counter with a lower background to reduce the minimum detectable activity until it was less than the desired goal. For other options when the minimum detectable activity is too high, refer to the section on "minimum average concentration."

Normally, measurement results (in terms of count rates) are compared with the decision level or other action levels. The minimum detectable activity, on the other hand, is normally compared with performance goals.

Bioassay programs are designed to detect intakes that would result in $H_{E,50}$ values of 1 mSv or more. There is a great deal of inference between a radioactivity measurement in a bioassay sample and the dose associated with it.

Similarly, because it is convenient to think of air-sampling programs (used for personnel dosimetry) in terms of concentrations, not activities, and because there are several other variables to be considered in determining concentrations, a more useful performance indicator for an air-sampling program (as contrasted with a counting system that is only a part of the program) is the minimum detectable concentration, described below.

3.5.3 Determining the Radioactivity Concentration

An integral part of an air-sampling program is the measurement of radioactivity and the subsequent interpretation of the data. Counts in a radioactivity measurement system come from both the background and samples. Under the assumption of constant concentration of radioactivity in the air during the time the sample is collected, and if sampling, decay, and counting times are short with respect to the half-life, the activity concentration is given by

$$C = \frac{R_n}{EFKt_S} \qquad (11)$$

where

C	=	concentration of radioactive material in the air in Bq m^{-3} (or µCi/cm^3)
R_n	=	net count rate in cps (or cpm)
E	=	fractional filter efficiency (% efficiency/100)
F	=	airflow rate through the sampler in $m^3\ s^{-1}$ (or cm^3/min)
K	=	counting efficiency in cps Bq^{-1} (or cpm/µCi)
t_s	=	duration of sample collection in s (or min).

3.5.4 Measuring Detection Capability for an Air-Sampling Program: Minimum Detectable Concentration

Suppose one wants to set a performance goal for an air-sampling program of being able to detect 0.1 × DAC. Such a choice would ensure that, for workers continuously present in the area, no intakes would occur that would result in a committed effective dose equivalent in excess of 1 mSv/y.

To determine if a program would meet this goal, one may calculate the minimum detectable concentration (*MDC*) of the equipment and procedures in the program. The *MDC* for any single measurement is

$$MDC \ in \quad \mathrm{Bq\,m^{-3}} \ (\text{or } \mu\mathrm{Ci/cm^3}) = \frac{3 + 3.29\sqrt{R_b t_g (1 + t_g/t_b)}}{E F K t_s t_g} \qquad (12)$$

where the symbols are as defined above. To have an air-sampling program that meets this detection capability goal, one may select procedures and equipment with values of flow rate, duration of sample collection, filter efficiency, counting efficiency, and gross and background counting times so that the *MDC* in Equation (12) is less than or equal to 0.1 × DAC (unless a weighted average of sample results for intervals less than 40 hours is used; see below).

3.5.5 Measuring Detection Capability for a Bioassay Program: Minimum Detectable Dose

The workplace scenario that one wishes to quantify begins with a worker who inhales a quantity of airborne radioactive materials in the workplace. If this occurs once, over a finite period of time, it is termed an acute intake. If it happens repeatedly or routinely, it is called a chronic intake. For simplicity, consider the acute intake case. In either case, a routine bioassay program may collect samples of excreta for analysis periodically.

After intake, some radioactive material translocates within the body, and is eliminated through natural biological turnover and, for materials whose half-life is short compared to the interval between bioassay samples, radioactive decay.

The fraction of the intake that remains in any bioassay compartment (e.g., a tissue or organ, or in excreta) is described by an intake retention function (*IRF*) for that compartment (International Commission on Radiological Protection (ICRP) 1979; Lessard et al. 1987; Raabe 1994). The *IRF* for urine or feces is the expectation value of excretion for a calculational model known as Reference Man (International Commission on Radiological Protection (ICRP) 1975). Bioassay compartment *IRF*s may decrease monotonically with time, or they may be more complex.

For individuals, even those who are well-described on the average by Reference Man-based *IRF*s, excretion rates are characterized by intra-subject variability. Inter-subject variability is seen in individuals whose excretion rates differ systematically from those predicted by the Reference Man

models. Such "biological variability," denoted by V_{bio}, can occur in excretion rates and volumes.

If radiochemical separation of radioactive materials is required, such as in preparation of samples for spectroscopy of alpha emitters, the fraction of the nuclide of interest that is recovered from the sample is known as the radiochemical yield, Y_{RC}. Radiochemical yield is variable, but is usually measured.

Intakes of radioactive material are related to $H_{E,50}$ by "dose per unit intake" factors published by various groups (Eckerman et al. 1988; International Commission on Radiological Protection (ICRP) 1994) For workers, the 50-year committed effective dose per unit intake for inhalation, $e_{inh}(50)$, is a function of radionuclide, lung clearance type, and particle size (International Commission on Radiological Protection (ICRP) 1994). Similar factors $h_{E,50}$ are available for $H_{E,50}$ in the EPA Federal Guidance Report 11.

The minimum detectable dose *MDD* for a bioassay program is directly proportional to the *Minimum Detectable Intake*, which is the *MDA* as modified by several other factors:

$$MDD = (Minimum\ Detectable\ Intake) \times h_{E,50} = \frac{MDA}{Y_{RC}\, V_{bio}\, IRF_u(\Delta t)} \times h_{E,50} \qquad (13)$$

A key difficulty is that $IRF_u(\Delta t)$ is a strongly-varying function of time Δt between intake and bioassay measurement. When the time of intake is unknown (as is usually the case for routine bioassay measurements, some reference value of time must be chosen for evaluating the *MDD*. The ICRP has recommended the midpoint of the interval, but in general this gives biased results. Another possibility is to assume that the intake occurred immediately after the previous bioassay sample, but this also leads to bias, usually significantly overestimating doses.

4 References

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Table 4. Summary of Symbols, Quantities, and Units

Symbol	Quantity	SI Unit	Traditional Unit
t_b	background counting time	s	min
t_g	gross counting time	s	min
Δt	time between intake and bioassay sample	d	d
N_b	number of background counts observed	-	-
N_g	number of gross counts observed	-	-
N_n	number of net counts observed	-	-
R_b	background count rate; estimator of ρ_b	s^{-1}	counts min^{-1}
R_g	gross count rate	s^{-1}	counts min^{-1}
R_n	net count rate; estimator of ρ_n	s^{-1}	counts min^{-1}
ρ_b	population count rate due to background	s^{-1}	counts min^{-1}
ρ_n	population count rate due to analyte	s^{-1}	counts min^{-1}
s_n	standard deviation of the net count rate	s^{-1}	counts min^{-1}
Y_{RC}	radiochemical yield (recovery)	-	-
V_{bio}	biological variability	-	-
K	counting efficiency	s^{-1} Bq^{-1}	counts min^{-1} μCi^{-1}
$p(A)$	unconditional probability	-	-
$p(A\|E)$	conditional probability of A given E	-	-
A	activity	Bq	µCi
s_A	standard deviation of the activity	Bq	µCi
C	activity concentration	Bq m^{-3}	µCi/cm^3
s_C	standard deviation of activity concentration	Bq m^{-3}	µCi/cm^3
α, β	probabilities of Type I and Type II errors		
k_α, k_β	standard normal deviates corresponding to α and β		
$DL(R_n)$	decision level for net count rate	s^{-1}	counts min^{-1}
MDA	minimum detectable activity	Bq m^{-3}	µCi/cm^3
MDC	minimum detectable concentration	Bq m^{-3}	µCi/cm^3
MDD	minimum detectable dose	Sv	rem
$h_{E,\,50}$	$H_{E,\,50}$ per unit intake	Sv/Bq	rem/µCi
$e_{inh}(50)$	committed effective dose per unit intake	Sv/Bq	rem/µCi
~	logical “not”	-	-
—	a bar over a symbol denotes “average,” e.g., $\overline{R}_n$, $\overline{C}$, $\overline{N}$, $MD\overline{C}$	-	-
IRF	intake retention function	-	-