MIDWEST RESEARCH INSTITUTE

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Gentlemen:

Please accept this as a quarterly progress report for the subject contract.

In the previous report we developed a continued fraction representation of the solution to the differential equation

$$\frac{d^2y}{dt^2} + ay + by^3 = 0 \ , \ y(0) = a_o \ , \ y'(0) = 0 \quad . \tag{1}$$

The analysis is easily extended to include the case of either a unit impulse or unit step input.

Following the analysis given in the first report, let

$$y = \cfrac{a_o}{1 + \cfrac{a_1 x}{1 + \cfrac{a_2 x}{1 + \cdot_{\cdot_{\cdot}}}}} \ , \ x = t^2 \tag{2}$$

be the continued fraction representation of the solution to (1).

If the continued fraction (2) is truncated, there results a rational approximation to the solution of (1). Generally, the sequence of convergents of the continued fraction converge much faster than the sequence of partial sums of the power series representation of the solution to (1). This is especially true when the function has a pole near the origin. For if the distance from the origin to the pole nearest the origin is r, then r is the radius of convergence of the Taylor series expansion. However, the rational approximation mimics the pole of the function and so gives a more powerful representation. See the numerical example included in this report.

Now the even approximants of (2) are known as the main diagonal Padé approximants which have the following property:

let $y = \sum_{k=0}^{\infty} c_k x^k$ be the power series solution to (1) and let $y_n = \frac{A_n}{B_n} = \frac{\sum_{k=0}^{n} a_{n,k} x^k}{\sum_{k=0}^{n} b_{n;k} x^k}$

be the n^{th} order main diagonal Padé approximant. Then if B_n is formally divided into A_n the resulting power series agrees with the power series solution for the first $2n+1$ terms.

Consider (1) where $a = 10$, $b = 100$ and $a_o = 1$. The solution is given by the Jacobian elliptic function $y = \mathrm{cn}(u,k)$ where $u = \sqrt{110}\, t$ and $k^2 = 5/11$.

Tables I and II compare the values of the Padé approximants and the corresponding partial sums of the power series solution to the true solution. Note that, since the solution to (1) has its smallest pole at approximately $u_o = 1.9i$, the power series is ineffective for computation if its argument has magnitude close to 1.9; while the Padé approximations are very efficient as is illustrated by the tables.

We now present a perturbation technique to analyze the response of (1) when a small amount of damping is present. Consider

$$y'' + ay' + y + by^3 = 0 \ , \ y(0) = A \ , \ y'(0) = 0 \quad , \qquad (3)$$

where a is a small parameter, $|a| < 1$. Assume that the response y is given by

$$y = \sum_{k=0}^{\infty} a^k y_k \quad . \tag{4}$$

Substitution of (4) into (3) and equating coefficients of a to zero yields an infinite set of differential equations, the first of which is the only nonlinear equation. The first two equations are

$$y_o'' + y_o + by_o^3 = 0 \ , \ y_o(0) = A \ , \ y_o'(0) = 0 \quad , \tag{5}$$

and

$$y_1'' + (1+3by_o^2)y_1 = - y_o' \ , \ y_1(0) = y_1'(0) = 0 \quad . \tag{6}$$

Note that we have developed approximations to the solution of (5) which is

$$y_o = A \ \mathrm{cn}(u,k) \ , \ u = (1+A^2b)^{1/2}t$$

and

$$k^2 = \frac{A^2b}{2(1+A^2b)} \quad . \tag{7}$$

Thus, the approximate solution to (3) up to and including the first correction term is

$$y \cong y_o + ay_1 \tag{8}$$

where y_1 is obtainable in terms of integrals of elliptic functions. Although the results are not yet in attractive form, work is progressing along these lines. Other techniques suggested by physical considerations are also being studied to incorporate the effects of damping.

Very truly yours,

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TABLE I

MAIN DIAGONAL PADÉ

u	y (True)	$y_1(t)$	$y_2(t)$	$y_3(t)$
0.1751	0.9848	0.9848	0.9848	0.9848
0.3524	0.9396	0.9397	0.9396	0.9396
0.5349	0.8658	0.8660	0.8658	0.8658
0.7247	0.7655	0.7662	0.7655	0.7655
0.9240	0.6417	0.6444	0.6417	0.6417
1.1327	0.4993	0.5070	0.4993	0.4994
1.3550	0.3399	0.3585	0.3397	0.3399
1.5837	0.1724	0.2108	0.1713	0.1724
1.8238	-0.0040	0.0668	-0.0077	-0.0041

TABLE II

TAYLOR'S SERIES EXPANSION

u	y (True)	Three Terms	Five Terms	Seven Terms
0.1751	0.9848	0.9848	0.9848	0.9848
0.3524	0.9396	0.9397	0.9396	0.9396
0.5349	0.8658	0.8666	0.8658	0.8658
0.7247	0.7655	0.7698	0.7656	0.7655
0.9240	0.6417	0.6587	0.6427	0.6418
1.1327	0.4993	0.5518	0.5061	0.5002
1.3550	0.3399	0.4778	0.3762	0.3495
1.5837	0.1724	0.4846	0.3258	0.2474
1.8238	-0.0040			