Energy deposition in TPC

Hans Bichsel

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1211 22^nd Ave East, Seattle, Wa, USA 98112-3534

Yuri: As I mentioned on the telphone, I wrote this paper in 1994. I have made a few changes just now.

1. Introduction

The detection of fast charged particles in the time projection chamber (TPC) is based on the measurement of their ionization in the gas. The passage of a particle through the TPC is along a "track". It is quite easy to calculate the *energy loss* (straggling) function $f(\Delta)$ [4], but the ionization observed in the TPC is given by the *energy deposited* in the gas, y, with a spectrum $\phi(y)$. The calculation of energy deposition is more complex than that of energy loss because the geometry of the detector must be taken into account. The reason can be seen in Fig. 1. A particle with rest mass M, charge ze, kinetic energy T and velocity v traverses padrows with limited dimensions. Secondary electrons (here called δ rays) produced in the collision may have ranges exceeding the dimensions of the volumes under observation. The padrow for which the ionization is calculated is region II in Fig. 1. Some of the energy lost by the particle in region II is carried out of the detector by the δ rays (here called "starters"), and the energy deposited is less than that lost [16]. On the other hand, some energy may be imported by particles created in other regions ("stoppers" and "crossers"). The starters of region II will be stoppers or crossers in region III. For large thin slabs of material, it is possible to use an analytic calculation to calculate the total energy deposited [14], but for the calculation of spectra, it appears that Monte Carlo calculations are the most suitable method [3, 4, 10, 17]. Such spectra have been calculated for a typical geometry of TPCs, and are presented here. The emphasis is on the determination of differences of the spectra $f(\Delta)$ and $\phi(y)$ relevant to particle identification (PID). It was shown in a previous paper [7] that for a large number of padrows (e.g., > 20) an efficient method to achieve PID is to select the smallest 60% of the observed y. This choice excludes y above about $1.8 y_p$, where y_p is the most probable energy deposition, then differences between $f(\Delta)$ and $\phi(y)$ for $y > 1.8 y_p$ are not important and no effort is made to calculate them accurately.

2. Concepts.

The Monte Carlo calculation simulates the physical process shown in Fig. 1. Random numbers are used to generate the location of the production of a δ ray and its energy E. The δ rays are represented by arrows indicating their initial velocity u. Those contributing to the ionization in the central volume II can be described as insiders i, starters s, stoppers p and crossers c. The ray a will not contribute anything.

For the calculation, the following data must be known:

1. the total macroscopic collision cross section Σ_t representing the probability for collisions by the particles, or, equivalently, the mean free path $\lambda = 1/\Sigma_t$ giving the average distance between collisions;

2. the differential collision cross section $\sigma(E, \beta)$ representing the probability for a collision with an energy loss E of particles with speed $v = \beta c$;

3. the angle of emission, θ, of the δ ray with respect to the trajectory of the particle;

4. an energy deposition function for the δ rays;

5. the geometry of the detector volume.

In order to simplify the present calculations, several assumptions were made:

1. the deceleration of particles during their traversal through the TPC is negligibly small,

2. on the *average*, electrons travel along a straight line given by their initial velocity;

3. the ionization, J, is proportional to the energy deposited, y [4, 11];

4. the particle velocity v is perpendicular to walls delineating the separation between regions, and no lateral boundaries are used.

5. backscattering of electrons from region III into region II is disregarded; it usually amounts to less than 10%.

Since the mean free path for nuclear reactions (of the order of 100 g/cm^2) is much larger than the dimensions of the detectors described here, they are not considered in the calculations.

The amount of calculation can be reduced if the smallest dimensions of the volumes of interest greatly exceed the ranges of electrons with energies less than a given value E_r. Effective ranges of low energy electrons are given in Table 1. Specifically, the energy deposition by δ rays with energy less than E_r is summed into a total energy deposition for all collisions with $E < E_r$ which is set equal to the energy loss of the particle, and a Monte Carlo calculation is only necessary for collisions in which $E > E_r$. Here padrows of width 4 cm were used. Calculations with different values of E_r can demonstrate how large the effects of this approximation are.

3. Collision spectrum and straggling functions

The collision spectrum $\sigma(E,\beta)$, giving the probability for an energy loss E, is shown in Fig. 2. An approximation based on the Fermi-virtual-photon method (FVP, [8, 1, 6, 15]) was used to calculate it. Energy loss straggling functions $f(\Delta)$ were calculated by convolutions [4]. If the collision spectrum is divided into two parts, the "restricted" straggling function $f_r(\Delta)$ is calculated with convolutions for the lower part, energy losses $E < E_r$, and Monte Carlo calculations are made for the upper part $E > E_r$.

For preliminary estimates, a rough, nonrelativistic approximation of Σ_t can be obtained from the Rutherford cross section for collisions with energy loss exceeding E_r (eV)

$$_R\Sigma_t \;=\; \frac{6.4}{\beta^2}\,\frac{z^2\;Z}{E_r}\quad /\;cm \tag{1}$$

for molecules with Z electrons. Thus for $E_r = 10$ keV, the mean free path, $\lambda = 1/\Sigma_t$, for relativistic particles (i.e., $\beta \sim 1$) with charge $z = 1$ in Ar is $\lambda \sim 90$ cm, and in $x = 4$ cm the mean number of collisions with energy loss greater than $E_r = 10$ keV is $x/\lambda = 0.05$. A better value is obtained with

$$\Sigma_t(E_r,\beta) \;=\; \int_{E_r}^{E_M} \sigma(E,\beta)\; dE \tag{2}$$

where $E_M = 2mc^2\beta^2\gamma^2$. Values of Σ_t are given in Table 2. The Rutherford approximation is good only for $E_r > 10$ keV. This is also evident from Fig. 2.

4. Energy deposition by δ rays

The angle θ of emission of the δ rays must be known. If their energy E is much larger than the binding energy in the atoms, θ can be approximated by the relativistic kinematic relation for the collision:

$$cos\ \theta \ = \frac{2E(m_0c^2 + T + M_0c^2)}{2\sqrt{E(E + 2m_0c^2)T(T + 2M_0c^2)}} \tag{3}$$

where m_0 is the rest mass of the electron. The non-relativistic approximation is:

$$cos\ \theta \ = \ \sqrt{E/E_M} \tag{4}$$

For a correct calculation of the path and energy deposition by the electrons, it would be necessary to use a Monte Carlo program such as ETRAN [2] which takes into account the angular deflections. A simpler method is used here, using "effective ranges" for the electrons, shown in Table 1. The distance τ from the point of origin of the δ ray to the planes separating regions is calculated. The energy E_l leaving the volume is obtained by reverse interpolation for $R(E_l) \ = \ R(E) - \tau$. The energy deposition in the detector then is $E_d \ = \eta(E, \tau)\ (E - E_l)$, where $\eta(E, \tau)$ accounts for the attenuation of the electron flux due to multiple scattering [13, 14].

5. Monte Carlo calculations

In a Monte Carlo calculation, the interactions of the particles are simulated with random numbers [5]. The energy loss of each particle consists of two parts: a total loss Δ_r due to collisions with $E < E_r$ and losses E_i for collisions with $E > E_r$. The loss Δ_r is selected at random from the straggling function $f_r(\Delta)$. It is assumed that this energy loss will be deposited completely in the detector. Added to this are losses with $E > E_r$.

The distance x between successive collisions is calculated with

$$x = -\lambda \ \ln \ \frac{1}{r_r} \tag{5}$$

where r_r is a random number between 0 and 1. A second random number is used to select the energy loss E. In the execution of the program, the following approach was used: from the probability density function for energy loss:

$$Q(E,\beta) = \int_{E_r}^{E} \sigma(E,\beta)\ dE \ / \int_{E_r}^{E_M} \sigma(E,\beta)\ dE \tag{6}$$

the inverse function $E(Q)$ is calculated with cubic spline interpolation for 10000 equally spaced values of Q. The value E is found at the index $j = 10000\ r_r$ in the table.

The energy deposited, y_d, by the δ ray then is calculated with one of the methods described in section 4, and for each particle the total energy deposited y is calculated, with contributions

$$y = \Delta_r + \sum \Delta_i + \sum y_s + \sum y_p + \sum y_c \tag{7}$$

where Δ_i are the energy losses of insiders, y_s, y_p and y_c are the energy depositions by starters, stoppers and crossers, respectively, each with probabilities given in Tables 3 and 5. Usually, for 600 MeV pions, one insider will occur, but only for about one in a hundred pions will there be a starter or stopper.

The contributions to Δ are

$$\Delta = \Delta_r + \sum \Delta_i + \sum \Delta_s \tag{8}$$

where Δ_s are the energy *losses* of starters (i.e. they include what goes into region III).

Spectra were calculated for various contributions:

ϕ_s for $y = \Delta_r + \sum \Delta_i + \sum y_s$

ϕ_p for $y = \Delta_r + \sum \Delta_i + \sum y_s + \sum y_p$

ϕ for $y = \Delta_r + \sum \Delta_i + \sum y_s + \sum y_p + \sum y_c$

$f(\Delta)$ for $\Delta = \Delta_r + \sum \Delta_i + \sum \Delta_s$

Calculations were made for 1,000,000 particles. The data were analyzed by summing the number of events in intervals $[y, y + dy)$ and $[\Delta, [\Delta + d\Delta)$ into bins ("multi channel analyzer").

6. Results

The influence of the δ rays crossing one of the boundaries of region II is as follows:

starters: because y_s is smaller than Δ_s, $\phi(y = \Delta)$ will be larger than $f(\Delta)$, their total number will depend on the width of region II only.

stoppers: the contribution y_p will increase y compared to Δ, and $\phi(y = \Delta)$ will be less than $f(\Delta)$, the number of stoppers increases with the size of region I, but mainly for $y > 15$ keV.

crossers: for $x_2 = 4$ cm the smallest possible value of y_c for pions with energies exceeding 600 MeV is $y_c = 15$ keV, for 30 MeV pions, it is 20 keV; the effect of crossers therefore is a reduction of the part of the spectrum relevant for PID by a constant fraction given by the number of crossers listed in Table 3. They amount to about half of the spectrum for $y > 20$ keV.

Calculations were made for 30, 600 and 10,000 MeV pions traversing argon, Fig. 1. The collision cross section $\sigma(E, \beta)$ depends only on particle speed, but the results will differ slightly for electrons, mesons and protons because the production angle θ, Eq. (3) depends on particle mass. The calculations are fast, for $x_1 = 10$ cm they take only one minute on a VAX-alpha 3000-400. Total numbers of δ rays contributing to the spectra are given in Table 3. Evidently, the choice of E_r influences the number of starters and stoppers very little.

Results for different widths of region I are given in Table 4. As expected, the numbers of starters are the same within stochastic fluctuations. The number of stoppers increases slightly with x_1, while the number of crossers increases with $x_1^{2/3}$.

A comparison of energy loss and deposition spectra for pions with kinetic energy $T = 600$ MeV and $E_r = 700$ eV for $x_1 = 100$ cm is made in Fig. 3. The spectrum for the energy losses calculated with eq. (8) agrees very closely with the spectrum calculated with convolutions for $E_r = E_M$. No differences between the most probable values, Δ_p and y_p, have been seen.

The differences of the numbers of events with $0 < y < 2 \cdot y_p$ and $0 < \Delta < 2 \cdot \Delta_p$ are given by

$$n_k = \int_0^{2\cdot y_p} \phi_k(y)\ dy \quad and \qquad n_f = \int_0^{2\cdot\Delta_p} f(\Delta)\ d\Delta. \tag{9}$$

and are shown in Table 5. The contributions from starters and stoppers are quite small for small y and in most cases are less than the stochastic fluctuations of count rates.

The major use of the knowledge of energy deposition is in particle identification. It was shown [7] that a practical method for identification in a TPC is the use of a reduced sample: typically, the low 60% of all measured ionization values are used. For the spectrum shown here, the upper limit of the energy loss then is about 6.7 keV. The net effect of the crossers will be a small increase in the mean value of the selected low values of experimental y used for PID, but this will occur for all particles and will have a minute influence on PID.

Clearly, the differences are quite small and can be disregarded for particle identification. For completeness, spectra extending to 150 keV are also given in Fig. 4.

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Table 1. Effective ranges R of electrons with kinetic energy E. The density, $\rho = 1.56$ mg/cm^3 for the P-10 mixture (90% Ar, 10% CH_4) at 760 torr and 293 K is used. Note that the difference between $\sigma(E, \beta)$ for argon and CH_4 is quite small [9]. Ranges R_c calculated with csda are also given [12]. The table was obtained as follows: for $E > 50$ keV, effective ranges were calculated with the algorithm given in [13], for $E < 10$ keV, experimental effective ranges measured for nitrogen [19, 18] were assumed to be the same for argon (with an uncertainty of maybe 10%). Between 10 and 50 keV, calculated ranges were reduced smoothly to the experimental value at 10 keV. The uncertainty of R is of order 20%.

$E(keV)$	$R(mg/cm^2)$	$R(cm)$	$R_c(mg/cm^2)$
0.1	0.0003	0.0002	–
0.2	0.0006	0.0004	–
0.4	0.0015	0.001	–
0.7	0.0028	0.0018	–
1.0	0.0048	0.0031	–
2.0	0.016	0.01	–
4.0	0.046	0.03	–
7.0	0.12	0.077	–
10	0.21	0.14	0.39
20	0.82	0.42	1.29
40	2.45	1.4	4.28
70	6.2	4	11.2
100	11.2	7.2	20.4
200	34.3	22	63
400	98	63	179
700	212	136	381
1000	335	215	594

Table 2. Values of $\Sigma(E_r, \beta)_t$ (cm^{-1}) calculated with Eqs. (1) and (2) for $\beta = 0.982$ (600 MeV pions, $z = 1$), Ar ($Z = 18$), $E_M = 26$ MeV. Since ${}_R\Sigma_t$ does not include relativistic effects, it is too large at 100 keV.

$E_r(keV)$	${}_R\Sigma_t$	Σ_t
0.1	1.2	2.46
0.2	0.6	1.77
0.4	0.3	0.63
0.7	0.17	0.26
1	0.12	0.162
3	0.04	0.0546
10	0.012	0.0124
30	0.004	0.0039
100	0.0012	0.0011

Table 3. Number of δ rays produced by the passage of 1,000,000 pions with energies T through a TPC (Fig. 1), calculated with two different cutoff energies E_r, for $x_1 = 10$ cm, $x_2 = 4$ cm. The subsets of events for which the initial δ ray energy was < 10 keV and < 2 keV are also given. The results from a single calculation (with 1,000,000 particles) are given for each case, they therefore have stochastic errors of order $\sqrt{N}$. For electrons with $\beta\gamma = 5.2$, the numbers of starters and stoppers are about 10% larger than for pions.

$T(MeV)$	$30\ MeV$		$600\ MeV$		$10\ GeV/c$	
$E_r(eV)$	700	3200	700	3200	700	3200
insider i	2,599,429	518,549	1,040,085	206,592	1,246,901	243,243
$E < 10keV$	2,480,414	400,233	996,596	162,975	1,203,116	199,468
$E < 2keV$	1,810,028	−−	735,848	−−	899,252	−−
starters	20,900	20,411	6633	6578	6398	6383
$E < 10keV$	859	646	185	149	228	169
$E < 2keV$	99	−−	13	−−	27	−−
stopper p	16,500	16,100	4524	4538	4508	4376
$E < 10keV$	843	662	164	133	203	179
$E < 2keV$	91	−−	24	−−	23	−−
crosser c	14,740	14,384	6080	5966		

Table 4. Number N contributions from various sources of δ rays from the passage of 1,000,000 pions with energies $T = 600$ MeV through a TPC with $x_2 = 4$ cm (Fig. 1), calculated with cutoff energy $E_r = 700$ eV, for different widths x_1 of region I.

$x_1(cm)$	*starters*	*stoppers*	*small p*	*crossers*
5	6536	3904	187	3479
10	6623	4556	190	6051
20	6610	5172	184	10054
40	6566	5626	174	16045
70	6471	5862	176	22304
100	6616	5898	182	26884

Table 5. Differences between different energy deposition spectra and the energy loss spectrum $f(\Delta)$, for $0 < y < 2 \cdot y_p$. The numbers are given by the integrals of Eq. (9).

$T(MeV)$	Δ_p	n_f	$n_f - n_s$	$n_f - n_p$
$10,000$	7.0	$856,784$	-661	2809
600	5.2	$868,291$	-519	2930
30	13.6	$796,733$	-2857	8310

Figure captions

Fig. 1. A charged particle traversing a layer of gas, in a TPC. The region of interest, a "padrow", is denoted as region II. Most δ rays will deposit their energy inside the region in which they are created (e.g. a in region I, i in region II), but some will escape from their region of origin and will deposit their residual energy in other regions: a stopper p starts in I, but stops in II; a crosser c starts in I, deposits some energy in II and stops in III; a starter s starts in II and stops in III. Calculations were made for several widths of region I, while region II which represents a "pad" in a TPC was 4 cm wide.

Fig. 2. Collision cross section $\sigma(E)$ for energy E in single collisions in argon gas. The ratio of $\sigma(E)$ to the Rutherford cross section is given. The spectrum extends to $E_M = 27$ MeV, and is the same for all particles with charge $z = 1$ and $\beta\gamma = 5.2$.

Fig. 3. Energy loss (dotted line) and deposition (solid line) spectra for particles with charge $z = \pm 1$ and $\beta\gamma = 5.2$ (approximately minimum ionization) traversing a TPC pad of width 4 cm argon. The bin size was 0.1 keV for both Δ and y. The length of region I was 100 cm. Most of the difference is due to crossers.