Chapter 2.
The Physics Background and Motivation

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2.1. Introduction

In this chapter we review the motivation for the experiment, as well as giving a brief history of the three measurements which have been carried out at CERN. [1-3] Before discussing the theoretical interest in the muon $(g-2)$ value or anomalous magnetic moment $a_\mu = \frac{1}{2}(g-2)$, we review spin motion in a magnetic field, and also discuss the electron g value.

2.1.1. The Muon

The muon is an extremely interesting lepton whose relatively long lifetime of 2.2 μs ($c\tau = 658.65m$) makes it possible to perform precision measurements. Although its mass of 105.658387 (34) MeV is 207 times the electron's, to the accuracy of current experiments the muon appears to be a pointlike lepton. Because its mass is so much heavier than the electron's there are measurable contributions to its anomalous moment from μ, τ, π (hadrons), and $W^\pm$ and Z^0, since as compared to the electron, the relative contribution of heavier objects to the anomalous moment [4] scales as $(m_\mu/m_e)^2 = 4 \times 10^4$.

The final CERN experiment [3] clearly established the presence of hadronic radiative corrections to the muon g value. One of the principal goals of the current experiment is to reach a sensitivity adequate to establish the presence of weak radiative corrections from single loop W and Z^0 graphs. The experimental goal of ± 0.35 *ppm* precision, which represents a factor of 20 improvement over CERN, is set by the predicted 1.3 *ppm* contribution to the muon anomaly from weak radiative corrections. This measurement will provide a clean test of the renormalization prescription of the unified electroweak theory.

The muon anomaly has provided a stringent test for new theories of particle physics, since any new field or particle which couples to the muon must contribute to a_μ. The magnitude of the effect is a quantitative question of the strength of the particle's coupling to the muon and its mass.

In order to achieve a deeper and less arbitrary theory for particle physics than the present Standard Model, many speculative theories have been proposed which include new particles such as additional gauge bosons, supersymmetric particles and excited leptons. Others include substructure for leptons, gauge bosons, or quarks. All of these modifications of the Standard Model would change a_μ(theor), often by an amount which would be detected by a measurement of a_μ(expt) to an accuracy of 0.35 *ppm*. The muon anomalous g-value can be regarded as a standard against which any speculative new theory can be tested; such precision tests are complementary to direct searches for new particles with the highest energy accelerators.

Historically, the Lamb shift and the electron and muon $(g-2)$ values have played a key role in the discovery and establishment of modern quantum electrodynamics. [5] Precision measurements of hyperfine structure intervals and the Zeeman effect in hydrogen, muonium and positronium have provided important tests of QED bound state theory as well as values for the magnetic moments of fundamental particles. High energy experiments can explore the physics of the W, Z^0, and other particles by direct production and are certainly the definitive way to identify new particles and new physics which are energetically accessible.

On the other hand, it is difficult in these high energy experiments to study the properties of more massive particles which cannot be produced by the accelerators. In principle, the existence of heavier particles can be detected through their effects via virtual radiative processes on the behavior of lighter observed particles. Precise and sensitive experiments such as the muon $(g-2)$ measurement will give us an important insight into the domain of physics which is not now accessible to high-energy experiments. In this sense low-energy, high-precision experiments play a role complementary to those at high energy colliders.

A high precision measurement of a fundamental quantity such as the muon $(g-2)$ value, for which a precise value can be calculated from basic theory, provides an important calibration point for modern particle theory. Not only is this valuable for the insight it provides about the very high energy regime beyond present accelerators, but also it may reveal new and deeper aspects of physics within the accessible energy regime, as was so dramatically illustrated for quantum electrodynamics by the Lamb shift in hydrogen and the anomalous g-value of the electron.

2.1.2. Spin Precession in a Magnetic Field

Charged particles which possess an angular momentum $\vec{L}$, have an associated magnetic dipole moment which is given by

$$\vec{\mu}_\ell = g_\ell \frac{e}{2mc} \vec{L}$$

where the gyromagnetic ratio for an orbital angular momentum $g_\ell = 1$. The energy associated with a dipole in a magnetic field is

$$H = -\vec{\mu} \cdot \vec{B}$$

and the torque it experiences is given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The latter equation leads to a (nonrelativistic) Larmor precession with frequency

$$\omega_\ell = \frac{g_\ell}{\hbar} \left(\frac{e\hbar}{2mc} \right) B$$

For spin angular momentum

$$\vec{\mu}_s = g_s \frac{e}{2mc} \vec{S}$$

where $g_s = 2$. The formulation of the Dirac equation, and its prediction that the gyromagnetic ratio of a spin 1/2 particle is exactly 2 (in the absence of a Pauli term) was a major step in the development of the theory of subatomic physics.

The nonrelativistic Larmor precession frequency for a positively charged particle is given by

$$\omega_s = \frac{g_s}{\hbar} \left(\frac{e\hbar}{2mc} \right) B = \left[1 + \frac{g_s - 2}{2} \right] \frac{e}{mc} B. \tag{2.1.1}$$

where the second term in brackets is the anomalous moment,

$$a = \frac{g_s - 2}{2} \tag{2.1.2}$$

and the (total) magnetic moment is given by

$$\mu = \left[\frac{q}{e} + a \right] \frac{e\hbar}{2mc} \tag{2.1.3}$$

which is the number quoted in the Particle Data Tables. [6] The first term in the square bracket is the Dirac moment (which is +1, 0 or −1 depending on the charge q), and the quantity a is the anomalous (Pauli) moment. We will drop the subscript s on g in the discussion below. Note that in the Particle Data Tables, three different magnetons are used: the Bohr magneton ($\mu_B = \frac{e\hbar}{2m_e c}$), the muon magneton ($\mu_\mu = \frac{e\hbar}{2m_\mu c}$) and the nuclear magneton ($\mu_N = \frac{e\hbar}{2m_p c}$).

Both relativistically (and nonrelativistically), one finds that a charged spin 1/2 particle moving in a magnetic field will precess relative to the momentum vector with a frequency ω_a which is given by the difference between the orbital cyclotron frequency ω_c and the spin precession frequency ω_s,

$$\omega_a = \omega_s - \omega_c \tag{2.1.4}$$

where the orbital cyclotron frequency is

$$\omega_c = \frac{eB}{mc\gamma} \tag{2.1.5}$$

and the spin precession frequency is

$$\omega_s = \frac{geB}{2mc} + (1-\gamma)\frac{eB}{mc\gamma}, \tag{2.1.6}$$

with the Larmor and Thomas precession terms explicitly separated.

Thus ω_a is given by

$$\omega_a = \frac{e}{mc} a_\mu B \tag{2.1.7}$$

and is directly proportion to the anomalous moment and independent of the muon momentum. For the muon, the spin vector leads the momentum vector by this amount.

In a region in which both magnetic and electric fields are present, the relativistic formula for the precession is given by [7]

$$\vec{\omega}_a = \frac{d\Theta_R}{dt} = \frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right] \tag{2.1.8}$$

where $\Theta_R = (\vec{s}, \vec{\beta})$ is the angle between the muon spin direction in its rest frame and the muon velocity direction in the laboratory frame. The other quantities refer to the laboratory frame.

Eq. (2.1.8) shows the key to the most recent CERN muon $(g-2)$ experiment, [3] as well as AGS E821. At the "magic γ" where

$$\left(a_\mu - \frac{1}{\gamma^2 - 1}\right) = 0 \tag{2.1.9}$$

the electric field does not contribute to the precession. One can thus achieve (weak) vertical focusing with electrostatic quadrupoles, and build the storage ring magnet to have a uniform dipole magnetic field. This condition is satisfied if $\gamma = 29.3$ and $p_\mu = 3.094$ GeV/c. The AGS is well suited to deliver a copious flux of pions or muons at this momentum.

2.2. The Three CERN Measurements of the Muon $(g-2)$ Value

In this section we review briefly the CERN experiments on the muon anomalous magnetic moment. In the period from about 1959 to 1977, measurements were made of $(g-2)$ for the muon in a series of three experiments at CERN. [1,2,3]. These experiments measured the $(g-2)$ precession frequency ω_a which is the difference between the spin precession frequency ω_s and the cyclotron frequency ω_c in a magnetic field B, see Eq. (2.1.4), Eq. (2.1.5), Eq. (2.1.6), and Eq. (2.1.7).

In these experiments the direction of the muon spin is determined through the parity-violating angular asymmetry in the decay $\mu \rightarrow e\nu_e\nu_\mu$ and the momentum direction was determined from knowledge of the muon trajectory in the magnetic field. The first experiment [1] utilized low-energy muons with momenta $p_\mu \approx 90$ MeV/c in a magnetic field $\mathbf{B} = 1.6\ T$, where inhomogeneities in $\mathbf{B}$ provided focusing and drifting for the muon orbits. This experiment gave $a_\mu = (1\ 162 \pm 5) \times 10^{-6}$ (4300 *ppm*). In an experiment which uses low-energy muons, such as this first CERN experiment, the observation time is of the order of the muon lifetime, $\tau_\mu = 2.2 \times 10^{-6} s$, which limits the number of precession cycles available for observation.

In the second CERN experiment, [2] polarized muons with momenta p_μ of 1.3 GeV/c ($\gamma = 12$) were produced by the decay of pions which were in turn produced by a primary proton beam hitting a production target inside the storage ring. Muon decay electrons were detected by several detectors placed in a shielded position 180^0 around the ring from the production target. The $(g-2)$ precession was measured in a storage ring with $\mathbf{B} = 1.7\ T$, with weak-focusing provided by the inhomogeneous magnetic field. Since the laboratory mean lifetime was dilated to $\gamma\tau_\mu = 26 \times 10^{-6} s$, twelve times more precession cycles were observable, compared with the first experiment. This experiment gave $a_\mu = (116\ 616 \pm 31) \times 10^{-8}$ (270 *ppm*). Further improvement would have been limited by the magnetic field gradient required for focusing, and the corresponding uncertainty in the mean field applicable to the muon population (see Eq. (2.1.7)).

The third CERN experiment [3] also measured the $(g-2)$ precession frequency of high energy muons in a storage ring, but its design was based upon the observation about the classical spin motion in a magnetic and electric field given by Eq. (2.1.8). The three experiments are summarized in Table 2.2.1.

With the "magic γ" the electric part of the bracketed factor given in Eq. (2.1.8) is zero,(see Eq. (2.1.9)) ($\gamma = 29.3$, $p_\mu = 3.094$ GeV/c), and the $(g-2)$ precession frequency is determined by $\mathbf{B}$ alone and is independent of $\mathbf{E}$. This observation allows for a separated function muon storage ring in which a homogeneous magnetic dipole field $\mathbf{B}$ determines

Table 2.2.1: The CERN Experiments

The First Experiment [1]

Method: $(g-2)$ precession in a bending magnet, $\mathbf{B} = 1.6$ T, with trajectories and focussing determined by field inhomogeneities. Performed at the Synchrocyclotron, $p_\mu = 90MeV/c$
$a_\mu \;=\; (1\ 162\ \pm\ 5) \times 10^{-6}\ \ (4300ppm)$

The Second Experiment [2]

Method: $(g-2)$ precession in a muon storage ring $\mathbf{B} = 1.7$ T, with weak focusing provided by higher multipoles in $\mathbf{B}$. Performed at the Proton Synchrotron, $p_\mu = 1.3Gev/c$, $\gamma = 12$
$a_\mu \;=\; (116\ 616\ \pm\ 31) \times 10^{-8}\ \ (270ppm)$

The Third Experiment [3]

Method: $(g-2)$ precession in a muon storage ring $\mathbf{B} = 1.5$ T, with weak focusing provided by electrostatic quadrupoles. Performed at the Proton Synchrotron, $p_\mu = 3.1Gev/c)$, $\gamma = 29.3$
$a_\mu \;=\; (1\ 165\ 924\ \pm\ 8.5) \times 10^{-9}\ \ (7.3ppm)$

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the $(g-2)$ precession frequency, and an electric quadrupole field provides vertical weak-focusing for the muons. This elimination of the magnetic field gradient in the third CERN experiment permitted a significant improvement in accuracy, since the average magnetic field which enters into Eq. (2.1.7) breaks down into an integration over the storage volume of each multipole of the field times the corresponding moment of the muon distribution, and little information existed on the muon distribution in the CERN experiments. Furthermore, the additional time dilation of the muon lifetime to from $26\mu s$ to $64\mu s$ at $\gamma = 29.3$ gave an additional improvement in accuracy.

The muon storage ring, magnet cross section, quadrupole electrodes, and a data sample from the third CERN experiment are shown in Fig. 2.2.1. The precession frequency ω_a (see Eq. (2.1.7)) of the spin relative to the momentum vector was measured by observing the decay electrons emerging on the inside of the ring. As the angular distribution in the decay rotated with the muon spin, the counting rate for the high energy electrons (emitted in the forward direction) was modulated at the frequency ω_a. The magnetic field

(a) (b)

(c) (d)

Figure 2.2.1: The Third CERN Experiment - (a) shows a plan view of the storage ring, which was composed of forty separate magnets excited by a single set of coils. Each of the sectors was stabilized separately. All dimensions are in mm. (b) A cross section of the magnet. (c) The electrostatic quadrupoles and vacuum chamber. (d) Muon decay electrons as a function of time. The $(g-2)$ oscillations are clearly visible superimposed on the muon lifetime curve.

was homogeneous to 20 *ppm* over the muon storage area which had a cross section 12 cm (horizontal) x 8 cm (vertical), and the effective mean magnetic field averaged over the muon orbits was known to 1.5 *ppm*. The electric quadrupole field was pulsed with an electrode voltage of + 24 kV on the top and bottom electrodes and -4 kV on the side electrodes for μ^+ storage.

The $(g-2)$ precession frequency ω_a is deduced from the decay electron counting data which, with no losses except from muon decay, is of the form

$$N = N_0 e^{(-t/\gamma\tau)}\{1 - A\cos(\omega_a t \;+\; \phi)\} \tag{2.2.1}$$

in which $\gamma\tau$ = laboratory muon lifetime. The asymmetry parameter A is the product of the stored muon polarization and the asymmetry in $\mu - e$ decay, which is a function of electron energy, and of the angular and energy spread of the electrons that are accepted by the detection system. Data showing the modulation of the decay electron counting rate at the frequency ω_a are given in Fig. 2.2.1(d).

In addition to values of ω_a and $\mathbf{B}$, determination of a_μ from Eq. (2.1.7) requires a value of the constant (e/mc). Since the magnetic field is measured by a proton resonance frequency ω_p (corrected for molecular and bulk diamagnetic shielding), the constant needed is

$$\lambda = \frac{\mu_\mu}{\mu_p}, \tag{2.2.2}$$

and a_μ is then given by

$$a_\mu = \frac{R}{(\lambda - R)}, \tag{2.2.3}$$

where $R = \omega_a/\omega_p$.

The errors quoted for the measurement of a_μ in the CERN experiment [3] are listed in Table 2.2.2. The overall one-standard-deviation error was 7.3 *ppm*, which was dominated by the statistical counting error of 7.0 *ppm* in the determination of ω_a. The overall systematic error was 2 *ppm*, which was due almost entirely to uncertainty in the mean magnetic field $\mathbf{B}$ in the storage ring, which involved an average over space, time, and muon distribution. Additional smaller systematic errors are associated with the deviation of the muon orbits from the ideal reference orbit.

The statistical error in ω_a determined from Eq. (2.2.1) is

$$\frac{\delta\omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a A\gamma\tau N_e^{\frac{1}{2}}}, \tag{2.2.4}$$

where N_e is the total number of decay electrons recorded. For a given number of decays the accuracy is improved by increasing the number of $(g-2)$ precession cycles per lifetime,

Table 2.2.2: Errors in CERN3 Experiment [3] The small systematic errors marked * were insignificant compared to the dominant errors, and thus were not studied extensively.

Source of Error	**Error (ppm)**	**Comments**
Statistics	7.0	
Magnetic Field	1.5	Includes absolute calibration of NMR probes (~ 0.8) and average over space, time and muon distribution
Electric Field Correction	0.3	1.7 *ppm* correction
Pitch Correction	0.12	0.6 *ppm* correction
Particle Losses*	< 1.0	
Timing Errors*	< 0.5	
Total	**7.3**	

$\omega_a \gamma\tau$, (hence the advantage of highly relativistic muons with dilated lifetime $\gamma\tau$) and by maintaining a large asymmetry parameter A. In the CERN experiment the total number of electron counts N_e was 1.34×10^8, and A varied from 0.17 for the lowest energy band to 0.38 for the highest. The overall statistical error in ω_a was 7.0 *ppm*.

The principal systematic error was the uncertainty in the knowledge of **B**. Since the magnetic field of the ring was not perfectly uniform, ω_p had to be a weighted average over the muons which contribute to the counting data. It also had to be averaged correctly over any drifts in the magnetic field between the proton NMR surveys and the muon runs. In the CERN experiment the field was automatically stabilized at 40 points in the ring (just outside the vacuum tank) and monitored during the runs by 37 plunging probes which could be driven radially into the vacuum chamber in the median plane. However, the main surveys of 250,000 points in the field were made between runs with the vacuum chamber removed. The removal of the chamber required that the magnet be switched on-and-off between the field surveys and the muon runs. Nevertheless, reproducibility to $\pm$ 1 *ppm* in the average field was achieved over periods of months. Drifts in magnetic field shape due to temperature effects meant that the field in the storage region varied even though 40 points just outside the storage region were stabilized. This was measured by the 37 plunging probes, and corrections applied. Including uncertainties in the distribution of muons inside the aperture, an overall uncertainty of 1.5 *ppm* was assigned to the mean value **B** of the magnetic field.

All other systematic uncertainties were below the level of 1 *ppm*, including those associated with particle losses, timing and changes in electronic gains, as well as with corrections for muon energy spread (electric field correction) and betatron oscillations (pitch correction).

It should be noted that the ratio of the muon and proton magnetic moments, μ_μ/μ_p, is needed as an auxiliary constant to deduce a_μ from the measured precession frequency ω_a and the field B measured in terms of a proton resonance frequency ω_p (Eq. (2.2.2) and (2.2.3)). The quantity μ_μ/μ_p was known to 0.3 *ppm* from measurement of the muonium Zeeman effect and from a μSR experiment. The best current value of λ utilizes in addition the accurately measured value of the hyperfine structure interval $\Delta\nu$ of muonium [8] and is accurate to 0.15 *ppm*. A still more precise value for λ may be obtained from more precise measurements of $\Delta\nu$ and the Zeeman effect of muonium in progress at LAMPF. [9] In addition, the muonium $1S \rightarrow 2S$ laser experiments at the Rutherford Laboratory [10] which will determine the muon mass, and thus λ is competitive with the LAMPF experiment.

The fine structure constant α which is required to obtain a_μ(theor) was known at the time of the CERN experiment [3] with adequate precision for the computation of a_μ(theor) at the time, and α is much better known now.

2.3. $(g-2)$ for the electron

The simplest Feynman diagram for the interaction of a lepton with an external field is shown in Fig. 2.3.1 1 and represents $g = 2$.

An anomalous moment can result from radiative corrections to Fig. 2.3.1 (see Fig. 2.3.2) and from internal structure if it is present. The proton anomalous moment of $+1.79\mu_N$ is a reflection of the its quark structure. For leptons, even in the absence of internal structure, radiative corrections are significant. The lowest order radiative correction to Fig. 2.3.1 is shown in Fig. 2.3.2.

Figure 2.3.1: The interaction of a spin 1/2 particle with an external magnetic field. (The $\times$ represents the static magnetic field of the storage ring and not a nucleus as it is sometimes used.)

Figure 2.3.2: The lowest order QED correction to $(g-2)$

The discovery of the anomalous moment for the electron was made in 1947 by Kusch and Foley, [11] who determined that

$$g_s = 2\,(1.00119 \pm 0.00005)\,,$$

consistent with a first-order correction of $\frac{\alpha}{2\pi}$ calculated by Schwinger. [12]

Since this seminal work, the anomalous magnetic moment of the electron, a_e, has been measured with ever-increasing precision. To the level of current measurements, QED diagrams alone suffice to determine its theoretical value, which can be written [13] as an expansion in powers of α:

$$a_e^{theory} = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + ... + \delta a \qquad (2.3.1)$$

where† $C_1 = 0.5$, $C_2 = -0.328\ 478\ 965\ ...$, $C_3 = 1.176\ 20(14)$, $C_4 = -1.569(60)$ and $\delta a = 4.46 \times 10^{-12}$. The correction δa comes from contributions of virtual muons, hadrons, $W^{\pm}$ and Z^0. The theoretical prediction is [13]

$$a_e^{theory} = 1\ 159\ 652\ 140\ \ (28) \times 10^{-12} \tag{2.3.2}$$

where the value for the fine structure constant

$$\alpha_{QH}^{-1} = 137.035\ 997\ 9\ \ (32)$$

is taken from measurements of the quantum Hall effect.

The values of $a_{e^\pm}$ from the Penning trap experiments are [14]

$$a_{e-} = 1\ 159\ 652\ 188.4\,(4.3) \times 10^{-12} \tag{2.3.3a}$$

and

$$a_{e^+} = 1\ 159\ 652\ 187.9\,(4.3) \times 10^{-12}. \tag{2.3.3b}$$

There is a 2σ agreement with QED at the level of 10^{-12} and, to the extent that the particle and anti-particle properties are the same, CPT invariance is verified at the 2×10^{-12} level. One should note that the contribution to the electron's anomaly from particles heavier than the electron is the same size as the current experimental uncertainty.

2.4. $(g-2)$ for the muon

The muon anomalous magnetic moment can be calculated in the same fashion as that of the electron. However, because the mass of the muon is much greater than that of the electron, diagrams with quark loops, or to a lesser extent, weak bosons, contribute measurably to the final result. The relative contribution of heavier particles to the muon's anomalous moment [4] goes as $\sim (m_\mu/m_e)^2$ which makes the contribution of such particles on the measured muon anomaly much larger ($\sim\ 4 \times 10^4$) than for the electron. The theoretical value for a_μ can be expressed [15]

$$a_\mu^{theory} = a_\mu^{QED} + a_\mu^{hadronic} + a_\mu^{weak} \tag{2.4.1}$$

and each of these contributions is discussed in detail below.

† The values of the higher order coefficients, both for the electron and the muon are constantly being refined, so these values represent a snapshot of this progress

2.4.1. The QED Contribution to a_μ

The QED contribution to the muon $(g-2)$ is again given by an expansion in α/π as for the electron anomaly Eq. (2.3.1). However, now virtual electrons, muons and tauons contribute in the lepton loops, so the coefficients are different. The coefficient $C_1 = 0.5$ is the same single photon Schwinger term shown in Fig. 2.3.2.

The expansion is given by

$$a_\mu^{QED} = C_1' \left(\frac{\alpha}{\pi}\right) + C_2' \left(\frac{\alpha}{\pi}\right)^2 + C_3' \left(\frac{\alpha}{\pi}\right)^3 + C_4' \left(\frac{\alpha}{\pi}\right)^4$$

where $C_1' = 0.5$, $C_2' = 0.765\ 857\ 39$, $C_3' = 24.045\ 11$, $C_4' = 126.12$ and $C_5' = 930$. The total QED term to eighth order is [15,16]

$$a_\mu^{QED} = 1\ 165\ 847\ 008\,(18)\,(28) \times 10^{-12} \tag{2.4.2}$$

The first uncertainty is from the calculation, the second from uncertainties in α (which was taken from the quantum Hall effect. The uncertainty in the QED contribution is negligible, compared with the proposed experimental accuracy.

2.4.2. Hadronic Contributions to a_μ

(a) (b) (c) (d)

Figure 2.4.1: Lowest order hadronic vacuum polarization diagrams

Loops where virtual hadrons are emitted and then reabsorbed contribute to the value of a_μ at a measurable level (60 parts per million), which was confirmed in the most recent CERN experiment. [3] The lowest order hadronic diagrams are shown in Fig. 2.4.1, the first order being shown in Fig. 2.4.1(a).

Figure 2.4.2: The hadronic light on light scattering. The contribution of this diagram cannot be obtained from the dispersion relation.

The contribution from these hadronic loops cannot currently be calculated reliably in QCD. However, the dominant contribution (shown in Fig. 2.4.1a) can be related, through dispersion theory, to the experimentally measured cross-section for e^+e^- annihilation into hadrons. The higher order diagrams shown in Fig. 2.4.1 can also be related to the quantity $R(s)$, defined by the expression

$$R(s) = \frac{\sigma_{tot}\left(e^+e^- \to \ hadrons\right)}{\sigma_{tot}\left(e^+e^- \to \mu^+\mu^-\right)}. \tag{2.4.3}$$

The single hadron loop contribution to $(g-2)$ is given by

$$a_\mu^{hadronic} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s)\, R(s) \tag{2.4.4}$$

where

$$K(s) = \frac{3s}{m_\mu^2}\left[x^2\left(1-\frac{x^2}{2}\right) + (1+x)^2\left(1+\frac{1}{x^2}\right)\left[ln\,(1+x) - x + \frac{x^2}{2}\right] + \frac{1+x}{1-x}x^2 lnx\right]$$

and

$$x = \frac{1-\beta}{1+\beta}, \qquad \beta = \sqrt{1-\frac{4m_\mu^2}{s}}$$

The integral is dominated by the kinematic regions around low-lying $\pi\pi$ resonances such as the ρ. There have been several evaluations of the hadronic contribution to a_μ, which are discussed below. To set the scale, we note that the contributions from the virtual hadron loops in Fig. 2.4.1 and Fig. 2.4.2 as evaluated by Kinoshita et al. [17] is

$$a_\mu^{had} = 7027\,(175) \times 10^{-11} \tag{2.4.5}$$

which corresponds to 60.6 ± 1.5 *ppm* of a_μ. The uncertainty is dominated by the knowledge of $R(s)$ (see Eq. (2.4.4)).

The dominant error arises from the accuracy with which R is known from experiment, principally in the range up to $\sqrt{s} = 1\ GeV$ but also in the higher energy range up to the J/ψ at $\sqrt{s} = 3.1\ GeV$.

The hadronic light on light contribution shown in Fig. 2.4.2 cannot be calculated from the dispersion relation, nor can it be calculated using QCD because of the low energies involved. A paper which discussed both a simple quark model calculation and a vector dominance model calculation was published by Kinoshita et al. [17] The value of the hadronic light-on-light contribution to $a_\mu^{hadronic}$ was $49(5) \times 10^{-11}$, which should be compared with the value of a_μ^{had} given in Eq. (2.4.5).

More recently there have been active discussions about the evaluation of this light by light scattering contribution, [18-20] and additional work is in progress to improve its evaluation using chiral pertubation theory. [21] This diagram represents the fundamental theoretical limitation to the value of $a_\mu^{hadronic}$ within the standard model.

In addition to the evaluation by Kinoshita [15,17] there have been several other evaluations of $a_\mu^{hadronic}$ in the literature, [22-26] as well as a recent evaluation by Worstell et al. [27] usingthe latest data and the methods of Kinoshita et al. [17], with the ρ meson contribution taken from the VEPP-2M data [22] published after Ref. 17), and with the 1992 Particle Data Group values for masses, widths and electron pair widths for the vector mesons. The Kinoshita et al, [17] values for nonresonant contributions have been used, although more recent data have reduced these errors as well, and they are in the process of including the new data in our evaluation of $a_\mu^{hadronic}$. The result obtained from this analysis is

$$a_\mu^{hadronic} = 6\ 797\ \ (88) \times 10^{-11} \tag{2.4.6}$$

. which has an uncertainty of half that of the earlier evaluation [17] given in Eq. (2.4.5).

These evaluations of $a_\mu^{hadronic}$ are listed in Table 2.4.1. We emphasize that each of these evaluations comes from a slightly different data set for R, see Eq. (2.4.3), and a slightly different form of the dispersion relation, see Eq. (2.4.4), as discussed below.

From the integral expression for $a_\mu^{hadronic}$ above, we can factor out the energy dependence of the integrand to get the differential contribution to the muon anomaly per unit of R as a function of s:

$$\left(\frac{1}{a_\mu R(s)}\right) \frac{da_\mu^{hadronic}}{ds} = \frac{1}{a_\mu} \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \frac{K(s)}{s^2} \tag{2.4.7}$$

This quantity is plotted in units of ppm/GeV^2 in Fig. 2.4.3. Since R is everywhere of order unity, this shows the relative contribution of hadronic vacuum polarization from

Table 2.4.1: Different Evaluations of first order $a_\mu^{hadronic}$. Higher Order Diagrams are estimated to be [17] $a_\mu^{hadronic}(higher\ order) = -41(7) \times 10^{-11} \quad (0.06\ ppm)$.

$a_\mu^{hadronic} \times 10^{11}$	Error in *ppm*	Reference
7068 (59) (164)	$(\pm 1.5\ ppm)$	Kinoshita et al. [15,17]
6840 (110)	$(\pm 0.9\ ppm)$	Barkov et al. [22]
7100 (105) (49)	$(\pm 0.9\ ppm)$	Casas et al. [23]
7245 (66) (259)	$(\pm 2.3\ ppm)$	Burkhardt et al. [24]
7048 (115)	$(\pm 0.9\ ppm)$	Martinovič et al. [25]
6986 (42) (16)	$(\pm 0.4\ ppm)$	Dubničková et al. [26]

Figure 2.4.3: Kinematic suppression factor $\frac{K(s)}{s^2}$ in units of *ppm* per GeV^2.

Table 2.4.2: Different evaluations of contributions to $a_\mu^{hadronic}$ in *ppm*, with errors.

ρ	$\omega + \phi$	J/Ψ + Υ	Nonresonant	**Total** (*ppm*)	Reference
43.43 ± 1.30	7.45 ± 0.47	0.63 ± 0.09	9.10 ± 0.55	60.61 ± 1.49	Kinoshita et al. [17,15]
42.00 ± 0.48				58.67 ± 0.94	Barkov et al. [22]
44.32 ± 0.37	7.83 ± 0.81	0.54 ± 0.04	8.21 ± 0.21	60.90 ± 0.92	Casas et al. [23]
43.61 ± 1.32	7.14 ± 0.43	0.66 ± 0.13	10.73 ± 1.82	62.14 ± 2.29	Burkhardt et al. [24]
43.79 ± 0.53	8.75 ± 0.36†	0.64 ± 0.05	9.94 ± 0.75	63.12 ± 1.03	Martinovič et al. [25]
42.39 ± 0.18	7.48 ± 0.24†	0.71 ± 0.02	9.35 ± 0.21	59.93 ± 0.37	Dubničková et al. [26]
42.00 ± 0.48	6.98 ± 0.14	0.78 ± 0.05	8.86 ± 0.50	58.62 ± 0.71	Worstell et al. [27]

†Includes the nonresonant 2 kaon contribution in the ω, ϕ energy range.

different energy ranges to the muon anomaly. Different evaluations of $a_\mu^{hadronic}$ correspond to different evaluations of R, either from different experimental data, different models, or different treatments of experimental errors.

Because different authors have broken down the hadronic vacuum polarization contributions into different energy ranges and among different categories of e^+e^- hadroproduction channels, a comparison between their analyses is not trivial. We have therefore organized hadronic contributions as tabulated by different authors into four broad categories for comparison: ρ, $\omega + \phi$, $J/\psi + \Upsilon$ (+ excitations), and non-resonant (continuum) hadroproduction. The contributions and errors associated with each contribution by each author are given in Table 2.4.2

The low energy behaviour of the physical cross section ratio, $R(s)$ is dominated mostly by the two pion spectrum which contains the ρ meson resonance with the small effect of interference with the ω meson resonance near its peak. The two pion production cross section, $\sigma_{\pi\pi}$ is calculated in terms of the electromagnetic pion form factor, $|F_\pi|$:

$$\sigma_{\pi\pi} = \frac{\pi\alpha^2}{3s}{\beta_\pi}^2 \left|F_\pi\right|^2 .$$

A model which fits the energy dependence of $|F_\pi|$ is given by the Gounaris-Sakurai formula which is a finite- and energy dependent-width correction to the usual Breit-Wigner form for

Figure 2.4.4: Experimental values of $|F_\pi|^2$ which come principally from the OLYA and CMD detectors at BINP, Novosibirsk. The solid curve corresponds to the Gounaris-Sakurai formula taking into account the $\rho-\omega$ interference. The dotted line is a parameterization with ω, ρ, $\rho'(1250)$, and $\rho''(1600)$ mesons.

a resonance. The high energy tail of the two pion spectrum is complicated by contributions from particular vector meson orbital excitations. The simplest method for inclusion of interference effects of the ρ, ω and further orbital excitations is to write the total $|F_\pi|$ as a sum of resonances:

$$|F_\pi| = |F_\rho| + a\,|F_\omega| + b\,\left|F_{\rho'}\right| + c\,\left|F_{\rho''}\right| + \dots \tag{2.4.8}$$

Figure 2.4.5: The most recent reproduction of all existing data on the electromagnetic pion form factor by means of the unitary and analytic VMD model.

where $a, b, c, \ldots$ are complex numbers whose amplitudes and phases characterize the interference effects, and the additional form factors are of the Breit-Wigner form. Approximating the form factor by such a sum is one example of the vector meson dominance (VMD) approach. Competing models to fit experimental data exploit more sophisticated expressions for the resonance shapes and different combinations of the form factors before squaring to get the cross section. Several such fits to the most recent experimental data for $|F_\pi|$ are presented in Fig. 2.4.4.

A new model of the pion and kaon form factors has been presented recently in Ref. 25 and Ref. 26. The vector meson dominance (VMD) form for the resonance shape is used and the unitary and analytic structure of the form factor is exploited. Interference effects between the ρ, ω, $\rho(1450)$, $\rho(1700)$, and $\rho(2150)$ are included as in Eq. (2.4.8). Significantly, this model allows for a simultaneous fit to data for timelike (colliding beam) and spacelike (fixed target) values of the square momentum transfer and is presented in

Fig. 2.4.5. In this analysis, as in all others except Ref. 24, the systematic errors from the different experiments are combined in quadrature to obtain the total error of 0.37 *ppm* for the hadronic contribution to a_μ. This result is compared with others in Table 2.4.1.

The next largest contribution to hadronic vacuum polarization, and also one of the next most significant sources of the error, is the three pion spectrum which includes the ω and ϕ meson resonances, and the kaon pair spectra (K^+K^- and K_LK_S). As the ω and ϕ mesons are much narrower resonances than the ρ ($\Gamma_\rho = 151$ MeV while $\Gamma_\omega = 8.43$ MeV $\Gamma_\phi = 4.43$ MeV) the resonance shapes are taken as strictly Breit-Wigner. The interference between them leads to an observable dip in the cross section just before the ϕ meson resonance. Integration over the Breit-Wigner line shapes for the ω and ϕ mesons does not differ appreciably from a narrow width approximation for the narrow vector meson contribution to a_μ:

$$a_\mu^V = \frac{{m_\mu}^2}{3{m_V}^2} K\left({m_V}^2\right) \Gamma^V_{e+e-}.$$

This approximation is used for handling the charm and bottom quark meson states as well and results for 1992 Particle Data Group values are presented in Table 2.4.2.

The non-resonant continuum background is handled differently by the different authors. The main difference is the choice of energy cutoff above which they use asymptotically free perturbative QCD and in the parameterization for the data for $R(s)$ below this cutoff. A particular (simple) parameterization for $R(s)$ used by Ref. 24 is presented in Fig. 2.4.6. This evaluation is the most pessimistic about systematic errors in experimental knowledge of R at center-of-mass energies above 1.4 GeV, often using a single experimental source and adding the systematic errors from different hadronic channels linearly. The resulting overall error assessment by these authors is therefore much more pessimistic than results from the other, more detailed, evaluations cited.

In order to utilize the more accurate value of a_μ expected from the BNL experiment, it is clearly of the greatest importance that more precise measurements of R be made. An improvement by a factor of about 4 in our knowledge of R in the most important $\sqrt{s}$ range is needed. The most accurate measurements of R below $\sqrt{s} = 1.4\ GeV$ have been made at the Budker Institute of Nuclear Physics (BINP) in Novosibirsk, Russia [22] with the VEPP-2M e^+e^- collider and the detectors OLYA and CMD. These data are shown in Fig. 2.4.4 where the square of the pion form factor F_π^2 is plotted vs $\sqrt{s}$; $[F_\pi^2 \sim R]$.

VEPP-2M has a useful energy range $E_{cm} = \sqrt{s}$ from 0.36 to 1.4 GeV. The luminosity of VEPP-2M has been upgraded by the addition a wiggler, and a luminosity of $5 \times 10^{30}\ cm^{-2}s^{-1}$ was demonstrated at the ϕ resonance energy during the 1993 run.

Moreover, a new detector Cryogenic Magnetic Detector-2 (CMD2) has been built [28] which employs a superconducting magnet and was designed to achieve accuracies of 0.5%

Figure 2.4.6: Parameterization of R including resonances up to $\sqrt{s} = 10$ GeV.

in the measurement of R. CMD2 has been carefully designed to minimize systematic errors associated with luminosity measurements, particle identification, energy measurement and detection efficiency. Fig. 2.4.7 indicates the principal features of the new detector, which now includes a drift chamber, a Z-chamber, a muon range system and CsI barrel calorimeter. The BGO end cap calorimeter crystals were installed during the 1993 shutdown which completed the detector construction.

The initial runs were made in 1992 both on the ρ ($25\ nb^{-1}$) and on the ϕ ($178\ nb^{-1}$) for initial debugging and off-line reconstruction development. The total integrated luminosity was about $200nb^{-1}$. Further data were collected in 1993 on the ϕ ($1.5\ pb^{-1}$) for the study of rare decays. These data were used for the first simultaneous measurements of the four main decay channels of the ϕ in a single experiment. The accuracy obtained was comparable with the world average, and the results are being prepared for publication.

Figure 2.4.7: The new CMD2 Detector at the Budker Institute for Nuclear Physics, Novosibirsk, Russia. CMD2 started taking data at VEPP-2M in 1992 with integrated luminosities of 25 nb^{-1} at the ρ and 178 nb^{-1} at the ϕ. In 1993 an additional integrated luminosity of 1.5 pb^{-1} was obtained on the ϕ.

The VEPP-2M dedicated run for the muon $(g-2)$ energy scan, going down in energy from the ϕ with steps of $\Delta E_{cm} = 10 MeV$, was started at the end of 1993 and should be completed at 0.36 GeV in the summer of 1995. About 1000 $e^+e^- \to \pi^+\pi^-$ events will be recorded at each energy point. A finer scan of perhaps 1 MeV steps is intended for the region around the ω meson in order to study the three pion contribution to $(g-2)$ and also $\rho - \omega$ interference in both the two and three pion channels. At each point the beam energy is being calibrated by the resonance depolarization method to eliminate cross section uncertainty on the resonance slopes. Special procedures will be performed to keep the trigger efficiency well understood and the detector elements' stability under control during runs.

The 1993-1994 run will give statistically sufficient data to have 0.3% accuracy in the determination of $a_\mu(had)$ and will also be used for extensive systematic errors studies. The results should be available in one to two years.

At Frascati a new ϕ factory is under construction and at this laboratory it would be possible to make accurate measurements of R in the $\sqrt{s}$ range ≤ 1 GeV [29].In addition, the Beijing Electron-Positron Collider (BEPC) is entering the planning stages for conducting measurements of $R(s)$ in the energy range above VEPP-2M and below the J/ψ: 1.4 - 3.1 GeV [30], although the luminosity will limit the lowest energy to somewhere in the 2 GeV region.

2.4.3. Weak Contribution to a_μ

(a) (b) (c)

Figure 2.4.8: Single loop weak contributions to the muon $(g-2)$ value.

The weak interaction contribution based on the standard electroweak theory is a weak radiative correction to an electromagnetic interaction and arises in lowest order from the single loop diagrams in Fig. 2.4.8 which involve ν_μ, Z^0 and H particle exchange and where two vertices are weak interaction vertices. The contributions are given by [31,4]

$$\Delta a_\mu\,(W) = \left(\frac{G_F m_\mu^2}{8\pi^2\sqrt{2}}\right)\left(\frac{10}{3}\right) = +3.89\times 10^{-9} \tag{2.4.9a}$$

$$\Delta a_\mu\,(Z) = \left(\frac{G_F m_\mu^2}{8\pi^2\sqrt{2}}\right)\left(\frac{1}{3}\right)\left[\left(3-4\cos^2\theta_W\right)^2-5\right] = -1.94\times 10^{-9} \tag{2.4.9b}$$

$$\Delta a_\mu\,(H) = \left(\frac{G_F m_\mu^2}{4\pi^2\sqrt{2}}\right)\int_0^1 \frac{y^2\,(2-y)\,dy}{y^2+(1-y)\,(m_H/m_\mu)^2} \tag{2.4.9c}$$

$$= \left(\frac{G_F m_\mu^2}{4\pi^2\sqrt{2}}\right)\left(\frac{m_\mu}{m_H}\right)^2 ln\left[\left(\frac{m_H}{m_\mu}\right)^2\right] \tag{2.4.9d}$$

$$\leq 0.01\times 10^{-9} \text{ if } M_H \gg m_\mu \tag{2.4.9e}$$

where we take $\sin^2\theta_W = 0.233 \pm 0.002$ for the weak mixing angle.

The diagrams with the W and Z bosons contribute appreciably as given in Eq. (2.4.9). For Higgs masses much greater than 300 MeV (and standard couplings) the contribution of

H to the muon anomaly is negligible. Since LEP searches [32] establish that $m_H > 58\ GeV$, the contribution from the diagram with H is indeed negligible.

The second order weak diagrams have been calculated by Kukhto et al. [33]

The total weak contribution is[‡]

$$a_\mu(weak) = (1.53 \pm 0.2) \times 10^{-9} \tag{2.4.10}$$

which represents 1.31 ± 0.04 *ppm* of $a_\mu(theor)$.

Comparison of theory and experiment on $a_\mu(weak)$ will constitute a new and sensitive test of the unified electroweak theory with its prescription for renormalizability. Just as virtual electromagnetic radiative corrections were critical to the development of modern renormalized quantum electrodynamics, virtual radiative corrections involving both the weak and electromagnetic interactions are most important to the renormalized unified electroweak theory. The first test of radiative corrections in the electroweak theory involved the electromagnetic radiative correction to neutron beta decay and hence involved quarks and a semi-leptonic process. [34]

The radiative correction included in the fundamental relation between m_W and m_Z is central to this topic, [35] where

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F\left(1 - \frac{m_W^2}{m_Z^2}\right)(1 - \Delta r)} \tag{2.4.11}$$

and m_W and m_Z are the physical masses of the W and Z bosons and G_F is the Fermi constant. The radiative correction parameter Δr arises from finite electroweak radiative corrections. The one loop radiative correction from vacuum polarization associated with the many fermions in the standard model amounts to $\Delta r \simeq +0.07$ and represents the running of α from 0 momentum to momentum m_Z. However, in addition there is an important contribution to Δr from the top quark mass m_t and a smaller one from the Higgs mass m_H. Until m_t and m_H are measured, a clean test of radiative corrections in the electroweak theory cannot be obtained from Eq. (2.4.11).

It has been pointed out to us [36] that the electroweak renormalizabliltyhas been tested in a comparison of high energy and atomic parity violation data. These tests have the complication of hadrons in the nuclei. This topic has been reviewed recently in the article by Langacker. [37]

Additional discussion on renormalization tests of the standard model is the review by Hollik. [38]

‡ There is some question as to the correctness of the second order weak correction. [33] It is being re-evaluated by Krause and Czarnecki at Karlsruhe. The error quoted above is that estimated by Kinoshita, rather than the 0.05 quoted in [33].

2.4.4. Current Theoretical and Experimental Values of a_μ

The total theoretical value of a_μ is

$$a_\mu^{theory} = 116\ 591\ 881\ \ (176) \times 10^{-11} \tag{2.4.12}$$

where the we take the value of the uncertainty from the published value of Kinoshita et al. [17] rather than the hadronic contribution from our analysis presented in Table 2.4.2, which employed the method of Kinoshita et al. [17] We note that the uncertainty on the hadronic term (Eq. (2.4.6)), as evaluated with the new data is roughly half the the contribution of the weak term(Eq. (2.4.10)). As is clear from the discussion in section 2.4.2, the hadronic radiative corrections are receiving substantial attention. Since these corrections are important for the interpretation of some of the LEP and SLD precision results, [39] as well as the $(g-2)$ experiment, there is considerable interest in the wider community in having the uncertainty on the magnitude of these low energy corrections resolved.

The current experimental values of the muon anomaly are

$$a_{\mu^-}^{exp} = 1\ 165\ 937\,(12) \times 10^{-9} \tag{2.4.13a}$$

and

$$a_{\mu^+}^{exp} = 1\ 165\ 911\,(11) \times 10^{-9} \tag{2.4.13b}$$

(a combined accuracy of 7.3 ppm), in good agreement with the calculated value. The goal of the new experiment (E821) is a measurement of $(g-2)$ to the level of 0.35 ppm, a factor of twenty improvement over the CERN precision. This sensitivity is $\sim$ 23% of the magnitude of the expected electroweak contribution to $(g-2)$ from single loop diagrams involving the intermediate vector bosons $W^\pm$, Z^0. A measurement at the proposed accuracy would thus provide an essential test of the renormalization of the unified electroweak theory.

2.5. New Physics Potential of E821

In the previous sections the standard model contributions to the muon $(g-2)$ value were discussed. If there are new particles which couple to the muon, they too will contribute to the anomalous moment.

A deviation from the $(g-2)$ value predicted by the standard model would be a signal for new physics. Among the possibilities would be: lepton substructure, new particles such as supersymmetric partners, additional gauge bosons, W substructure, etc. A deviation from the standard model prediction would contain two pieces of information, a magnitude and a sign. For example, supersymmetry would produce a measured value smaller than the standard model value. [15] This point is disputed in a recent paper [40] where it is claimed thatthe previous supersymmetric calculations need to be redone, and that either-sign deviation from the standard model can be accommodated by their (flipped $SU(5)$ model) supersymmetric calculation.

Relative to other probes of these same new physics possibilities, the muon g value is most sensitive to muon and W boson substructure. This topic is discussed in the literature by a number of authors, [41-45] with the paper by Méry et al. [45] having the most comprehensive.

The review by Kinoshita and Marciano [15] summarizes a range of possible new physics contributions to the muon anomalous moment, which are listed in Table 2.5.1. Limits on various parameters which will come from E821, LEP II and the LHC are given. Except where noted, these values come from the review by Kinoshita and Marciano [15].

More recently Arzt, Einhorn and Wudka [46] have developed a systematic approach, using the language of effective Lagrangians, to discuss how new physics can contribute to the muon $(g-2)$ value. They separate the discussion of new high-energy effects into two cases: those which decouple from low-energy physics, and those which do not. If Λ is the (large) energy scale at which the new physics manifests itself, in the decoupling case at energies $E << \Lambda$ observable corrections are suppressed by powers of $1/\Lambda$, whereas in the nondecoupling case, "the corrections to standard model processes are given by a chiral expansion in powers of p/Λ, where p is a typical momentum for the process at hand." [46] They conclude that in the decoupling case under some conditions the E821 sensitivity limit is $\Lambda \leq 50 TeV$, but there would be no new sensitivity to effects of an anomalous triple gauge boson vertex. On the other hand, in the nondecoupling case, E821 may be sensitive to effects from an anomalous triple gauge boson vertex, and could set interesting limits if no deviation is observed.

We emphasize that a precision measurement of $(g-2)$ is complementary to other high energy measurements. In some cases, such as composite W bosons, the limit which will

Table 2.5.1: Sensitivity to new physics of the new muon $(g-2)$ experiment

New Physics	E821 Sensitivity	Comments
μ substructure[a]	$\Lambda \geq 5\ TeV$	LHC domain
μ Form factors[b] $(1-k^2/\Lambda^2)$	$(W^{\pm})\ \Lambda > 450 GeV$ $(Z^0)\ \Lambda > 64 GeV$ $(\gamma)\ \Lambda > 180 GeV$	$\Lambda_{LEP\ I} > 300 GeV$
excited muon[c]	$m_{\mu^*} \geq 6.3\ GeV$	Already Excluded
excited muon[d]	$m_{\mu^*} \geq 400\ GeV$	LEPII comparable
$W^{\pm}$ substructure[e]	$\Lambda \geq 400\ GeV$	LEPII $\sim 100 - 200\ GeV$
$\frac{(g_W-2)}{2} \neq 0$[f]	≥ 0.02	LEPII$\sim$ 0.05, LHC$\sim$ 0.2
Light Higgs	$\leq$ 300MeV	$O(10^{-3}g)$ coupling
Heavy Higgs	$\leq\ 500 GeV$	$O(g)$ coupling
Supersymmetry[g]	$\leq 130\ GeV$	FNAL $\bar{p}p$ collider
$W_R^{\pm}$	$\leq 250\ GeV$	FNAL $\bar{p}p$ collider
$Z^{'}(E6)$	$\leq 100\ GeV$	FNAL $\bar{p}p$ collider

[a]For substructure $\Delta a_\mu \sim m_\mu^2/\Lambda^2$
[b]See Ref. 45
[c]Chiral Invariant Interaction
[d]Traditional $\mu \cdot \mathbf{B}$ interaction (non-chiral)
[e]Goes to 0 if $M_W \to 0$ to get proper coupling to conserved current.
[f]The LEPII sensitivity is taken from the report of a 1992 Delphi working group on LEP200.
[g]Signature for SUSY in 100-200 GeV region is a value of a_μ below the SM value, although some authors take exception to this statement [40]

be placed by the new $(g-2)$ experiment is considerably better that that which will come from LEP II or the LHC. [15] Even before the measurement of $R(s)$ is completed (see Eq. (2.4.3)) there is a window of opportunity for new physics in going from the 7.5 *ppm* measurement at CERN to better than a 1 *ppm* measurement at Brookhaven. To exploit fully the additional factor of two to three in precision available at the AGS, we will need an improved value for the hadronic contribution to $(g-2)$.

One should remember that nuclear β decay told us about the existence of the $W^{\pm}$ boson long before its direct observation. The neutral current events observed in neutrino scattering at CERN told us about the Z^0. The ingredient which was needed to make their existence convincing was the development of the renormalizeable Weinberg-Salam-Glashow theory. Once WSG was firmly in place, the discovery of the $W^{\pm}$ and Z^0 were just as expected as the discovery of the antiproton was many years before.

With the standard model now firmly in place, there is common agreement on which signatures would indicate physics beyond the standard model. The muon $(g-2)$ experiment, along with the program of rare kaon decay experiments at Brookhaven, will provide some of the most sensitive probes of new physics in the TeV mass range during the pre-LHC era.

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