PARTICLE CORRELATIONS IN THE SATURATED QCD MATTER

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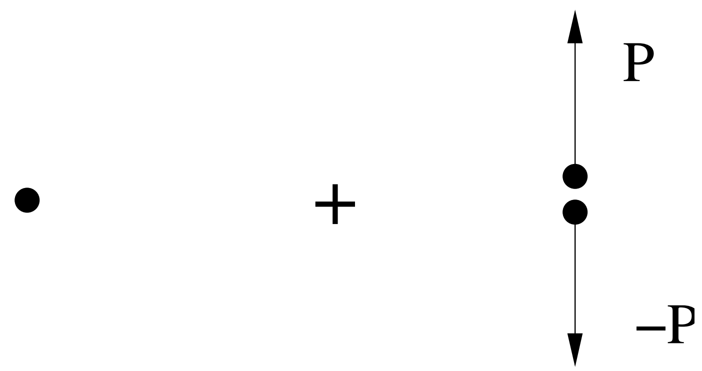
Phys.Rev.D72:094013,2005 (hep-ph/0506126) R. Baier, A.K., M. Nardi and U. Wiedemann

Where did the away side jets go? - Initial state perspective

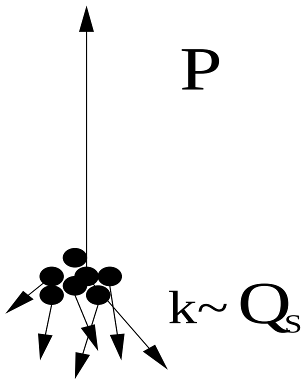
Kharzeev-Levin-McLerran, Nucl.Phys.A748:627 (hep-ph/040327160) :
as always provocative and as most of the time intriguing

What is partonic content of the wave function of a small perturbative object



What about a "large" "saturated" hadron (like maybe nucleus?) - especially at low x.

Well, we might expect that as there are many partons in the wave function, a large transverse momentum of one of them is balanced by many partons with $p_T \sim Q_S$, which are the most populous dwellers in the wave function.



Simple minded picture: scatter this wave function on a target - partonic wavefunctions decohere and partons go on shell. A saturated projectile produces jets with unbalanced transverse momentum.

Is this true? Not obvious. A wave function of a saturated projectile is not uniformly dense - it is only dense on transverse scales larger than Q_S^{-1}. Certainly if you look for quantum evolution to produce high $p_T \gg Q_S$ partons in the wave function, they are produced in the "dilute" component of the wave function - by quantum emission very close to emitting color charges. Thus for $p_T \gg Q_S$ one should still expect strong back-to back correlations. However for $p_T = 2 - 5\, Q_S$ one indeed can expect such an effect.

Additionally, when traversing a dense target the partons do not simply decohere, but also get kicks from the target again of about Q_S.

All in all one expects back-to-back correlations to be much weaker in the case of saturated QCD matter at transverse momenta of several Q_S.

Can we calculate double inclusive cross section?

We would like to calculate the double gluon inclusive cross section: AA$\rightarrow$ ggX.

We cannot calculate everything, but we can calculate something.

"AA" is very difficult. (Semi)Analytically intractable for the moment. Strictly speaking we are even missing the formalizm for such calculations.

Plan B: "pA". Here we can only hope to see the "square root" of the effect - broadening due to transverse momentum imparted by the target. But the formal framework exists.

- Jalilian-Marian, Kovchegov: Phys.Rev.D70:114017,2004 (hep-ph/0405266) - tour de force calculation of "$pA \rightarrow \mathrm{g}(y_1)\mathrm{g}(y_2)X$" including effects of quantum evolution $(y_1 \gg y_2)$

- Baier, A.K., Nardi and Wiedemann: Phys.Rev.D72:094013,2005 (hep-ph/0506126) - double inclusive gluon - no quantum evolution

- Nikolaev and/or Schafer and/or Zakharov and/or Zoller: Phys.Rev.D72:114018,2005 (hep-ph/0508310) plus several other recent papers - variety of double inclusive observables gg, qg, $\mathrm{q\bar{q}}$ - no quantum evolution

Can we make sense of these formulae?

We have now several VERY LONG EXPRESSIONS for a variety of observables with and without evolution. Can we understand (have some number+some pictures) the effects of saturation ?

I wish that my room had a floor,
I don't care too much for a door,
But this walking around
Without touching the ground
Is getting to be quite a bore

It's not easy, and especially difficult for double gluon emission.

A much simpler observable of this type is the double inclusive quark-gluon distribution. Given a simple but reasonable model for the interaction with the target we have analysed the double inclusive spectrum for qA$\rightarrow qgX$.

I will very briefly outline the setup for calculation of this type of observables in the saturated environment, and will present some results of numerical evaluation.

Eikonal cross sections

Consider a "small" projectile with wave function

$$|\Psi_{in}\rangle = \sum_{\{\alpha_i,\mathbf{x}_i\}} \psi(\{\alpha_i,\mathbf{x}_i\})\,|\{\alpha_i,\mathbf{x}_i\}\rangle\ .$$

α_i - color indices, x_i - transverse coordinates of partons.

This propagates eikonally through the target and emerges with the wave function:

$$|\Psi_{out}\rangle = \mathcal{S}|\Psi_{in}\rangle = \sum_{\{\alpha_i,\mathbf{x}_i\}} \psi(\{\alpha_i,\mathbf{x}_i\}) \prod_i W(\mathbf{x}_i)_{\alpha_i\beta_i}\,|\{\beta_i,\mathbf{x}_i\}\rangle$$

Here $\mathcal{S}$ is the S-matrix, and the W's are Wilson lines

$$W(\mathbf{x}_i) = \mathcal{P}\exp\{i \int dz^-\,T^a\,A_a^+(\mathbf{x}_i,z^-)\}$$

with A^+ - the gauge field in the target.

This wave function further evolves after the interaction to asymptotic time, and any observable has to be calculated only allowing for the final state emissions during this propagation. Perturbatively this is neatly taken into account by the following trick. The incoming wave function in perturbation theory can be written as

$$|\Psi_{in}\rangle = C|\Psi(\mathrm{valence})\rangle$$

with the perturbative "Cloud" operator which dresses the valence degrees of freedom by the Weizsacker-Williams gluon field

$$C = P\exp \quad i\frac{g}{2\pi^{3/2}} \qquad d\mathbf{x}\, d\mathbf{z}\, d\xi \frac{z_i - x_i}{(\mathbf{z}-\mathbf{x})^2}[a_i^d(\mathbf{z},\xi) + a_i^{d\dagger}(\mathbf{z},\xi)]\, \rho_\xi^d(\mathbf{x}) \quad ,$$

where P stands for rapidity ordering, and $\rho_\xi^d(\mathbf{x})$ denotes the total charge density operator integrated from the rapidity of the valence components of the projectile to the rapidity of the Weizsacker-WIlliams gluon ξ. The cloud operator C works like the evolution operator. Thus the recipe to calculate the expectation value of an oservable $O(a, a^\dagger)$ is

$$\boxed{\langle\Psi_{out}|C O(a, a^\dagger) C^\dagger|\Psi_{out}\rangle}$$

The simplest process to consider is gluon emission off a single quark projectile

$$\boxed{q\,A \longrightarrow q(\mathbf{k})\,g(\mathbf{p})\,X}$$

To leading order in α_s, the incoming wave function is

$$|q(x)\rangle \sim |q(x)\rangle + \frac{g}{2\pi^{3/2}} \qquad d^2z\,\frac{(x-z)_i}{(x-z)^2}|q(x), g(z)\rangle$$

Direct application of the above formalizm gives for the probability of emitting a gluon with momentum p and a quark with momentum k

$$\begin{aligned}
d\mathbf{b}\,\frac{dN}{dy\,d\mathbf{k}\,d\mathbf{p}} \quad &= \quad \frac{\alpha_s}{\pi}\frac{1}{(2\pi)^4} \quad {}_{\mathbf{x}\,\bar{\mathbf{x}}\,\mathbf{z}\,\bar{\mathbf{z}}}\, e^{-i\mathbf{k}\cdot(\mathbf{x}-\bar{\mathbf{x}})-i\mathbf{p}\cdot(\mathbf{z}-\bar{\mathbf{z}})}\frac{(z_i-x_i)(\bar{z}_i-\bar{x}_i)}{(\mathbf{z}-\mathbf{x})^2(\bar{\mathbf{z}}-\bar{\mathbf{x}})^2} \\
&\times \quad \left[Q(\mathbf{z},\mathbf{x},\bar{\mathbf{x}},\bar{\mathbf{z}})S(\bar{\mathbf{z}},\mathbf{z}) + S(\mathbf{x},\bar{\mathbf{x}}) - S(\mathbf{x},\bar{\mathbf{z}})S(\bar{\mathbf{z}},\bar{\mathbf{x}}) - S(\mathbf{x},\mathbf{z})S(\mathbf{z},\bar{\mathbf{x}})\right]\ .
\end{aligned}$$

The entire information about the target nucleus is contained in two target averages of products of Wilson lines, - dipole and quadrupole cross sections

$$\begin{aligned}
S(\bar{\mathbf{u}},\mathbf{u}) \quad &= \quad \langle\frac{1}{N}\mathrm{Tr}\ \ W^{F\dagger}(\bar{\mathbf{u}})W^{F}(\mathbf{u})\ \ \rangle_T\,, \\
Q(\bar{\mathbf{u}},\mathbf{u},\mathbf{z},\bar{\mathbf{z}}) \quad &= \quad \langle\frac{1}{N}\mathrm{Tr}\ \ W^{F\dagger}(\bar{\mathbf{u}})W^{F}(\mathbf{u})W^{F\dagger}(\mathbf{z})W^{F}(\bar{\mathbf{z}})\ \ \rangle_T\,.
\end{aligned}$$

Our aim is to understand the effects of saturation in this expression. We use the (by now standard) eikonal expressions for the Wilson loop averages

$$S(\bar{\mathbf{u}}, \mathbf{u}) = \exp\left[-v(\bar{\mathbf{u}} - \mathbf{u})\right] \ ,$$

$$\begin{aligned} Q(\bar{\mathbf{y}}, \mathbf{x}, \bar{\mathbf{x}}, \mathbf{y}) \quad = \quad & \frac{v(\mathbf{x}-\bar{\mathbf{x}}) + v(\mathbf{y}-\bar{\mathbf{y}}) - v(\mathbf{x}-\mathbf{y}) - v(\bar{\mathbf{x}}-\bar{\mathbf{y}})}{v(\mathbf{x}-\bar{\mathbf{x}}) + v(\mathbf{y}-\bar{\mathbf{y}}) - v(\mathbf{x}-\bar{\mathbf{y}}) - v(\mathbf{y}-\bar{\mathbf{x}})} e^{[-v(\mathbf{x}-\bar{\mathbf{x}}) - v(\mathbf{y}-\bar{\mathbf{y}})]} \\ & - \frac{v(\mathbf{x}-\bar{\mathbf{y}}) + v(\mathbf{y}-\bar{\mathbf{x}}) - v(\mathbf{x}-\mathbf{y}) - v(\bar{\mathbf{x}}-\bar{\mathbf{y}})}{v(\mathbf{x}-\bar{\mathbf{x}}) + v(\mathbf{y}-\bar{\mathbf{y}}) - v(\mathbf{x}-\bar{\mathbf{y}}) - v(\mathbf{y}-\bar{\mathbf{x}})} e^{[-v(\mathbf{x}-\bar{\mathbf{y}}) - v(\mathbf{y}-\bar{\mathbf{x}})]} \end{aligned}$$

This is a simple Glauber approximation (also known as McLerran-Venugopalan model) where $v(\mathbf{x})$ is the cross section of the dipole-nuclean scattering (scaled by the atomic number).

The function $v(\mathbf{x})$ also has the meaning of the target gluon field correlation function and is directly proportional to the gluon density in the target, thus it has the logarithmic dependence on the transverse separation.

$$v(\mathbf{x}) = \mathbf{x}^2 \, \frac{\tilde{Q}_s^2(\mathbf{x})}{8} \equiv \mathbf{x}^2 \, \frac{Q_{s,0}^2}{8} \log \left(\frac{1}{\mathbf{x}^2 \Lambda^2} + a \right) .$$

We take $\Lambda \equiv \Lambda_{\mathrm{QCD}} = 0.2\,\mathrm{GeV}$ and the small regulator $a = 1/(x_c^2\,\Lambda^2)$, $x_c = 3\,\mathrm{GeV}^{-1}$.

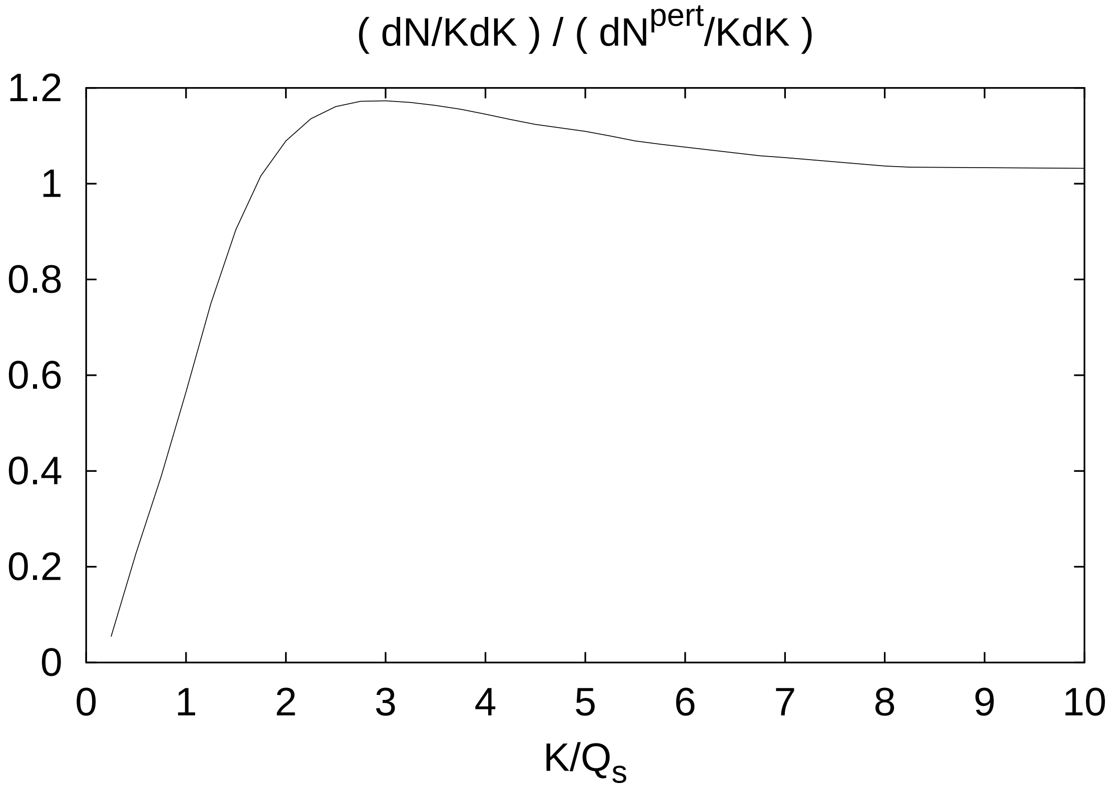
The saturation scale Q_s is defined implicitly as

$$v(Q_s^{-1}) \sim 1; \qquad Q_s^2 \equiv \tilde{Q}_s^2(\mathbf{x}^2 = 1/Q_s^2) \, .$$

With this definition, $Q_s^2 = 2\,\mathrm{GeV}^2$ corresponds to $Q_{s,0}^2 \simeq 0.5\,\mathrm{GeV}^2$.

Total recoil momentum distribution

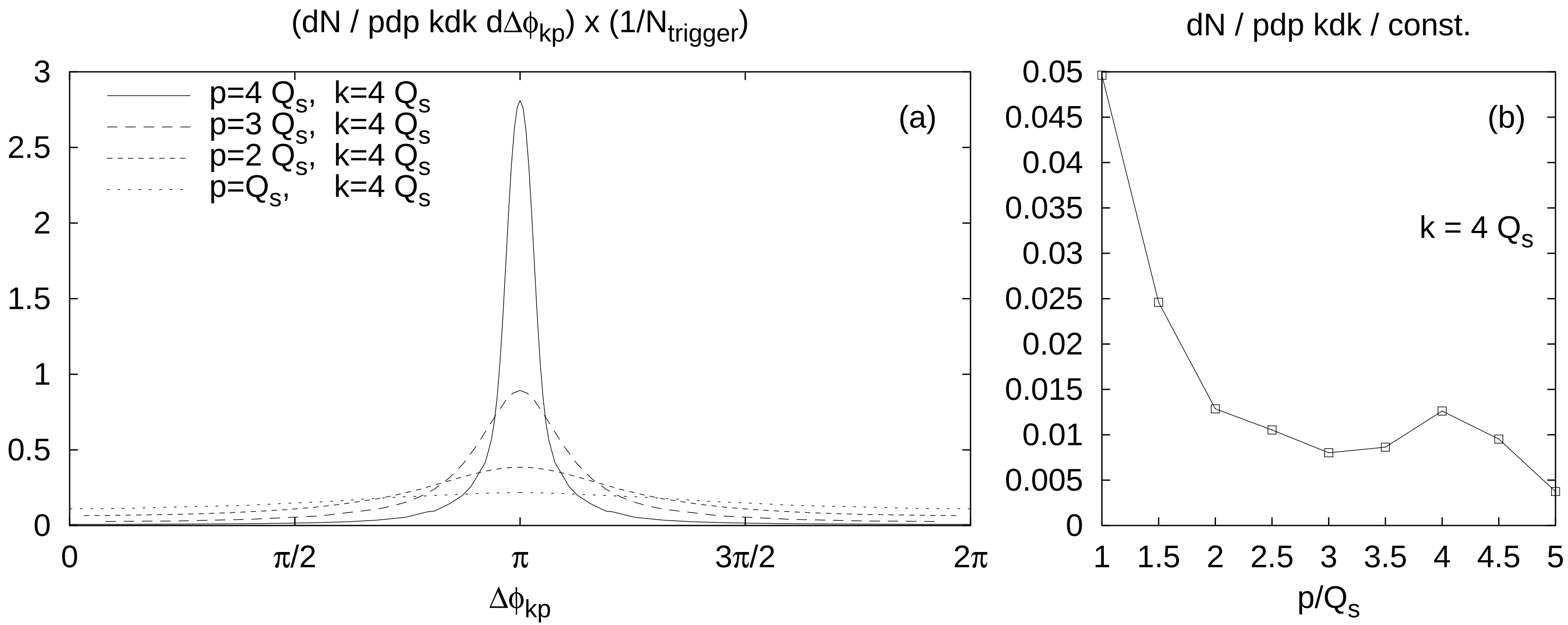
For the total momentum transmitted by the target to the qg state $K = k + p$ we find a typical Cronin like shape



Target imparts momentum of order Q_s very efficiently - very little small K final states.
At $K \sim 2 - 3\,Q_s$ there is enhancement.
At $K \gg Q_s$ recover perturbative distribution.

Angular correlation - large trigger momentum

Quark trigger



The projectile is only one quark and up to one gluon. The total transverse momentum of the incoming system is zero - q and g are exactly correlated back-to-back. While travelling through the target, each parton gets a transverse kick mostly $\sim Q_s$. But with small probability a parton can scatter off the perturbative high momentum tail of the target field.

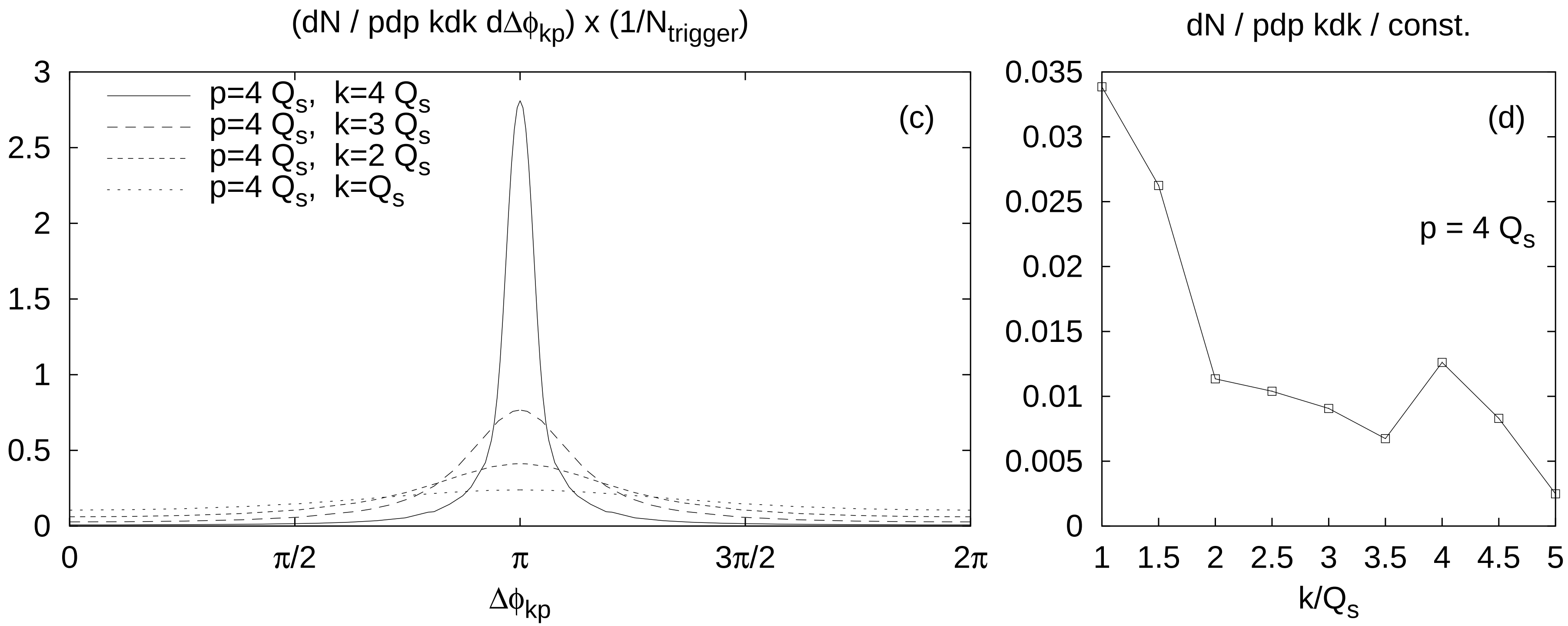
• Large p and k final states come from two sources: either large momenta in the initial state kicked by Q_s,

or low k quark that scatters off the hard tail ("kicking" a gluon out of the target).

These events are back-to-back correlated (slightly randomized direction).

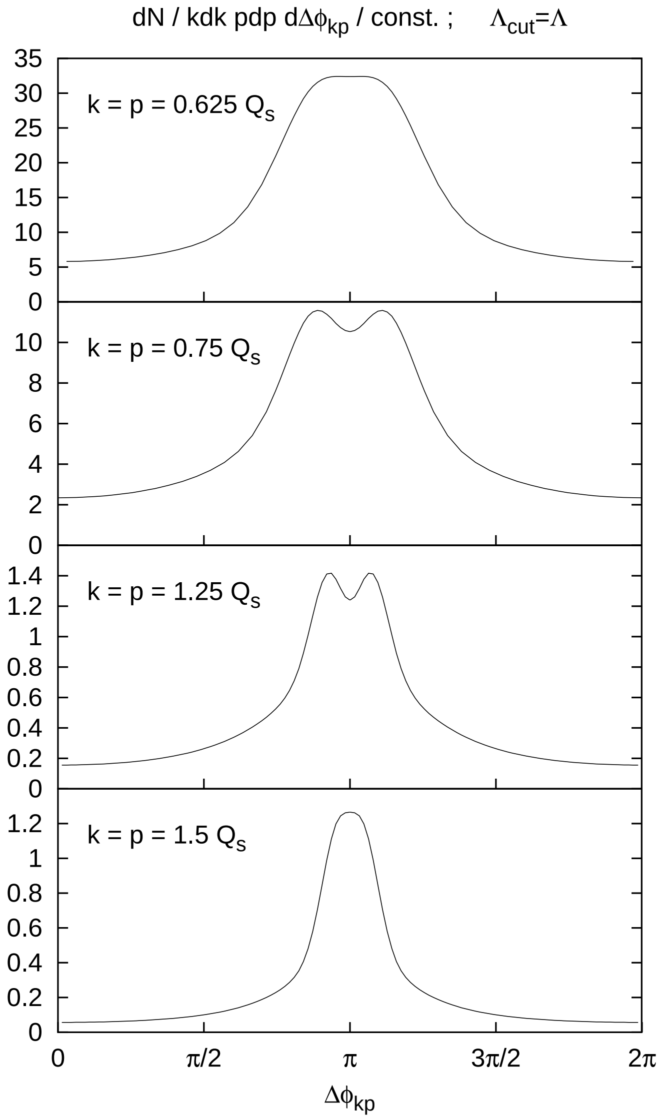
• Large k with $p = Q_s$ only arise due to hard scattering of the quark. The direction of the hard kick is random and uncorrelated with the direction of momentum of the gluon. The angular distribution in this regime is almost completely flat.

Gluon trigger



Qualitatively similar.

Angular correlation - trigger momentum $\sim Q_S$

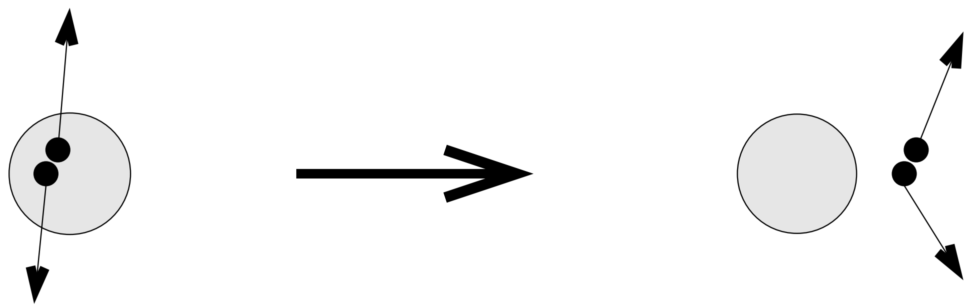


The angular correlation is not peaked at $\phi = \pi$, but instead at $\phi = \pi - \delta$. The dip at π disappears for trigger momenta $k < 0.6\,Q_s$ and $k > 1.4\,Q_s$.

Coherent scattering effect: soft multiple scattering of the initial quark-gluon components of small transverse size. If the pair is smaller than the transverse correlation length of the target fields, it scatters as one single object. It picks up a typical soft momentum $Q_{s,0}$ which is shared equally between the quark and the gluon.

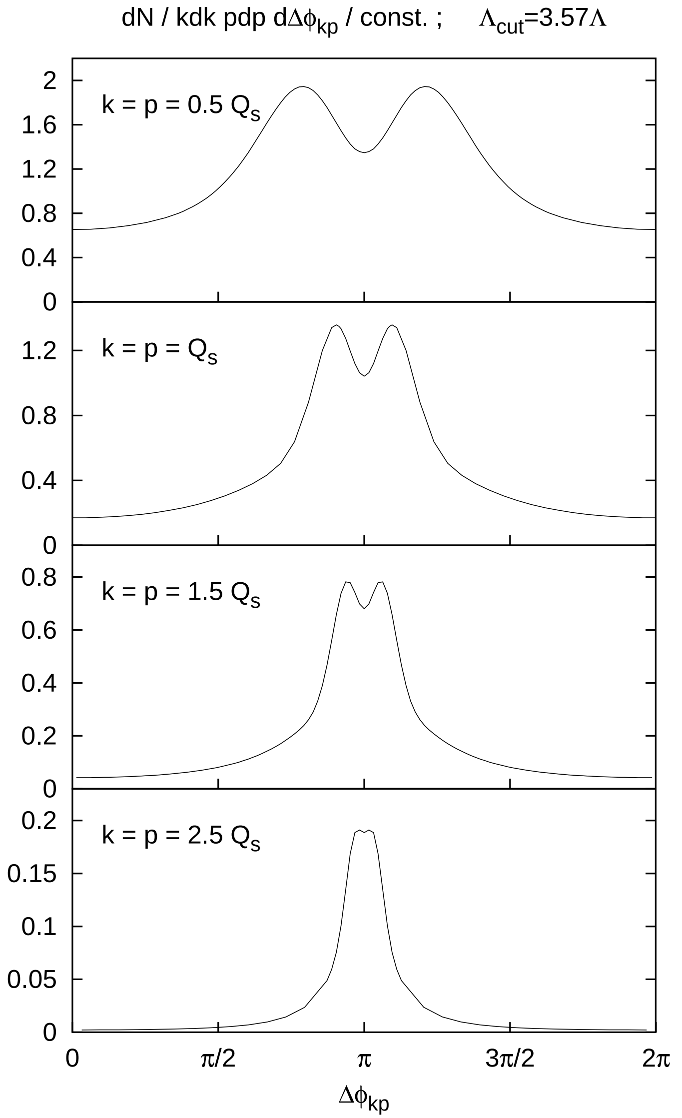
Start with the initial momenta of the quark and the gluon $\mathbf{k}_{in}$, $-\mathbf{k}_{in}$. In the final state $\mathbf{k} = \mathbf{k}_{in} + \delta\mathbf{k}$ and $\mathbf{p} = -\mathbf{k}_{in} + \delta\mathbf{k}$ with $|\delta\mathbf{k}| = Q_{s,0}/2$ and $\mathbf{k}_{in} \cdot \delta\mathbf{k} = 0$. For $k \gg Q_{s,0}$ the angle between the momenta in the final state

$$\phi = \pi - \frac{Q_{s,0}}{\sqrt{2}k}\,.$$



For large trigger momenta there is not enough phase space for coherent scattering - most of such final states are produced by hard scattering of the quark. For small trigger momentum again there is not enough phase space - most of scattering is incoherent soft - correlations disappear and so does the dip.

IR cutoff, or poor man's dipole



Caution: the numerical integration required an IR cutoff on the size of the $q(x)g(z)$ component in the wave function. This is quite obvious - we start with a coloured projectile - its radiation is Coulomb and spreads all over the transverse plane. Without cutoff there is a logarithmic divergence. We have restricted the sizes to

$$\boxed{(\text{x-z})^2 \leq \Lambda_{QCD}^{-2}}$$

What if the projectile is small?

We expect the relative importance of the small size configurations to increase, and therefore the weight of final states produced by final state radiation to be reduced. The range of trigger momenta which show the dip structure should increase. E.g. cut at $Q_{s,0} \sim$ constituent quark size

Summary

We are developing tools for first principle calculations in the saturated environment.

The two particle correlations in the simplest process we have looked at indeed exhibit the expected property of angular decorrelation even at fairly high momenta: $k = 4Q_s$, $p = 2Q_s$ there is practically no angular correlation to talk about.

A double hump structure in the angular spectrum for $k, p \sim Q_s$ - the maximum of the distribution is not back to back, but can be as fas as $\sim \frac{\pi}{4}$.