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Preface
The Mining of Missing Treasure

Most appealing—and sometimes even stirring—is a well-constructed case showing that, without doubt, some given assertion holds. Typically, such a case is based on logical and flawless reasoning, on a sequence of steps that follow inevitably from the hypotheses used to deduce each. In other words, a *proof* is given establishing that the assertion under consideration indeed holds.

Such proofs are clearly crucial to logic and to mathematics. Not so obvious, but true, proofs are crucial to circuit design, program writing, and, more generally, to various activities in which reasoning plays a vital role. Indeed, most desirable is the case in which no doubt exists regarding the absence of flaws in the design of a chip, in the structure of a computer program, in the argument on which an important decision is based. Such careful reasoning is even the key factor in games that include chess and poker.

This book features one example after another of flawless logical reasoning; the context is that of finding proofs absent from the literature. The means for finding the missing proofs is reliance on a single computer program, William McCune's automated reasoning program OTTER.

One motivating force for writing this book is to interest others in automated reasoning, logic, and mathematics. As the text strongly indicates, we delight in using OTTER equally in two quite distinct activities, both featured in this volume: finding a proof where none is offered by the literature (which includes the case when an open question is answered), and finding a proof far more appealing and simpler than any the literature provides. The latter is directly pertinent to solving Hilbert's 24th problem, recently found in a notebook in file 600 of Hilbert's mountainous Nachlass at Gottingen University Library. We believe that many other individuals, if introduced to the program OTTER, will also derive substantial enjoyment. Further, we believe that the challenge offered by the type of problem featured in this book can be as engrossing as solving puzzles and playing various games that appeal to the mind. Indeed, sometimes inexpressible is the excitement engendered when seeking a proof with fewer steps than was found by one of the great minds of the twentieth century. Some proofs are simply beautiful to behold.

A second motivating force rests with our obvious enjoyment of the type of research featured in this book. Like the fancier of fine wines, we continually seek new open questions to attack, whether (at one end of the spectrum) they concern the settling of a conjecture or (at the other end) the focus is on proof refinement, sometimes radical refinement. We encourage readers to send us additional open questions, challenging problems, and proofs to refine.

Proof refinement, in addition to its pertinence to the newly discovered Hilbert 24th problem, may offer substantial interest for circuit and chip design, program synthesis, and other practical applications. If, for example, the refinement concerns a sharp reduction in proof length, the corresponding structure may evince far more attractive properties in the context of efficiency. Of a different nature, if the refinement focuses on the avoidance of some type of term, useful structural information might be revealed.

Another factor that motivated us was our wish to collect in a single volume a surprisingly large number of proofs, most of which were previously absent from the literature. In some cases, no proof was offered of any type; in some cases, the proof that was offered was far from axiomatic. And even where an axiomatic proof was presented, we have found and offer in many, many cases a proof that is in one or more ways a refinement: fewer steps, the absence of thought-to-be-required lemmas, the avoidance of certain types of term, and the like. We present axiomatic proofs only, some in the text and many more on the included CD-ROM. None of the proofs rely on induction, or on metaargument, or on higher-order logic. In one sense, the book can serve as an encyclopedia of proofs—many new

and many refined—a work that sometimes extends, sometimes replaces, and sometimes supplements the research of more than a century. These proofs offer the implicit challenge of finding others that are further refinements.

In a rather different sense, the book may serve as the key to eventually answering one open question after another, whether the context is logic, mathematics, design, synthesis, or some other area relying on sound reasoning. In that regard, we include in detail numerous diverse methodologies we have found effective for settling a conjecture or for refining a proof. The methodologies are themselves intriguing. For an example, one methodology asks for two independent paths that lead to success and, rather than emphasizing what is common to both (their intersection), instead heavily focuses on what is not shared (their symmetric difference). A quite different and counterintuitive example (in effect) has the program avoid the attack successfully taken by one of the masters.

Although the emphasis here is on their use in the context of logic and mathematics, we conjecture that the methodologies we offer will prove most useful in a far wider context. We also suspect that, especially for those who enjoy solving puzzles and unraveling the mysteries of science, the nature of the methodologies will provide substantial stimulation.

If we succeed, this volume will introduce some readers to the excitement of discovering new results, increase the intrigue of those already familiar with such excitement, and (for the expert) add to the arsenal of weapons for attacking deep questions and hard problems.