Calculation of T-odd fragmentation functions in semi-inclusive processes

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The study of the nonperturbative features of quark and gluon dynamics inside hadrons is based on information extracted from distribution and fragmentation functions. At leading order, the state of a quark with respect to a dominant longitudinal direction is parametrized by three functions: the momentumf_1, the helicity g_1 and the spin-transverse distributions h_1 (or transversity). The first two ones are rather well known from experiments, while h_1 is chiral-odd and, therefore, unaccessible in inclusive processes. The possibility to detect more exclusive channels allows for a larger set of fragmentation functions (FF) in the final state, among which there are possible chiral-odd partners that could allow extraction of the transversity.

These classes of FF are also odd with respect to a special transformation, the naive time-reversal [1]. In other words, they are forbidden in absence of final-state interactions (FSI) and are called "T-odd". Therefore, they are interesting objects by themselves because they represent a tool to microscopically study the phenomena occurring in the final jet and leading to the detected particles. From this perspective, it is certainly not convenient to consider semi-inclusive processes where just one leading hadron is detected, because it requires the ability of describing the whole jet dynamics. It is easier to assume the jet as a spectator and consider two leading hadrons to be detected [2].

By generalizing the Collins-Soper light-cone formalism [3] for fragmentation into multiple hadrons and in analogy with semi-inclusive hard processes involving one detected hadron in the final state [4], the cross section for two-hadron semi-inclusive emission is a linear combination of projections $\Delta^{[\Gamma]}$ by specific Dirac structures Γ, where Δ is the quark-quark correlation function describing the decay of a quark with momentum k into two hadrons P_1, P_2, namely

$$\Delta_{ij}(k;P_1,P_2) = \sum_X \int \frac{d^4\zeta}{(2\pi)^4}\, e^{ik\cdot\zeta} \langle 0|\psi_i(\zeta)\, a^\dagger_{P_2}\, a^\dagger_{P_1}|X\rangle\, \langle X|a_{P_1}\, a_{P_2}\, \overline{\psi}_j(0)|0\rangle\ . \tag{1}$$

The projections involve integration over the (hard-scale suppressed) light-cone component k^+ and, consequently, ζ is light-like [2]. The sum in Eq. (1) runs over all the possible intermediate states involving the two final hadrons P_1, P_2. Since the three external momenta k, P_1, P_2 cannot all be collinear at the same time, we choose for convenience the frame where the total pair momentum $P_h = P_1 + P_2$ has no transverse component.

When $\Gamma = \gamma^-, \gamma^-\gamma_5, i\sigma^{i-}\gamma_5$, four different interference FF appear at leading twist, $D_1, G_1^\perp, H_1^\perp, H_1^{\sphericalangle}$, that depend on how much of the fragmenting quark momentum k is carried by the hadron pair ($z = z_1 + z_2$), on the way this momentum is shared inside the pair ($\xi = z_1/z$ and $1-\xi = z_2/z$), and on the "geometry" of the pair, namely on the transverse relative momentum of the two hadrons ($\mathbf{R}_T^2$) and on the relative orientation between the pair plane and the quark jet axis, i.e. on the transverse momentum of the quark $\mathbf{k}_T$ with respect to the P_h^- direction and the scalar product $\mathbf{k}_T \cdot \mathbf{R}_T$.

The different Dirac structures Γ are related to different spin states of the fragmenting quark and lead to the nice probabilistic interpretation at leading order [2]: D_1 is the probability for an unpolarized quark to produce a pair of unpolarized hadrons; $G_1^\perp$ is the difference of probabilities for a longitudinally polarized quark with opposite chiralities to produce a pair of unpolarized hadrons; $H_1^{\sphericalangle}$ and $H_1^\perp$ both are differences of probabilities for a transversely polarized quark with opposite spins to produce a pair of unpolarized hadrons. $G_1^\perp$, $H_1^{\sphericalangle}$ and $H_1^\perp$ are (naive) "T-odd" and do not vanish only if there are residual interactions in the final state. In this case, the constraints from time-reversal invariance cannot be applied. $G_1^\perp$ is chiral even; $H_1^{\sphericalangle}$ and $H_1^\perp$ are chiral odd and can, therefore, be identified as the chiral partners needed to access the transversity h_1. Given their probabilistic interpretation, they can be considered as a sort of "double" Collins effect [5].

In order to make quantitative predictions, we adopt the formalism of the spectator model, specializing it to the emission of a hadron pair. The basic idea is to replace the sum over the complete set of intermediate states in Eq. (1) with an effective spectator state with a definite mass M_D, momentum

P_D. Consequently, the correlator simplifies to

$$\Delta_{ij}(k; P_1, P_2) \sim \frac{\theta(P_D^+)}{(2\pi)^3}\, \delta\left((k-P_h)^2 - M_D^2\right) \langle 0|\psi_i(0)|P_1, P_2, D\rangle\langle D, P_2, P_1|\overline{\psi}_j(0)|0\rangle \;, \qquad (2)$$

where the additional δ function allows for a completely analytical calculation of the Dirac projections $\Delta^{[\Gamma]}$. For the hadron pair being a proton and a pion with invariant mass the mass of the Roper resonance, results have been published in Ref. [1]. In this case, the spectator state has the quantum numbers of a scalar or axial diquark. FSI arise from the interference between the direct production and the decay of the Roper resonance. For the hadron pair being two pions with invariant mass in the range $[m_\rho - \Gamma_\rho, m_\rho + \Gamma_\rho]$ with $m_\rho = 768$ MeV and $\Gamma_\rho \sim 250$ MeV, the spectator state becomes an on-shell quark with mass $m_q = 340$ MeV. Naive "T-odd" FF now arise from the interference between the direct production of the two π, which exchange a quark in t channel, and the decay of the ρ. We have explicitly checked that the former contribution reproduces the experimental transition probability for $\pi - \pi$ production in the relative S-channel, while the latter one is known to do the same job in the relative P-channel. Hence, we believe this choice represents most of the $\pi - \pi$ strength for invariant mass in the considered interval.

By defining specific Feynman rules for the $\rho\pi\pi$, $q\pi q$ and $q\rho q$ vertices, we perform actual calculations of the interference FF with microscopical ingredients. Cut-offs are introduced in the vertices to exclude large virtualities of the quark while keeping the asymptotic behaviour of FF at large z consistent with the quark counting rule. We infer the vertex form factors from previous works on the spectator model [6]. However, there numbers should be taken as indicative, since the ultimate goal is to verify that nonvanishing "T-odd" FF occur at leading twist, particularly when integrating on some of the kinematical variables and possibly washing all interference effects out.

Results of analytical calculation of Eq.(2) show that $H_1^\perp = 0$ and $H_1^{\sphericalangle} = -2m_q G_1^\perp / m_\pi$, where m_q is the quark mass. After integrating over $\mathbf{k}_T, \mathbf{R}_T^2$ while keeping $\mathbf{R}_T$ in the horizontal plane of the lab (usually identified with the scattering plane), we still get nonvanishing $H_1^{\sphericalangle}(z, M_h)$ and $G_1^\perp(z, M_h)$.

The cross section for the deep-inelastic scattering of an unpolarized electron on a polarized proton target where two pions are detected in the final state, contains, after integrating all the transverse dynamics $(\mathbf{P}_{hT}, \mathbf{k}_T, \mathbf{R}_T^2)$, an unpolarized contribution proportional to $D_1(z, M_h)$ and a term proportional to $H_1^{\sphericalangle}(z, M_h)$ which depends on the transverse target polarization S_T. Therefore, by flipping the polarization of the target, it is possible to build the following azimuthal asymmetry

$$\mathcal{A} \propto \frac{S_T}{2m_\pi} \sin(\phi_{R_T} + \phi_{S_T}) \frac{h_1(x)}{f_1(x)} \frac{H_1^{\sphericalangle}(z, M_h)}{D_1(z, M_h)} , \tag{3}$$

where ϕ_{R_T}, ϕ_{S_T} are the azimuthal angles of $\mathbf{R}_T, \mathbf{S}_T$ with respect to the scattering plane, respectively. The asymmetry shows indeed the familiar sinusoidal azymuthal dependence. Noteworthy is the factorization of the chiral-odd, naive "T-odd" $H_1^{\sphericalangle}$ from the chiral-odd transversity h_1. Therefore, such asymmetry measurement allows for the extraction of h_1 using a model input for the FF.

References

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