Model independent constraints on leptoquarks from rare processes

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Abstract

We present model independent constraints on the masses and couplings to fermions of B and L conserving leptoquarks. Such vector or scalar particles could have masses below 100 GeV and be produced at HERA; we list the generation dependent bounds that can be calculated from rare lepton and meson decays, meson-anti-meson mixing and various electroweak tests.

1 Introduction

A leptoquark is a scalar or vector particle that couples to a lepton and a quark. It may or may not have well-defined baryon and lepton number, depending on the choices of coupling. Those with B violating interactions would in general mediate proton decay, so their masses are expected to be very large ($\sim 10^{15}$ GeV). In this paper we only consider leptoquarks with B and L conserving renormalizable couplings consistent with the symmetries of the Standard Model.

There are no interactions involving a quark, a lepton and a boson in the Standard Model. There is a scalar Higgs doublet with electroweak quantum numbers, and vector bosons that are either coloured or charged, but no boson carrying colour and charge. This is a reflection of the fact that classically the leptons and quarks appear to be independant unrelated ingredients in the Standard Model. However, in each generation of the quantum theory, they have equal and opposite contributions to the hypercharge anomaly, which must vanish for the quantum theory to make sense. It would therefore seem natural to have interactions between the quarks and leptons in any extension of the Standard Model, and, in consequence, bosons coupling to a lepton and a quark.

At ordinary particle-anti-particle colliders, leptoquarks could be pair-produced, leading to a lower bound on their mass of roughly half the accelerator centre of mass energy. However, heavier ones can only be exchanged in the t channel, giving a small cross-section for the interactions they mediate. This problem is not present at a machine colliding electrons and protons, since a quark and an electron could form an s channel leptoquark; HERA, the e-p collider at DESY, which has already published some bounds on leptoquarks [48], should be able to see [1, 2] those with $m_{lq} < 314$ GeV (the collider centre-of-mass energy) as peaks in the x distributions of inclusive processes, if such leptoquarks exist. They also expect to see virtual effects due to heavier leptoquarks [2, 3, 4].

Present and upcoming results that could give constraints on leptoquarks were discussed in [5]. At low energies, leptoquarks could induce two-lepton two-quark interactions, like those mediated by the electroweak four-fermion vertices. This suggests that the leptoquark yukawa coupling squared ($\equiv \lambda^2$), divided by the mass squared ($\equiv m_{lq}^2$) is at least as small as G_F. However, for certain combinations of generation indices on λ, there should be much stronger bounds because leptoquarks could mediate interactions forbidden in the Standard Model by lepton family number conservation and the absence of flavour-changing neutral currents (FCNC).

In this paper, we concentrate on bounds from existing accelerators, meson-anti-meson ($K\bar{K}$, $D\bar{D}$, $B\bar{B}$) mixing, rare lepton and meson decays, and a few electroweak tests. Most of these, have, of course, been previously calculated in various models, [6, 7, 8, 9, 10, 11, 12, 2, 13, 14, 15, 3] so we wish to list the bounds in as complete and model independent a fashion as possible. We neglect bounds from CP violation, because these constrain the imaginary part of the coupling, not its magnitude.

In the remainder of this section, we introduce the interactions we are allowing the leptoquarks to have. There is then a short review of some theoretical models in which leptoquarks appear, followed by sections on pre-HERA accelerator mass bounds, constraints from meson decays, meson anti-meson mixing, muon physics and τ decays, and a section of bounds that are none of the above.

We will assume that the couplings are real, so we neglect any constraints arising from leptoquark contributions to CP violation. (This simplifying assumption was not made in

[7, 13, 15, 16, 17, 18].) We will discuss each calculation in the text, and usually list the bounds (without generation indices). Most of the numerical upper bounds are collected in the tables at the end.

We neglect QCD corrections to all the rates that we calculate, except $b \to s\gamma$. This may be a poor approximation, but we would not expect this to change our bounds on λ^2 by more than a factor of 2.

There are seven renormalizable B and L conserving quark-lepton-boson couplings consistent with the SU(3)×SU(2)×U(1) symmetries of the Standard Model for both scalar and vector leptoquarks, and each coupling carries generation indices for the two fermions (these are suppressed in equations (1) and (2)). The scalar and vector interaction lagrangians are therefore [2]

$$\begin{aligned} \mathcal{L}_S \;=\; & \{\, (\lambda_{LS_o}\bar{q}^c_L i\tau_2\ell_L + \lambda_{RS_o}\bar{u}^c_R e_R)S^\dagger_o + \lambda_{R\tilde{S}_o}\bar{d}^c_R e_R\tilde{S}^\dagger_o + (\lambda_{LS_{1/2}}\bar{u}_R\ell_L + \\ & \lambda_{RS_{1/2}}\bar{q}_L i\tau_2 e_R)S^\dagger_{1/2} + \lambda_{L\tilde{S}_{1/2}}\bar{d}_R\ell_L\tilde{S}^\dagger_{1/2} + \lambda_{LS_1}\bar{q}^c_L i\tau_2\vec{\tau}\ell_L\cdot\vec{S}^\dagger_1 \,\} + h.c. \end{aligned} \tag{1}$$

and

$$\begin{aligned} \mathcal{L}_V \;=\; & \{\, (\lambda_{LV_o}\bar{q}_L\gamma_\mu\ell_L + \lambda_{RV_o}\bar{d}_R\gamma_\mu e_R)V^{\mu\dagger}_o + \lambda_{R\tilde{V}_o}\bar{u}_R\gamma_\mu e_R\tilde{V}^{\mu\dagger}_o + (\lambda_{LV_{1/2}}\bar{d}^c_R\gamma_\mu\ell_L \\ & +\lambda_{RV_{1/2}}\bar{q}^c_L\gamma_\mu e_R)V^{\mu\dagger}_{1/2} + \lambda_{L\tilde{V}_{1/2}}\bar{u}^c_R\gamma_\mu\ell_L\tilde{V}^{\mu\dagger}_{1/2} + \lambda_{LV_1}\bar{q}_L\gamma_\mu\vec{\tau}\ell_L\cdot\vec{V}^{\mu\dagger}_1 \,\} + h.c. \end{aligned} \tag{2}$$

where the SU(2) singlet, doublet and triplet leptoquarks respectively have subscipts 0, 1/2 and 1, and the L/R index on the coupling reflects the lepton chirality. We have written these lagrangians in the so-called "Aachen notation". In the first column of tables 3 and 4, we have written the interactions in this notation, and also {in curly brackets}, in the notation of reference [2]. (In this alternative formulation, fermion number two leptoquarks have couplings g, fermion number 0 leptoquarks have couplings h, and SU(2) singlet, doublet and triplet leptoquarks respectively have subscipts 1, 2 and 3.) The fermions in our notation are chiral: here and in the tables, $\bar{f}_R = (Rf)^\dagger\gamma^o$ and $f^c_L = (Lf)^c$.

Generation indices are superscripts and the lepton family index comes first: λ^{ij} couples a leptoquark to an ith generation lepton and a jth generation quark. If leptoquarks also carry generation indices, cancellations between the different generations of leptoquark are conceivable (vaguely similiar to the GIM mechanism). In this case our bounds would apply to the sum over leptoquark-induced amplitudes, but not to the coupling constants and masses of each generation of leptoquark.We ignore this possibility. In the sections on rare meson decays, we quote bounds on e^2 (see (15) and (16)), defined as λ^2 for vectors and $\lambda^2/2$ for scalars.

2 Theory

We present here a short review of some extensions of the Standard Model that could contain leptoquarks. The first possibility is Grand Unified Theories (GUTs) [19], where leptons and quarks usually appear in the same multiplet. In consequence it is quite normal to have gauge and Higgs bosons coupling to a quark and a lepton (with or without B and L violation). Secondly, there are extended technicolour theories[24], where quarks and leptons individually appear in multiplets of the dynamically broken extended technicolour group. The other particles in each multiplet are new fermions, that would appear at low energies in fermion-anti-fermion bound states, some of which are leptoquarks. And finally in substructure models [25], the "preons" in a quark and lepton could combine to form a scalar or vector leptoquark.

2.1 Grand Unified Theories

Leptoquarks first appeared in Pati and Salam's SU(4) model [20], where lepton number was treated as a fourth colour: the four weak doublets of each generation are arranged as a four of SU(4). SU(4) is then spontaneously broken, so that the gluons remain massless and the leptoquark gauge bosons become heavy. In this model, the leptoquarks induce flavour-changing neutral currents and lepton family number violation; they can also violate B, but their contribution to proton decay is severely suppressed [21].

Constraints on Pati-Salam type leptoquarks have been discussed in [20] and [7, 8]. One would expect these vector leptoquarks to have full-strength couplings to a lepton and a quark of the same generation, giving, for instance, a large ($\sim \lambda^2/m_{lq}^2, \lambda \sim 1$) contribution to $K \to e\mu \;\; (s + \mu \to d + e)$. They could also interact with quarks and leptons of different generations, but one would expect this to be Cabbibo suppressed. So the best bounds on these leptoquarks should come from interactions involving lepton-quark pairs of the same generation. SU(5) [22] GUT models contain vector leptoquarks, but these acquire grand unification scale masses when the gauge group breaks to $SU(3)\times SU(2)\times U(1)$, so are of little interest to us. This is a generic feature of GUTs: the quarks and leptons share multiplets, so there are necessarily leptoquark gauge bosons; however the lepton-quark symmetry is broken at the GUT scale, so these gauge bosons have large masses. (Note, however, that these are not neccessarily of order 10^{15} GeV; the "GUT-scale" for Pati-Salam could be $\sim 10^5$ GeV.) SU(5) also contains scalar leptoquarks in the Higgs sector. The Standard Model Higgs doublet that breaks SU(2) shares an SU(5) **5** with an SU(2) singlet leptoquark. Given the fermion representation in SU(5), this leptoquark necessarily has B-violating interactions because it couples to two quarks and also to a lepton and a quark. The scalar potential must therefore be arranged to give these leptoquarks very large masses, in which case they are of no interest to us. Alternatively, one can break SU(5) invariance by setting the leptoquark-quark-quark couplings to zero, as was done in reference [11], which allows all five components of the scalar **5** to be light. It is of course possible to have scalar leptoquarks in SU(5) with B and L conserving interactions by putting them in a representation other than the **5**. An example of this can be found in [23],where it is shown that the minimal SU(5) model plus a light ($m_{lq} \sim 100$ GeV) SU(2) doublet leptoquark would be compatible with present experimental results on proton decay and $\sin^2\theta_W$. This B and L conserving leptoquark ($\equiv \tilde{S}_{1/2}$, see table 3) occupies the same position in the SU(5) **10** as the quark doublet, so its interactions with light Standard Model fermions are of the form $\bar{d}_R l_L \tilde{S}^\dagger_{1/2}$. The other members of the **10** are assumed to be very heavy.

The field theory limit of the heterotic superstring is some "GUT-like" gauge field theory, which is broken at or below the compactification scale to the Standard Model and possibly some extra $U(1)$s: $SU(3)\times SU(2)\times U(1)^{n+1}$. This low energy limit may contain leptoquarks for the same reasons that GUTs do. For instance, certain Calabi-Yau compactifications have $G = E_6/H$ as the four-dimensional gauge group below the compactification scale, where H is a discrete symmetry group that can be chosen such that G is the Standard Model ($\times U(1)^n$). The low energy fields are then expected to sit in the **27** of E_6,

and for each generation consist of quarks and leptons, a right-handed neutrino, the two Higgs doublets of the Minimal Supersymmetric Standard Model, an extra SO(10) singlet, and a pair of SU(2) singlets, which may have thr couplings of either diquarks or leptoquarks, but not both simultaneously [13], as this would destabilize the proton. The constraints on leptoquarks in such superstring-derived models were calculated in [13], and are extensively reviewed in [14].

2.2 Technicolour

If our knowledge of quantum field theory is extrapolated to energies far above those experimentally accessible today, one discovers that the masses of fundamental scalars are generically the same size as the cutoff energy. So to keep the mass of the Higgs at the electroweak scale, one must either fine-tune the parameters of the theory to many decimal places, or introduce new physics that explains the comparatively small scalar mass. Both supersymmetry and technicolour [24] do this.

In a technicolour theory, all scalars are bound states of fermion anti-fermion pairs, like the mesons of QCD. One introduces new electroweak doublets and singlets (technifermions) which are multiplets of a non-abelian gauge interaction called technicolour. The new interaction becomes strong at the electroweak scale, and, by analogy with QCD, one expects a techni-fermion condensate to form, breaking some of the global symmetries of the theory. Three of the goldstone bosons arising from this global symmetry breaking become the longitudinal components of the electroweak gauge bosons. This neatly gives masses to the W and the Z, without a fundamental scalar Higgs. In fact, if one temporarily ignores the problem of generating fermion masses, one can simply introduce an electroweak doublet of left-handed technifermions, and their right-handed singlet counterparts. This has an $SU(2)_L \times SU(2)_R$ global symmetry if one neglects the (comparatively weak) standard model interactions. When the technifermion condensate forms, the global symmetry is assumed to be broken to $SU(2)_V$, and the three goldstone bosons associated with the broken axial currents ($\sim$ pions in QCD) are eaten by the electroweak gauge bosons.

To be a viable substitute for the scalar Higgs, technicolour must also produce fermion masses; this is done in extended technicolour (ETC) models. The idea is to introduce more technifermions, and gauge interactions involving ordinary and technifermions that are broken at very high energies. The gauge bosons of these new interactions (which we refer to as ETC gauge bosons) presumably acquire mass by some dynamical symmetry breaking mechanism of their own, and afterwards induce four-fermion interactions involving two technifermions and two ordinary fermions. After the formation of the technifermion condensate, the four-fermion interaction becomes a "standard model" mass term.

It is desirable to introduce a whole standard model generation of technifermions so that the ETC gauge bosons do not have to carry $SU(3)\times SU(2)\times U(1)$ quantum numbers. This means that there will be "leptoquark technimesons" made up of a techniquark and an anti-technilepton. It is easiest to see why leptoquarks naturally arise in ETC models by looking at a simple example. An extra generation, plus right-handed neutrino, who each transform as an **N** under the technicolour SU(N), are added to the Standard Model, and one removes the Higgs sector. One also must add other interactions and particles (including the ETC gauge bosons) at higher energies. The new technifermions are assumed to interact with each other via the techniforce, and with other Standard Model particles via colour and electroweak interactions, and through the effective four-fermion vertices induced by ETC gauge bosons. Technicolour becomes strong around 1 TeV, a technifermion condensate is assumed to form, and some of the global symmetries of the technicolour sector are dynamically broken.

In QCD, if one neglects the masses of the u and d quarks, there is a global $SU(2)_L \times SU(2)_R$ symmetry. This is broken by the condensate to $SU(2)_V$, and the pions are the (pseudo-)goldstone bosons of the broken axial symmetry. Similar behaviour is expected in a technicolour theory; in the model considered here, if one neglects electroweak, colour and four-fermion interactions, there is a global $SU(8)_L \times SU(8)_R$ symmetry (assuming complex representations under the technicolour group) which is expected to break to $SU(8)_V$. This produces 63 goldstone bosons. Three will be eaten by the electroweak gauge bosons, and the others acquire masses due to the neglected interactions. It is clear that among the 60 goldstone bosons, there will be "leptoquark mesons" composed of a techniquark and an anti-technilepton, bound together by the technicolour force. This scalar leptoquark will interact with a quark and an anti-lepton carrying the same Standard Model quantum numbers (it would be easy to construct a model where the leptoquark had fermion number 2).

The coupling of a technimeson leptoquark to a lepton and a quark occurs when the technifermion constituents of the leptoquark exchange an ETC gauge boson and turn into an ordinary quark and lepton. The "Yukawa" coupling can only be predicted in a specific model, but it is expected to be of order m_l/Λ or m_q/Λ, where Λ is the technicolour scale. Constraints on leptoquarks in technicolour models were computed in [7, 8, 6].

2.3 Composite Models

Leptoquarks frequently arise in composite, or substructure, models [25, 26, 27, 28, 29].

In these scenarios, the leptons and quarks, and sometimes the gauge bosons, are assumed to be bound states of more fundamental particles. In principle, one could hope that this would explain the quark and lepton spectrum, and reduce the number of fundamental fields (as quarks did for the hadronic spectrum). However, at the moment, most models do not require fewer constituents ("preons") than there are particles in the Standard Model.

One of the fundamental questions in building theories where the Standard Model particles are composite, is to explain why their masses are much smaller than the compositeness scale. A solution for the fermion masses is to have them forbidden by a global chiral symmetry in the strongly interacting sector of the theory. This symmetry is broken by the Yukawa and $SU(3)_c \times U(1)$ couplings, so one would expect the masses generated by these symmetry-breaking effects to be small. Models depending on preon chiral symmetries to preserve the masslessness of composite fermion bound states must, however, satisfy the 't Hooft anomaly matching conditions [30]. These require that chiral anomalies calculated in the effective composite theory match those calculated in the preon subtheory [30, 31, 32]. A solution of the anomaly matching conditions is to reinterpret the "tumbling" gauge theories of dynamical symmetry breaking [33] as composite models satisfying the 't Hooft conditions via the principle of complementarity. Although solutions of the anomaly matching conditions have been pursued by a variety of methods, (for a review, see [35]), the construction of a fully realistic model remains an open problem. Another method for keeping the composite fermions light is to make them the supersymmetric partners of goldstone bosons in a theory with a spontaneously broken approximate global symmetry. Their masses must then remain small even after the breaking of supersymmetry and the global symmetry. It is even more difficult to explain how composite gauge bosons could have masses much below the substructure scale; the Case-Gasiorowicz-Weinberg-Witten theorem forbids the presence of interacting massless spin-1 composite particles, suggesting that all composite vector bosons should have masses of order the compositeness scale [36].

If the quarks and leptons are composite particles, they should have form factors. The "substructure contribution" to $g-2$ of the muon is expected to be of order $(m_\mu/\Lambda_c)^2$, where Λ_c is the compositeness scale. This constrains $\Lambda_c \gtrsim 1$ TeV, which makes it difficult to associate the weak scale with the substructure scale (as was done in [26]) in realistic models. There are a number of stronger, but more model-dependant bounds on Λ_c [38].

Leptoquarks appear naturally in many substructure models[39]. This is not surprising: if a constituent particle of a Standard Model fermion carries quark or lepton number, then a composite quark could turn into a lepton in the presence of a composite lepton (that turned into a quark) by exchanging the appropriate constituents.The bound state consisting of the exchanged constituents would be a leptoquark. This suggests that leptoquarks in composite models could naturally induce interactions where flavour or generation number was conserved overall, but separately violated in the quark and lepton sector.

3 Mass Bounds

There have been numerous searches for leptoquarks at existing accelerators. Possible searches at $e\bar{e}$ colliders were discussed by Hewett and Rizzo [37] who point out that the measurement of the R-ratio from PEP and PETRA constrains all scalar leptoquarks to have $m_{lq} \geq 15$ GeV. The leptoquarks are assumed to be pair-produced in the decay of a virtual photon, so this bound depends only on their electric charge which is known to be $\geq 1/3$ (see tables 1 and 2). The same measurement from AMY [40] gives

$$m_{lq} \geq 22.6 \text{ GeV} \qquad \text{(for scalars)}. \tag{3}$$

We expect a similiar bound for vectors

$$m_{lq} \gtrsim 20 \text{ GeV} \qquad \text{(for vectors)}. \tag{4}$$

These are the best model independant lower bounds on the mass (that we are aware of). There have also been leptoquark searches at LEP and the $p\bar{p}$ colliders, but these look for leptoquark decay products, so only apply if the leptoquark-lepton-quark Yukawa coupling is "sufficiently" large (although this is not a significant constraint; see below). The LEP lower bound [41] is

$$m_{lq} \geq 44 \text{ GeV} \tag{5}$$

for scalars, and a similiar bound should apply to the vectors. This was calculated assuming that the Z_o turned into a pair of leptoquarks, which then decayed to jets and two leptons ($e\bar{e}, \mu\bar{\mu}, \tau\bar{\tau}, \nu\bar{\nu}$; the mass bound actually depends slightly on the type of lepton [41]—(5) is an approximation). Since all the leptoquarks are charged, they all couple to the Z_o, although this is rather weak in the case of the $Q_{em} = 1/3$ scalar singlet (see tables 1 and 2).

Delphi [43] has also looked for Z decays into an on-shell and an off-shell scalar leptoquark. They conclude that

$$m_{lq} > 77 \quad \text{GeV} \ \ (\lambda > e) \tag{6}$$

for leptoquarks decaying to a quark and an electron or a muon. Note that unlike the bounds (5) and (9), this only applies for large ($\lambda > e$) leptoquark-lepton-quark couplings. In principle, leptoquarks could contribute at one loop to the decays $Z_o \to \bar{\ell}_1 \ell_2$, with $\ell_1 \neq \ell_2$, but the rates for such processes are so small that LEP would not expect to see them even if the leptoquark yukawas were of order 1 [42].

The decay rate of a leptoquark to a lepton and a quark is

$$\Gamma = \frac{\lambda^2 m_{lq}}{16\pi} \ \ \text{(for scalars)} \tag{7}$$

$$\Gamma = \frac{\lambda^2 m_{lq}}{24\pi} \ \ \text{(for vectors)} \tag{8}$$

so one would need the coupling constant to be very small ($\lesssim 10^{-8}$) for the leptoquark to decay outside a detector. Such a small renormalizable coupling would be unexpected, so it should be safe to assume $m_{lq} \geq 44$ GeV. (Note that equations (7) and (8) give the leptoquark decay rate to a specific flavour of quark and lepton. The total decay rate would be the sum over all types of kinematically accessible quarks and leptons.)

The UA2 bound [44] on scalar leptoquarks decaying to $e+$ jets is $m_{lq} > 67$ GeV, and CDF has a bound of 113 GeV (95%) [45]. The strongest $\bar{p}p$ collider bound is a D0 result [46] for leptoquark pairs decaying to two electrons plus jets:

$$m_{lq} > 126 \ \ \text{GeV} \qquad (\text{lq} \to eq) \tag{9}$$

where the leptoquark is assumed to decay entirely to electron plus jets. The cross section for vector leptoquark production is being calculated [47], so bounds on vector leptoquarks from CDF and DO should be possible in the future.

It is not clear that leptoquarks that could be seen at HERA must satisfy these bounds; although they must be produced in a quark-electron (or positron) collision, they could have stronger couplings to muons or taus, decay to μ or τ plus jets, and thereby avoid this constraint. We will nonetheless use $m_{lq} = 100$ GeV in our numerical results.

The ZEUS and H1 Collaborations at HERA have (mass-dependent) limits on the couplings of scalar and vector leptoquarks produced in an ep collision and decaying to $e + X$ or $\nu + X$ [48]. The exact bounds depend somewhat on the type of leptoquark, and will of course improve with time. The present limits rule out roughly from

$$\lambda > .05 \quad m_{lq} = 25 \text{ GeV} \tag{10}$$

to

$$\lambda > 1 \quad m_{lq} = 225 \text{ GeV} \tag{11}$$

for the SU(2) singlet and triplet leptoquarks (scalar and vector), and from

$$\lambda > .1 \quad m_{lq} = 25 \text{ GeV} \tag{12}$$

to

$$\lambda > 1 \quad m_{lq} = 150 \text{ GeV} \tag{13}$$

for the doublets. Note that these are just very rough approximations to the present data. See [48] for accurate plots.

4 Meson Decays

Since leptoquarks have not been directly observed at any high energy colliders, one must look for traces of them in other interactions (meson decays, for instance). At energies well below their mass, the leptoquarks can be "integrated out", leaving effective four-fermion vertices involving two leptons and two quarks, and a photon-lepton-lepton (or quark-quark) magnetic moment vertex. We will come back to the latter in sections 6 and 7. The four fermion vertices contract the spinor indices of a lepton and a quark, but can be Fierz-rearranged to contract the quark indices together; in this form one can estimate the quark matrix elements between initial and final meson states. The leptoquark induced effective four-fermion vertices are listed in tables 3 and 4. The hermitian conjugates of the listed operators are of course also possible. Most of the leptoquarks couple to quarks of only one chirality, so the Fierz-rearranged vertices must be of the form $V \pm A$, similiar to the weak interactions. The only exceptions to this are $S_o, S_{1/2}, V_o^\mu$ and $V_{1/2}^\mu$, which have two interactions: a "left-handed" one with coupling constant λ_L, involving the leptoquark, a left-handed lepton and a quark, and a "right-handed" interaction coupling the leptoquark to a right-handed lepton and a quark. The Fierz-rearranged effective four-fermion vertex arising when these two interactions appear at opposite ends of the leptoquark propagator contains tensor, scalar and pseudoscalar pieces. We will ignore the tensor operators, because their matrix elements are difficult to estimate, and just list the scalar $\pm$ pseudoscalar parts. Some experimental results will give much better constraints on the coefficients of the $S \pm P$ matrix elements than the $V \pm A$ ones. In these cases, we will list bounds on the products $\lambda_L \lambda_R$. In general, however, we will avoid the "mixed coupling" constraints because they do not apply if one of the interactions does not exist.

In the Standard Model, many mesons are forbidden from decaying via the strong or electromagnetic interactions. Since such mesons decay at a rate consistent with the weak interactions, one would roughly expect the effective dimensionful coupling $\sim \lambda^2/m_{lq}^2$ of the leptoquark four-fermion vertices to be as small as that of the weak interactions:

$$\frac{\lambda^2}{m_{lq}^2} \lesssim \frac{4G_F}{\sqrt{2}}|V| \tag{14}$$

where V is the appropriate CKM matrix element.

However, we can get much stronger constraints on the leptoquark couplings that induce interactions not present in the Standard Model. A variety of meson decay constraints were considered in references [2, 3, 7, 8, 9, 10, 11, 13, 15]. The first "non-Standard" type of leptoquark-induced behaviour we will consider is the lack of chiral suppression in meson decays due to pseudoscalar matrix elements. The V–A vertices of the weak interactions give a matrix element squared for the leptonic decay of pseudoscalar mesons proportional to $E_i E_n - \vec{k}^{\,2}$ (where E_i, E_n and $\vec{k}$ are the energies and momentum of the outgoing leptons), because the helicity ($\sim$ chirality for a relativistic particle) of one of the outgoing leptons must be flipped via a mass term to conserve angular momentum. The decay rate is therefore strongly suppressed for relativistic leptons. There is no such chiral suppression in meson decays induced by leptoquarks with both left and right-handed couplings (pseudoscalar matrix elements, see below).

One would also expect leptoquarks, if they exist, to induce lepton family number violating decays; even if the leptoquark Yukawa interactions are "generation diagonal" in the sense that they only couple first generation quarks to first generation leptons, this would still allow, for instance a kaon to decay to $\bar{e}\mu$. It would similiarly be very natural for leptoquarks to induce flavour-changing decays suppressed in the Standard Model.

Leptoquarks can induce effective four fermion vertices of the form

$$\frac{e^{ij}e^{mn}}{m_{lq}^2}(\bar{q}^j\gamma^\mu P^q q^n)(\bar{\ell}^m\gamma_\mu P^\ell \ell^i) \tag{15}$$

and/or

$$\frac{e^{ij}e^{mn}}{m_{lq}^2}(\bar{q}^j P^q q^n)(\bar{\ell}^m P^\ell \ell^i) \tag{16}$$

(see tables 3 and 4) where P^q and P^ℓ are chiral projection operators = L or R, and m_{lq} is the leptoquark mass. The coefficients of the effective operators in the third column of tables 3 and 4 are generally twice as large for vectors as for scalars. In equations (15) and (16) we have introduced a coupling $e^2 = \lambda^2$ ($e^2 = \lambda^2/2$) for vectors (scalars); the constraints we compute on e^2 will therefore be bounds on the square of the vector coupling, or half the square of the scalar coupling. As is clear from tables 3 and 4, $V \pm A$ quark currents are induced by leptoquarks with couplings of the same chirality at both ends of the propagator. The effective coupling of $V \pm A$ 4-fermion vertices is therefore e_L^2/m_{lq}^2 or e_R^2/m_{lq}^2. In all the following, we will quote bounds on e_L^2 with the understanding that the constraint also applies to e_R^2. (Remember that the coupling constant subscript applies to the lepton chirality.) The scalar and pseudoscalar vertices are induced by the leptoquarks $S_o, S_{1/2}, V_o^\mu$ and $V_{1/2}^\mu$ with couplings of opposite chiralities at either end of the propagator. The effective coupling of the $S \pm P$ vertices is therefore $e_L e_R/m_{lq}^2$.

The matrix element squared for the decay of a pseudoscalar meson M ($\bar{q}^j q^n$ bound state) to the leptons ℓ^i and $\bar{\ell}^m$ is

$$\begin{aligned} |\mathcal{M}|^2 &= \tfrac{2}{m_{lq}^4}\{[(e_L^{ij}e_L^{mn})^2 + (e_R^{ij}e_R^{mn})^2]m_M^2\tilde{A}^2(E_iE_m - \vec{k}^{\,2}) \\ &+[(e_L^{ij}e_R^{mn})^2 + (e_L^{ij}e_R^{mn})^2]\tilde{P}^2(E_iE_m + \vec{k}^{\,2}) \\ &+2(e_L^{ij}e_L^{mn} + e_R^{ij}e_R^{mn})\tilde{P}\tilde{A}m_M(e_L^{ij}e_R^{mn}E_im_m - e_R^{ij}e_L^{mn}E_im_m)\} \end{aligned} \tag{17}$$

where

$$\tilde{P} = \frac{1}{2} < 0|\bar{q}^j\gamma^5 q^n|M > \sim \frac{f_M m_M}{2}\frac{m_M}{m_{q^j} + m_{q^n}} \tag{18}$$

and

$$\tilde{A}\,p^\mu = \frac{1}{2} < 0|\bar{q}^j\gamma^\mu\gamma^5 q^n|M >= \frac{f_M p^\mu}{2} \tag{19}$$

(p^μ is the meson 4-momentum and f_M the decay constant). The approximation for $\tilde{P}$ is a current algebra result, so not appropriate for the heavier mesons; we will use it anyway to get qualitative results. Equation(17) gives a meson decay rate

$$\begin{aligned} \Gamma(M \to \ell^i\bar{\ell}^m) &= \tfrac{k}{4\pi m_M^2}\tfrac{1}{m_{lq}^4}\{[(e_L^{ij}e_L^{mn})^2 + (e_R^{ij}e_R^{mn})^2]m_M^2\tilde{A}^2(E_iE_m - \vec{k}^{\,2}) \\ &+[(e_L^{ij}e_R^{mn})^2 + (e_L^{ij}e_R^{mn})^2]\tilde{P}^2(E_iE_m + \vec{k}^{\,2}) \\ &+2(e_L^{ij}e_L^{mn} + e_R^{ij}e_R^{mn})\tilde{P}\tilde{A}m_M(e_L^{ij}e_R^{mn}E_im_m - e_R^{ij}e_L^{mn}E_im_m)\} \end{aligned} \tag{20}$$

where k is the magnitude of the lepton 3-momentum in the centre-of-mass frame, and the absence of chiral suppression in the pseudoscalar matrix element is clear.

4.1 Pion Decays

In the Standard Model, charged pions decay principally to $\mu\nu$ because the decay to $e\nu$ is suppressed by angular momentum conservation, as discussed in the previous section. The experimental ratio [49]

$$R_{exp} \equiv \frac{\Gamma(\pi^+ \to \bar{e}\nu)}{\Gamma(\pi^+ \to \bar{\mu}\nu)} = 1.231 \pm .006 \times 10^{-4} \tag{21}$$

agrees with the Standard Model prediction [50]

$$R_{th} = \frac{m_e^2}{m_\mu^2} \frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} (1 + \Delta) = 1.235 \pm .004 \times 10^{-4} \tag{22}$$

where Δ is a radiative correction. Using (20) to compute the contribution to this ratio from the interference of the Standard Model axial vector amplitude and the presumably small leptoquark-induced effective pseudoscalar operators, one can require

$$\frac{\tilde{P}}{m_\pi \tilde{A}} \left(\frac{e_L^{n1} e_R^{11}}{\sqrt{2} G_F m_{lq}^2} \frac{m_\pi}{m_e} - \frac{e_L^{n1} e_R^{21}}{\sqrt{2} G_F m_{lq}^2} \frac{m_\pi}{m_\mu} \right) < \frac{|R_{exp} - R_{th}|}{R_{th}} \quad . \tag{23}$$

If we assume that there is no spurious cancellation between the pseudoscalar contributions to $\pi \to \mu\nu$ and $\pi \to e\nu$ (first and second terms of equation (23)), then this gives

$$(e_L^{n1} e_R^{11})^2 < 5 \times 10^{-7} \left(\frac{m_{lq}}{100 \text{ GeV}} \right)^2 \quad , \quad (e_L^{n1} e_R^{21})^2 < 10^{-4} \left(\frac{m_{lq}}{100 \text{ GeV}} \right)^2 \tag{24}$$

where we have used the current algebra result $\tilde{P} \simeq 7 f_\pi m_\pi$ [7], and n is a light neutrino index ($m_\nu \ll m_e$).

We get bounds on $(e_L^{11}/m_{lq})^2$ from the interference between W and leptoquark exchange, which, if small, must satisfy

$$\frac{1}{\sqrt{2} G_F m_{lq}^2} (e_L^{11} e_L^{n1} - e_L^{21} e_L^{n1}) < \frac{|R_{th} - R_{expt}|}{R_{th}} \tag{25}$$

where n is again an arbitrary light ($m_\nu \ll m_e$) neutrino index. Assuming that leptoquark couplings to muons and electrons are not equal ($e_L^{11} \neq e_L^{21}$), we can conclude that:

$$e_L^{21} e_L^{n1}, e_L^{11} e_L^{n1} < 2 \times 10^{-3} \left(\frac{m_{lq}}{100 \text{ GeV}} \right)^2 \quad . \tag{26}$$

If the leptoquark coupling is independant of the lepton generation, so that $e^{11} = e^{21}$, then leptoquarks will not affect the R ratio. However, $(e^{11})^2 [= (e^{21})^2$ in this case], is still constrained by quark-lepton universality (see section 8.1), which gives a better bound than this one.

Equations (24) and (26) give bounds on the couplings of the effective vertices of tables 3 and 4 that involve u, d, e (or μ) and any flavour of light ($m_\nu \ll m_e$) neutrino. We list the exact constraints in the tables. Since the notation in the tables is rather obscure, we will discuss in detail the first four rows of the table listing bounds on vector leptoquark couplings (table 5). The four-fermion vertices have an effective coupling constant e^2/m_{lq}^2, so the bounds on e^2 depend on m_{lq}^2. We take $m_{lq} = 100$ GeV to compute the numbers in the tables, so the numerical bounds scale as $(m_{lq}/100 \text{ GeV})^2$. The rows are labelled by experiments, and the columns by coupling constants; an entry in the table is the bound on a particular coupling constant from the given experiment. These constraints are in general generation dependent, so we list the coupling constant generation indices in parentheses. For example, we have just found that the upper bound on R constrains axial-vector quark operator couplings to be less than $2 \times 10^{-3} (m_{lq}/100 \text{ GeV})^2$. Only leptoquarks inducing an interaction between u, d, ν and e or μ could contribute to R, so we see from table 4 that we have bounds on $\lambda^2_{LV_o}$ and $\lambda^2_{LV_1}$. Since the neutrino in the decay $\pi^+ \to \bar{e}\nu$ could be any species providing that it is light, we have the bounds

$$\lambda^{11}_{LV_o} \lambda^{n1}_{LV_o} < 2 \times 10^{-3} \left(\frac{m_{lq}}{100 \text{ GeV}} \right)^2 \tag{27}$$

$$\lambda^{11}_{LV_1} \lambda^{n1}_{LV_1} < 2 \times 10^{-3} \left(\frac{m_{lq}}{100 \text{ GeV}} \right)^2 \tag{28}$$

where n is the generation index of a light neutrino. This is the first row of the table. From (26), the same bounds apply to leptoquark couplings inducing the decay $\pi \to \mu\nu$, which gives the second row of the table. We also have bounds on pseudoscalar operators from the R ratio. Again looking in table 4, we see that there are such operators with effective couplings $\lambda_{LV_o} \lambda_{RV_o} / m^2_{lq}$ and $\lambda_{LV_{1/2}} \lambda_{RV_{1/2}} / m^2_{lq}$. Checking carefully in the second column to get the right generation indices on the right coupling constants, we see that the bound (24) gives

$$\lambda^{n1}_{LV_o} \lambda^{11}_{RV_o} < 5 \times 10^{-7} \left(\frac{m_{lq}}{100 \text{ GeV}} \right)^2 \tag{29}$$

$$\lambda^{n1}_{LV_{1/2}} \lambda^{11}_{RV_{1/2}} < 5 \times 10^{-7} \left(\frac{m_{lq}}{100 \text{ GeV}} \right)^2 \tag{30}$$

as listed in the third row of the table. Note that bounds that cross two columns apply to the product of the couplings in those columns, with the first set of generation indices applying to the first column coupling constant and the second set to the second column. The fourth row of the table is the constraint from (24) applied to pseudoscalar operators involving muons.

From the upper bound $\text{BR}(\pi^+ \to \bar{\mu}\nu_e) < 8 \times 10^{-3}$, one can constrain leptoquark induced effective interactions of the form

$$\frac{e_L^2}{m^2_{lq}} (\bar{u}\gamma^\mu P d)(\bar{e}\gamma_\mu L \nu) \ , \quad \frac{e_L e_R}{m^2_{lq}} (\bar{u} P d)(\bar{e} L \nu) \tag{31}$$

to satisfy

$$\frac{e_L^2}{m^2_{lq}} < .25 G_F \ , \quad \frac{e_L e_R}{m^2_{lq}} < 5 \times 10^{-3} G_F \frac{m_\mu}{m_\pi} \tag{32}$$

where we have again used $\tilde{P} = 7 f_\pi m_\pi$.

There is also an upper bound on the lepton flavour violating decay $\pi^o \to \mu^\pm e^\mp$: $\text{BR}(\pi^o \to \mu^\pm e^\mp) < 1.6 \times 10^{-8}$, and the branching ratio for $\pi^o \to e^\pm e^\mp$ has been measured [51] to be $\sim 7 \times 10^{-8}$. Unfortunately, since the neutral pion decays electromagnetically $\pi^o \to \gamma\gamma$ with a lifetime eight orders of magnitude smaller than that of the charged pions, these do not translate into very strong bounds on the leptoquark couplings. We will get much better constraints, with the same generation indices, from muon conversion on nuclei, and atomic parity violation.

4.2 Kaon Decays

Charged kaons decay to $\mu\nu_\mu$ (BR = 63.5 %) and a variety of other final states, most involving pions. The decay to $e\nu_e$ is suppressed by angular momentum conservation as in the pion case, so if we require that the leptoquark contribution to the branching ratio be less than the experimental error, we get the results listed in the tables. We assume, as in the pion case, that there is no cancellation between the leptoquark contributions to $K \to \mu\nu$ and $K \to e\nu$, and we use $\tilde{P} \sim f_K m_K/2$ to estimate the pseudoscalar contribution.

If kaons decayed via a leptoquark, it would be just as natural to have $\mu\nu_e$ in the final state as $\mu\nu_\mu$. The upper bound on the ratio [38]

$$\frac{BR(K \to \mu\nu_e)}{BR(K \to \mu\nu_\mu)} < 6.3 \times 10^{-3} \tag{33}$$

implies

$$\frac{e_L^4}{8m_{lq}^4 G_F^2 |V_{su}|^2}, \frac{(e_L e_R)^2 \tilde{P}^2}{2m_{lq}^4 G_F^2 |V_{su}|^2 f_K^2 m_\mu^2} < 6.3 \times 10^{-3} \tag{34}$$

where we again use $\tilde{P} \sim m_K f_K/2$ to calculate the numerical constraints listed in the tables.

There are also bounds from the absence of the flavour-changing decays $K \to \pi \ell_1 \bar{\ell}_2$, where $\ell_1\bar{\ell}_2 = e\bar{e}, \mu\bar{\mu}, \mu\bar{e}$ or $e\bar{m}u$. The Standard Model does not allow a strange quark to turn into a down quark at tree level (FCNC), whereas leptoquarks could easily induce an $\bar{s}d\ell\bar{\ell}$ vertex. If we assume, using isospin symmetry, that

$$< K^+|\tilde{O}|\pi^o >= \frac{1}{\sqrt{2}} < K^+|\tilde{O}|\pi^+ > \tag{35}$$

(where $\tilde{O}$ is some isospin 1/2 operator) and neglect all lepton masses, we can require

$$\frac{e_L^4}{m_{lq}^4} < 4G_F^2 |V_{su}|^2 \frac{BR(K \to \pi^+ \ell\bar{\ell})}{BR(K \to \pi^o \bar{\ell}\nu)} \tag{36}$$

giving the constraints listed in the tables. Neglecting the masses should introduce an error of less than a factor of 2, because $BR(K \to \pi^o \bar{e}\nu) \simeq 1.4\ BR(K \to \pi^o \bar{\mu}\nu)$.

Finally we consider constraints from leptonic decays of neutral kaons. Most scalar and vector leptoquarks would allow the decays $K_L \to \mu\bar{\mu}, e\bar{e}, \mu\bar{e}$, which are suppressed in the Standard Model by the absence of flavour-changing neutral currents (and lepton family number violation). There are both axial vector and pseudoscalar quark matrix elements that can contribute to these decays. We will neglect the pseudoscalar contributions in the decays to $\mu\bar{e}$ and $\mu\bar{\mu}$ because they constrain the product $e_L e_R$, and the bounds on the individual coupling constants (e_L^2, e_R^2) are only a factor of m_μ/m_K weaker. Using equation (20) with $\tilde{A} = f_K/2$ we find that

$$\Gamma(K_L \to e^\pm \mu^\mp) \simeq \frac{(m_K^2 - m_\mu^2)^2}{64\pi m_K^3} \frac{e_L^4}{m_{lq}^4} f_K^2 m_\mu^2 < 1.2 \times 10^{-27} \text{ GeV} \tag{37}$$

which implies

$$e_L^2 < 6 \times 10^{-7} \left(\frac{m_{lq}}{100 \text{ GeV}} \right)^2 \tag{38}$$

One can similiarly compute the decays $K_L \to \mu\bar{\mu}$ and $K_L \to e\bar{e}$ from equation (20). This gives constraints as listed in the tables. We have included the bounds from effective pseudoscalar operators in the decay to $e\bar{e}$, using $\tilde{P} \simeq m_K f_K/2$, because they are considerably stronger (not suppressed by the small electron mass). Note, however, that scalar leptoquarks do not induce pseudoscalar operators of the form $(\bar{d^j} P d^m)(\bar{e^i} P e^m)$ (see table 3), so these bounds only apply to $\lambda_{LV_o}\lambda_{RV_o}$ and $\lambda_{LV_{1/2}}\lambda_{RV_{1/2}}$.

4.3 D decays

Since the mass of the D^+ meson is much greater than that of the muon, the decay $D^+ \to \mu^+\nu$ is suppressed by angular momentum conservation, like $\pi^+ \to e^+\nu$. The present upper bound on the rate for $D^+ \to \mu^+\nu$ is roughly a factor of 2 larger than the Standard Model prediction, so is not the best place to get bounds on leptoquarks that induce axial vector effective vertices. However, it does strongly constrain effective pseudoscalar quark vertices involving c, d, μ and any neutrino:

$$\frac{(m_D^2 - m_\mu^2)^2}{16\pi m_D} \tilde{P}^2 \frac{(e_L e_R)^2}{m_{lq}^4} < 4.5 \times 10^{-16} \text{ GeV} \tag{39}$$

or, (assuming $\tilde{P} \simeq f_D m_D/2$, with [52] $f_D \sim .2$ GeV)

$$e_L e_R < 6 \times 10^{-3} \left(\frac{m_{lq}}{100 \text{ GeV}}\right)^2 \quad . \tag{40}$$

A much weaker bound can be calculated for the effective coupling of vertices involving c and s quarks, a muon and a neutrino, from the upper bound $BR(D_S^+ \to \mu^+\nu) < .03$. We do not list this in the tables.

The CKM matrix element V_{cd} is measured in $\Delta C = 1$ neutrino interactions ($\nu + d \to \mu + c$) to be .204 $\pm$.017 [38]. Requiring that leptoquarks contribute less than the total observed rate (a conservative assumption) gives

$$e_L^2 < \frac{4G_F}{\sqrt{2}} |V_{cd}| m_{lq}^2 \tag{41}$$

for vertices involving c, d, μ and ν_μ. The upper bound on the branching ratio for $D^+ \to \mu^+\nu$ gives slightly weaker constraints, but applies to all flavours of neutrino, so is included in the tables.

The CKM matrix element V_{cs} is determined to be $1.0 \pm .2$ from the decay $D^o \to K^- e^+ \nu$, so one can require

$$\frac{e_L^2}{m_{lq}^2} < \frac{4G_F}{\sqrt{2}} \tag{42}$$

where e_L^2/m_{lq}^2 is the effective coupling for an axial vector vertex involving c, s, e, and any neutrino.

We can constrain the effective coupling of the flavour-changing operator $(\bar{u}\gamma^\mu P c)(\bar{l}\gamma^\mu P l)$ (l is an electron or a muon) from the upper bound on the branching ratio $BR(D^+ \to \pi^+ l\bar{l})$, if we assume, by isospin symmetry, that

$$< D^+ | \bar{u}\gamma^\mu P c | \pi^+ > \quad = \quad < D^o | \bar{d}\gamma^\mu P c | \pi^- > \tag{43}$$

(where $P = L, R$). The D^o is observed to decay to $\pi^- e\nu$ with a branching ratio of 3.9 $\times 10^{-3}$, so if we neglect all lepton masses we can require

$$\frac{e_L^2}{m_{lq}^2}\frac{\sqrt{2}}{4G_F|V_{cd}|} < \sqrt{\frac{\tau_{D^o}}{\tau_{D^+}}\frac{BR(D^+ \to \pi^+ l\bar{l})}{BR(D^o \to \pi^- \nu \bar{e})}} \tag{44}$$

which gives the constraints listed in the tables.

As in the case of kaons, one expects leptoquarks to induce flavour-changing D^o decays to lepton pairs ($\bar{\mu}\mu, \bar{e}e, \bar{\mu}e$). Most of the leptoquark interactions induce effective axial vector quark and lepton currents, and since $m_D \gg m_\mu, m_e$, the contribution of these currents to the decays $D \to \bar{\mu}\mu, \bar{e}e, \bar{\mu}e$ is suppressed by angular momentum conservation. One gets strong bounds on the coupling constants of effective pseudoscalar operators from the leptonic decays, but better bounds on the axial vector operators from the previously considered $D^+ \to \pi^+ l\bar{l}$ decays. Using (20) to calculate the rates for D^o decay with the approximation (18), the limit BR($D^o \to \mu\bar{e}$) $< 10^{-4}$ [38] gives

$$e_L e_R < 4 \times 10^{-3}\left(\frac{m_{lq}}{100 \text{ GeV}}\right)^2 \quad (\text{for } D^o \to \mu\bar{e}) \tag{45}$$

Similiarly for BR($D^o \to \mu\bar{\mu}$) $< 1.1 \times 10^{-5}$ [38] one gets

$$e_L e_R^2 < 10^{-3}\left(\frac{m_{lq}}{100 \text{ GeV}}\right)^2 \quad (\text{for } D^o \to \mu\bar{\mu}) \tag{46}$$

and from BR($D^o \to e\bar{e}$) $< 1.3 \times 10^{-4}$ [38]

$$e_L e_R^2 < 4 \times 10^{-3}\left(\frac{m_{lq}}{100 \text{ GeV}}\right)^2 (\text{for } D^o \to e\bar{e}) \tag{47}$$

4.4 B decays

One can get constraints on the leptoquark couplings to b quarks from the upper bounds on the flavour-changing decays $B \to \ell\bar{\ell}X$ ($\ell\bar{\ell} = \mu\bar{\mu}, e\bar{e}$), $B \to \ell_1\bar{\ell}_2 K$ ($\ell_1\bar{\ell}_2 = \mu\bar{\mu}, \mu\bar{e}, e\bar{e}$), and $B^o \to \bar{\mu}\mu, \bar{e}e, \bar{\mu}e$. If we approximate the hadronic matrix element in the $B \to \ell_1\bar{\ell}_2 K$ decays via heavy quark effective theory [54], we get some of our best constraints from these processes. We nonetheless calculate and list the bounds from $B \to \ell\bar{\ell}X$ and $B^o \to \ell_1\bar{\ell}_2$ first because they should be more dependable.

The upper bounds on the leptonic branching ratios are stronger than those for the inclusive decays $B \to \ell\bar{\ell}X$, but the constraints on the couplings are weaker because the theoretical rates are suppressed by angular momentum conservation (small lepton masses). We therefore list, for the leptonic decays, bounds on the coupling constants of effective pseudoscalar vertices, and bounds on coupling constants whose generation indices would allow $B^o \to \bar{\mu}e$, because the latter are not constrained by the inclusive decay data.

The decay $B^- \to e\bar{\nu}X$ ($B^- \to \mu\bar{\nu}X$) is observed to have a branching ratio of 12 % (11%). This is the rate expected in the Standard Model with $|V_{cb}| = .05$, so we can require that the leptoquarks mediating this interaction satisfy

$$\frac{e_L^2}{m_{lq}^2} \lesssim \frac{4G_F}{\sqrt{2}}|V_{cb}| \tag{48}$$

The branching ratio for $B \to \tau\nu X$ has recently been measured to be $4.2 + .72 - .68 \pm .46\%$ by Aleph [55]. One would expect the rate to be suppressed by about a factor of 2.5 with respect to $B \to \mu\nu X$ by kinematics, so leptoquarks coupling a tau, a neutrino, a b and a c or u quark must also satisfy (48). If we neglect differences in the kinematics between $B^+ \to \ell\bar{\ell}X$ and $B^+ \to \nu\bar{\ell}X$, where $\ell = e, \mu$, we can also constrain

$$\frac{e_L^4}{m_{lq}^4} < 8G_F^2|V_{cb}|^2 \frac{BR(B^+ \to \ell\bar{\ell}X)}{BR(B^+ \to \nu\bar{\ell}X)} \tag{49}$$

(48) and (49) give bounds on four fermion vertices involving $e\nu\ b\ c$ (or $e\nu\ b\ u$, but we will get better bounds on this from (50), $\ell\bar{\ell}bs$ and $\ell\bar{\ell}bd$.

Taking $V_{cb} = .06$, we obtain the constraints listed in the tables. The ratio of CKM matrix elements V_{ub}/V_{cb} is measured to be [56] .10 $\pm .03$ by fitting the lepton energy spectrum in semi-leptonic B decays. We can therefore constrain leptoquarks coupling a b, u, e and a ν to satisfy

$$\frac{e_L^2}{m_{lq}^2} \lesssim \frac{4G_F}{\sqrt{2}}|V_{ub}| \quad . \tag{50}$$

We can get constraints on four fermion interactions involving b, τ, a down-type quark (d or s) and another lepton ($\ell = e$, μ or τ) from the decays $B \to \ell\bar{\ell}X$ because the τ decays very rapidly. Suppose first that $\ell = e$, and the leptoquark mediates the decay $B \to \tau\bar{e}X$. We are interested in the τ decaying to ν + hadrons, in which case this would contribute to $B \to \nu\bar{e}X$, or the tau decaying to $e\nu\bar{\nu}$, giving $e\bar{e}\nu\bar{\nu}X$ as the final state. The experimental limit on the branching ratio for $B \to e\bar{e}X$ is smaller than the measured rate for $B \to \nu\bar{e}X$, so we would get better constraints if we could assume $B \to e\bar{e}\nu\bar{\nu}X \subset B \to e\bar{e}X$. However, the CLEO Collaboration [57] required that the $e\bar{e}$ pair in its upper bound on $B \to e\bar{e}X$ come out back to back, and since the tau is not extremely relativistic, it is not clear that the electron from its decay would be at 180^o from the $\bar{e}$.We will therefore calcalate bounds from the rate for $B \to \nu eX$. We assume that the phase space suppression due to the tau mass contributes about a factor of 1/2.5, so

$$\Gamma(B \to \tau eX) = \frac{1}{2.5}\frac{e^4}{m_{lq}^4}\frac{\Gamma(B \to e\nu X)}{8G_F^2|V_{cb}|^2} \quad . \tag{51}$$

We can therefore require that couplings involving b, τ, e and an s or d satisfy

$$\frac{e^2}{m_{lq}^2} < 2\sqrt{5}G_F \frac{V_{cb}}{[BR(\tau \to \nu X')]^{1/2}} \quad . \tag{52}$$

If the b decayed to τ, μ and a s or a d quark, and the τ decayed to $\mu\nu\bar{\nu}$, this would probably have been seen by UA1 in their search [58] for $B \to \mu\bar{\mu}X$. We can therefore require

$$\frac{e^2}{m_{lq}^2} < \frac{4G_F V_{cb}}{\sqrt{2}}\sqrt{\frac{BR(B \to \mu\mu X)}{BR(B \to \mu\nu X)BR(\tau \to \mu\nu\bar{\nu})}} \tag{53}$$

for couplings involving b, τ, μ and a d or an s quark.

Finally we consider the decays $B \to \tau\tau X$. We assume that both taus decay to muons, and we include an approximation to the the phase space suppression due to the masses of the final state particles [1]. The effective coupling of four-fermion vertices involving a

[1] We thank Andrzej Czarnecki for creating a simple analytic approximation for us.

b, two τs, and a d or an s quark must therefore satisfy

$$\frac{e^2}{m_{lq}^2} < \frac{4G_F}{\sqrt{2}} \frac{V_{cb}}{BR(\tau \to \mu\nu\bar{\nu})} \sqrt{\frac{BR(B \to \mu\mu X)}{BR(B \to \mu\nu X)\ PS}} \tag{54}$$

where PS is the phase space suppression factor:

$$PS = \left[1 - \left(\frac{2m_\tau}{m_b}\right)^2\right]^{2.48} . \tag{55}$$

The rate for the decay $B^o \to \bar{e}\mu$ via an effective axial vector quark operator will be (37) with the obvious modifications:

$$\Gamma(B^o \to e^\pm \mu^\mp) = \frac{(m_B^2 - m_\mu^2)^2}{64\pi m_B^3} \frac{e_L^4}{m_{lq}^4} f_B^2 m_\mu^2 < 3 \times 10^{-18} \text{ GeV} \tag{56}$$

where we have used the recent CLEO bound $BR(B^o \to \bar{e}\mu) < 6 \times 10^{-6}$ to get the numerical constraint, and we assume $f_B \sim .2$ GeV to get the constraints in the tables. Using $\tilde{P} \sim f_B m_B/2$ in (20), we can estimate the decay via a pseudoscalar vertex to be

$$\Gamma_P \sim .03 \frac{(e_L e_R)^2}{m_{lq}^4} \text{ GeV}^5 \lesssim 2 \times 10^{-17} \text{ GeV} \tag{57}$$

which gives the bounds on $\lambda_L \lambda_R$ listed in the tables. These constraints, however, are only rough estimates.

The most stringent experimental bounds on flavour-changing neutral currents involving b quarks come from the exclusive decays $B^+ \to X^+ \ell_1^+ \ell_2^-$, where $\ell_1^+ \ell_2^-$ can be $e\bar{e}$, $\mu\bar{e}$ or $\mu\bar{\mu}$, and X is a K or a π. However, to calculate a rate for this decay, we need to evaluate the matrix element $< X^+|\bar{s}\gamma^\mu b|B^+ >$. We do this by relating it at zero recoil to $< X^+|\bar{s}\gamma^\mu c|D^o >$ via the heavy quark formulism [54], and then making very simple approximations for the momentum transfer dependence of the form factors.

The B to X matrix element can be written

$$< X^+|\bar{s}\gamma^\mu b|B^+ >= f_+^{B\to X}(p_B^\mu + p_X^\mu) + f_-^{B\to X}(p_B^\mu - p_X^\mu) \tag{58}$$

where $f_\pm$ are functions of the momentum transfer $t = (p_B - p_X)^2$. A similiar expression can be written for the $D^o \to X^+$ matrix element, and the B and D form factors can then be related by the heavy quark symmetry [59]. Neglecting corrections of order m_c/m_b and Λ/m_b, this gives

$$f_+^{B\to X}(t_{max}) = -f_-^{B\to X}(t_{max}) = \sqrt{\frac{m_b}{m_c}} \left[\frac{\alpha(m_b)}{\alpha(m_c)}\right]^{-6/25} f_+^{D\to X}(t_{max}) \tag{59}$$

where t_{max} is the maximum momentum transfer between the mesons ($(m_B - m_X)^2$ for the B decays, $(m_D - m_X)^2$ for the Ds).

The decay rate for $B \to X\ell^+\ell^-$ is proportional to f_+^2 [60], so if we were to ignore the t dependance of the form factors, and the mass of all the final state particles in both the B and D decays, we could require that

$$\frac{e^4}{m_{lq}^4} < 8G_F^2|V|^2 \frac{m_D^5}{m_B^5} \frac{m_c}{m_b} \left[\frac{\alpha(m_b)}{\alpha(m_c)}\right]^{12/25} \frac{\Gamma(B^+ \to X\ell^+\ell^-)}{\Gamma(D^o \to X\ell\nu)} \tag{60}$$

where $|V|$ is the appropriate CKM matrix element, and $\Gamma(B^+ \rightarrow X\ell^+\ell^-)$ is the experimental upper bound on this rate.

These approximations are rather extreme; the form factors can vary by up to an order of magnitude as t varies from t_{min} to t_{max}, and m_K/m_D is far from negligable. To model the momentum dependence of the form factors, we assume that they vary with t as

$$f_+(t) = \frac{f_+(0)}{1 - t/m^2} \tag{61}$$

(where we take the mass to be that of the decaying pseudoscalar meson for convenience; the vector meson mass might be more accurate, but for b and c mesons, these are very similiar), then (59) becomes (if we neglect $m_X/m_B, m_X/m_D$)

$$f_+^{B\rightarrow X}(0) = \frac{m_D}{m_B}\sqrt{\frac{m_b}{m_c}}\left[\frac{\alpha(m_b)}{\alpha(m_c)}\right]^{-6/25} f_+^{D\rightarrow X}(0) \tag{62}$$

and the bound (60) is

$$\frac{e^4}{m_{lq}^4} < 8G_F^2|V|^2\frac{m_D^3}{m_B^3}\frac{m_c}{m_b}\left[\frac{\alpha(m_b)}{\alpha(m_c)}\right]^{12/25}\frac{\int d\Pi_3(1-t/m_D^2)^{-2}\mathcal{F}_D}{\int d\Pi_3(1-t/m_B^2)^{-2}\mathcal{F}_B}\frac{\Gamma(B^+\rightarrow X\ell^+\ell^-)}{\Gamma(D^o\rightarrow X\ell\nu)} \tag{63}$$

where the integrals are over final state phase space, and $\mathcal{F}$ is the appropriate integrand (which is independent of the decaying meson mass in the approximation that final state masses are neglected).The ratio of integrals will depend on the masses of the final state mesons: if the integrals are dominated by $t \sim t_{max}$, then their ratio could be of order m_D^2/m_B^2, giving the constraint (60). If instead, the small momentum transfer region is dominant, the ratio should be ~ 1. We can skirt these difficulties to some extent by comparing $B^+ \rightarrow K^+\ell^+\ell^-$ to $D^o \rightarrow \pi^+ e\nu$ because $m_\pi/m_D \approx m_K/m_B$. If we assume that $f_+^{D\rightarrow K}(0) = f_+^{D\rightarrow\pi}(0)$ [62], then we get (60) but with the branching ratio for $D^o \rightarrow \pi^+ e\nu$ and the upper bound on $B^+ \rightarrow K^+\ell^+\ell^-$. For the decays $B^+ \rightarrow \pi^+\ell^+\ell^-$, we will use (63), assuming that the ratio of integrals is 1, because this gives the weakest bounds. This series of approximations gives the constraints listed in the tables, but we emphasize that they are very sloppy.

CLEO has recently observed the decay $B^* \rightarrow K^*\gamma$ with a branching ratio of $(4.5 \pm 1.5 \pm .9) \times 10^{-5}$ [63], which is consistent with the rate expected from the Standard Model. Leptoquarks could also contribute via the diagrams of figures 3 and 4, with b and s quarks on the external legs, and a charged lepton ℓ in the loop. We can therefore calculate bounds on couplings $\lambda^{\ell s}\lambda^{\ell b}$ by requiring that the leptoquark contribution to $b \rightarrow s\gamma$ be "sufficiently small". The leptoquark contribution to $b \rightarrow s\gamma$ in a string inspired supersymmetric model was calculated in [64], but is not applicable to the simple leptoquark case because of the presence of the supersymmetric partners.

In section 6.2, we calculate bounds on leptoquarks from the decay $\mu \rightarrow e\gamma$. By making the initial fermion a b, the loop fermion a lepton, and dividing by three (no colour sum) in equations (98),(101),(104) and (106), we can calculate $\Gamma(b \rightarrow s\gamma)$ at a scale $\mu \sim m_W \sim m_{lq}$ from equation (96). To get a rough idea of the magnitude of the bound on leptoquarks from $b \rightarrow s\gamma$, we can simply add the Standard Model and leptoquark amplitudes, run the sum down to m_b [80], and require that the total rate fit into CLEO's allowed window. Since the renormalisation group running mixes $b \rightarrow s\gamma$ with $b \rightarrow sG$, we need the leptoquark contribution to $b \rightarrow sG$. Instead, we will assume that it is smaller than the Standard Model amplitude, and neglect it. With these approximations, the $b \to s\gamma$ amplitude at m_b can be written

$$\mathcal{A}(b \to s\gamma) = a[\mathcal{A}_{SM}(b \to s\gamma) + \mathcal{A}_{LQ}(b \to s\gamma) + b] \tag{64}$$

where a and b parametrize the QCD running. The relative sign between the Standard Model and leptoquark amplitudes is unknown, as is the exact value of the Standard Model amplitude (it depends on m_t), and the relationship between the measured rate for $B \to K^*\gamma$ and the calculated one for $b \to s\gamma$. So to get a rough idea of the magnitude of the $b \to s\gamma$ bound on leptoquarks, we will require

$$|\mathcal{A}_{LQ}(b \to s\gamma)| \lesssim |\mathcal{A}_{SM}(b \to s\gamma)| \tag{65}$$

where we take $m_t = 2m_W$ in the Standard Model amplitude.

Following reference [65], we get for scalar leptoquarks:

$$\lambda_L^{\ell b}\lambda_L^{\ell s}, \lambda_R^{\ell b}\lambda_R^{\ell s} < \frac{3 \times 10^{-2}}{Q_\ell + Q_{lq}/2}\left(\frac{m_{lq}}{100\mathrm{GeV}}\right)^2 \quad \text{(scalars)} \tag{66}$$

where ℓ is a charged lepton, and Q_ℓ and Q_{lq} are defined as the quark and leptoquark electric charges coming out of the incident b vertex.

There is a similiar bound for non-gauge vectors:

$$\lambda_L^{\ell b}\lambda_L^{\ell s}, \lambda_R^{\ell b}\lambda_R^{\ell s} < \frac{2 \times 10^{-2}}{2Q_\ell + \frac{5}{2}Q_{lq}}\left(\frac{m_{lq}}{100\mathrm{GeV}}\right)^2 \quad (\mathrm{non-gauge\ vectors}) \tag{67}$$

where $2Q_\ell + \frac{5}{2}Q_{lq} = 1/3$ $(2/3)$ for λ_{LV_o}, λ_{RV_o} and λ_{LV_1} ($\lambda_{LV_{1/2}}$, $\lambda_{RV_{1/2}}$). We do not list the non-gauge bounds in the tables. The bounds on gauge vectors are to weak to be interesting. Equations (66) and (67) are really just estimates of the magnitude of the bound that could be derived; however, since these couplings are better constrained by other B decays (see table 15), this is not a serious problem.

5 Meson-anti-meson mixing: K^o-$\bar{K}^o$, D^o-$\bar{D}^o$ and B^o-$\bar{B}^o$

Weak interaction box diagrams (see figure 1) neatly account for the small mass term that mixes the K^o and $\bar{K}^o$ mesons. Similiar box diagrams, with leptoquarks instead of W bosons and the internal quarks replaced by leptons would also contribute to the K^o-$\bar{K}^o$ mass difference, and one can therefore derive bounds on the leptoquark couplings from these amplitudes. This has been done in [7, 10, 13, 15] and more recently in [53].

The Standard Model boxes are GIM suppressed (of order $G_F\alpha(m_q/m_W)^2$ rather than $G_F\alpha$) because the qqW coupling constant matrix in generation space is unitary. This may also be the case for vector leptoquarks, if they are the vector bosons of some spontaneously broken gauge symmetry (this is the only renormalisable way of making spin-1 particles; however, composite and technicolour models do not neccessarily reduce to renormalizable low-energy effective theories.). The gauge boson leptoquark contribution to meson-anti-meson mixing would be of the same form as the Standard Model one, giving upper bounds on the vector leptoquark couplings of order (14). If the vector leptoquarks are not gauge bosons, GIM-type suppression is unlikely, and the box diagrams for vectors would be of order λ^4/m_{lq}^2. This would give stronger constraints and will be briefly discussed at the end of the next section. We assume in the tables however, that the vector leptoquarks are gauge bosons, and list the weaker bounds.

Scalar leptoquark box diagrams will of course be of order λ^4/m_{lq}^2 and we should get interesting constraints from these.

5.1 Vector leptoquarks

The Standard Model W boxes contributing to the K^o-$\bar{K}^o$ mass difference give [69]

$$\frac{\Delta m_K}{2} = \frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi \sin^2\theta_W}[\sum_{q=c,t}(V_{qs}V_{qd}^*)^2\frac{m_q^2}{m_W^2} \tag{68}$$
$$+V_{cs}V_{cd}^*V_{ts}V_{td}^*\frac{2m_c^2m_t^2}{m_W^2(m_t^2-m_c^2)}\ln\left(\frac{m_t^2}{m_c^2}\right)] < K|(\bar{d}\gamma^\mu Ls)(\bar{d}\gamma_\mu Ls)|\bar{K}>$$

where V is the CKM matrix, and we have used the identity $(\bar{q}\gamma^\alpha\gamma^\mu\gamma^\beta Lq')(\bar{q}\gamma_\beta\gamma_\mu\gamma_\alpha Lq')$ $= 4(\bar{q}\gamma^\mu Lq')(\bar{q}\gamma_\mu Lq')$. If one neglects the top quark contributions because $V_{td} \ll 1$, and approximates the matrix element (vacuum saturation) as

$$< K|(\bar{d}\gamma^\mu Ls)(\bar{d}\gamma_\mu Ls)|\bar{K} > \simeq \frac{1}{3}f_K^2 m_K \tag{69}$$

this agrees quite well with the measured mass difference $\Delta m_K = 3.5\times 10^{-15}$ GeV. Gauge leptoquarks will give the same contribution to Δm_K, with $g_W/\sqrt{2}$ replaced by λ_{lq} and lepton masses instead of quarks. We therefore require

$$\frac{1}{32\pi^2 m_{lq}^2}[\,|\lambda^{ld}\lambda^{ls}|^2\frac{m_l^2}{m_{lq}^2}+\lambda^{ld}\lambda^{ls}\lambda^{l'd}\lambda^{l's}\frac{2m_l^2m_{l'}^2}{m_{lq}^2(m_l^2-m_{l'}^2)}\ln\left(\frac{m_{l'}^2}{m_l^2}\right)] < \frac{G_F}{\sqrt{2}}\frac{\alpha\sin^2\theta_c\cos^2\theta_c}{4\pi\sin^2\theta_W}\frac{m_c^2}{m_W^2} \tag{70}$$

where l and l' are leptons, and θ_c is the Cabbibbo angle. If we neglect the second term on the left hand side, this gives bounds on all vector leptoquarks coupling down-type quarks to massive (= charged) leptons l:

$$\lambda^{l1}\lambda^{l2} < .1\left(\frac{m_{lq}}{100\text{ GeV}}\right)^2\left(\frac{1\text{ GeV}}{m_l}\right) \tag{71}$$

We list the bounds for couplings to muons and taus in the tables. Neglecting the term corresponding to the exchange of different flavoured leptons (l and l') in the box should be an acceptable approximation, because it would be surprising if it cancelled against the first term, and including it makes the bounds more complicated.

This analysis has been for leptoquarks with left- or right-handed couplings. One might hope that for V_o^μ and $V_{1/2}^\mu$, who have both, one could escape the GIM-type suppression by having left- and right-handed couplings at either end of the lepton propagator. This would give $\sum_l \lambda_L^{ld}\lambda_R^{ls} \neq 0$. However, the lepton chiralities must be flipped in the propagators to do this, giving a contribution to Δm proportional to $\lambda_L^2\lambda_R^2 m_l^2/m_{lq}^4$, so we will neglect these mixed bounds (no stronger; more difficult to calculate).

There has been no observed mixing between the D^o and its CP-conjugate the $\bar{D}^o$. The present upper bound on the mass difference ($\Delta m_D < 1.3\times 10^{-13}$ GeV) is considerably larger than the Standard Model prediction, but can still be used to constrain leptoquark couplings if we approximate the matrix element as in (69) (with $f_D \sim .2$ GeV) . This gives

$$\frac{1}{32\pi^2}|\lambda^{lu}\lambda^{lc}|^2\frac{m_l^2}{m_{lq}^4} < \frac{3\Delta m_D}{2f_D^2 m_D} \tag{72}$$

implying

$$|\lambda^{lu}\lambda^{lc}| < .3\left(\frac{m_{lq}}{100\text{ GeV}}\right)^2\left(\frac{1\text{ GeV}}{m_l}\right) \tag{73}$$

A similiar argument for the observed B^o-$\bar{B}^o$ mass difference $\Delta m_B = 3.6 \times 10^{-13}$ GeV gives (using $f_B \sim .2$ GeV)

$$|\lambda^{ld}\lambda^{lb}| < .3 \left(\frac{m_{lq}}{100 \text{ GeV}}\right)^2 \left(\frac{1 \text{ GeV}}{m_l}\right) \tag{74}$$

The limits for muons and taus ($l = 2, 3$ on the internal lines) are listed in the tables.

We now briefly consider non-gauge leptoquarks. The second term of the massive vector propagator:

$$\frac{i}{k^2 - m_{lq}^2}(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_{lq}^2}) \tag{75}$$

is a nuisance in calculations, and the box diagrams we just discussed were done in a gauge where it is absent. If the vector leptoquark is not a gauge boson, we do not have this option, and the loop momentum integration over the $k^\mu k^\nu / m_{lq}^2$ terms diverges. However, the new physics that produced the vector leptoquark (compositeness?) should appear at some energy scale Λ, which can be used as a cutoff for the divergent integral. The non-gauge vector leptoquark contribution to Δm is therefore model-dependent. We can nonetheless get conservative bounds on the couplings by only considering the contribution to the box amplitude from the $g_{\mu\nu}$ terms. This is

$$\begin{aligned}\Delta m_M = \sum_{ij} (\lambda^{iq'}\lambda^{iq})(\lambda^{jq}\lambda^{jq'}) \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu k^\nu}{(k^2 - m_{lq}^2)^2 (k^2 - m_i^2)(k^2 - m_j^2)} \times \\ < M|(\bar{q}\gamma^\alpha\gamma_\mu\gamma^\beta P q')(\bar{q}\gamma_\beta\gamma_\nu\gamma_\alpha P q')|\bar{M} > \end{aligned} \tag{76}$$

where M is a K, D or B, q and q' are the meson constituent quarks, P is L or R, and i and j are lepton indices. If there are no cancellations in the coupling constant sums (no GIM-type suppression), the lepton masses can be neglected and the integral approximated as $g^{\mu\nu}[64\pi^2 m_{lq}^2]^{-1}$. Taking $i = j$, and approximating the matrix element as in (69), we get

$$|\lambda^{iq}\lambda^{iq'}|^2 < \frac{48\pi^2 m_{lq}^2}{f_M^2 m_M} \Delta m_M \tag{77}$$

for non-gauge vector leptoquarks. This gives the numerical bounds

$$\lambda^{ld}\lambda^{ls} \;<\; 6 \times 10^{-4} \left(\frac{m_{lq}}{100\text{GeV}}\right) \quad (K\bar{K}) \tag{78}$$

$$\lambda^{lu}\lambda^{lc} \;<\; 2 \times 10^{-3} \left(\frac{m_{lq}}{100\text{GeV}}\right) \quad (D\bar{D}) \tag{79}$$

$$\lambda^{ld}\lambda^{lb} \;<\; 2 \times 10^{-3} \left(\frac{m_{lq}}{100\text{GeV}}\right) \quad (B\bar{B}) \tag{80}$$

where l is an arbitrary lepton flavour index. We do not list the bounds in the tables because they only apply if there is no "GIM" suppression.

5.2 Scalar leptoquarks

As we mentioned, the constraints on scalars are more interesting. The box diagrams for a scalar leptoquark (see figure 2) with couplings to leptons of only one chirality (we neglect diagrams proportional to $(\lambda_L\lambda_R)^2$) give a contribution to Δm_M of

$$\begin{aligned}\Delta m_M = \sum_{ij} (\lambda^{iq'}\lambda^{iq})(\lambda^{jq}\lambda^{jq'}) \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu k^\nu}{(k^2 - m_{lq}^2)^2 (k^2 - m_i^2)(k^2 - m_j^2)} \times \\ < M|(\bar{q}\gamma_\nu P q')(\bar{q}\gamma_\mu P q')|\bar{M} > \end{aligned} \tag{81}$$

with the same notation as in (76). This is the same integral and expectation value as for vectors, but the scalar coupling constant matrix is not required to be unitary, so the integral can be approximated as $g^{\mu\nu}[64\pi^2 m_{lq}^2]^{-1}$, as in the non-gauge vector case. If we take take $i = j$ and assume there are no cancellations between the different lepton contributions to Δm_M, then we get the constraint

$$|\lambda^{iq}\lambda^{iq'}|^2 < \frac{196\pi^2 m_{lq}^2 \Delta m_M}{f_M^2 m_M} \tag{82}$$

where we have used equation (69) to approximate the matrix element. Note that these constraints are independant of the lepton mass, so apply equally to neutrinos and all generations of charged leptons. We use $f_D \simeq .2$ GeV $\sim f_B$ to get the numbers in the tables.

6 Muon Physics

Muon family number violating processes provide some of the strongest constraints on leptoquarks from leptonic physics. Bounds on scalar leptoquarks from muon conversion on nuclei, $\mu \to e\bar{e}e$, and $\mu \to e\gamma$ have been considered in [8, 9, 10, 13, 15].

There is a very strong upper bound on the lepton flavour-changing rate for a muon to turn into an electron when scattered off a nucleus. The experimental limit [66] on muon conversion on titanium is

$$\frac{\Gamma(\mu \mathrm{Ti} \to e \mathrm{Ti})}{\Gamma(\mu \mathrm{Ti} \to \nu \mathrm{Ti}')} < 4.6 \times 10^{-12} \quad . \tag{83}$$

If we neglect the difference in kinematics of the neutrino and the electron, this implies

$$\left| \frac{e_L^2}{m_{lq}^2} < e\mathrm{Ti}|(\bar{\mu}\gamma^\mu P e)(\bar{q}\gamma_\mu P q)|\mu\mathrm{Ti} > \right|^2 < 4.6\times 10^{-12} \left| \frac{4G_F}{\sqrt{2}} < \nu\mathrm{Ti}|(\bar{\mu}\gamma^\mu L\nu)(\bar{u}\gamma_\mu L d)|\mu\mathrm{Ti} > \right|^2 \tag{84}$$

where P is a chiral projection operator. The matrix elements are not necessarily the same, because the isospin structure could be different, and μ-e conversion could be a coherent process [67]. The ratio of the matrix elements was calculated in [67, 68] for a coherent E-μ conversion interaction, and the $N\mu \to Ne$ matrix element was found to be larger than that for $N\mu \to N'\nu$. However, the interaction can only be coherent if it is vector, and one can arrange the couplings of some of the leptoquarks such that they are only axial vector. We will therefore assume that the matrix elements are comparable and require

$$\frac{e_L^4}{m_{lq}^4} < 4 \times 10^{-11} G_F^2 \tag{85}$$

which applies to all the leptoquark interactions because they all couple u or d quarks to charged leptons. As usual, the precise limits are in the tables.

6.1 $\mu \to ee\bar{e}$

Leptoquarks could allow the decay of a muon to $ee\bar{e}$ via box diagrams (see figures 1 and 2), or through the effective coupling of a muon to an electron and a Z (or a γ^*)

which subsequently decays to $e\bar{e}$. The box amplitudes are similiar to those for meson-anti-meson mixing, with quarks and leptons interchanged. The effective $Z\mu e$ vertex is induced by leptoquark triangle diagrams, is divergent, and is non-trivial to calculate. By analogy with the Standard Model [70], we expect the triangle amplitude to be larger than the box by a factor of $\ln(m_{lq}^2/m_q^2)$; we will nonetheless only discuss the box diagrams because they are easier to compute; this will give us conservative limits.

Scalar and vector leptoquark boxes will induce effective four-lepton vertices

$$C_{LL}(\bar{\mu}\gamma^\mu Le)(\bar{e}\gamma_\mu Le)\ ,\quad C_{RR}(\bar{\mu}\gamma^\mu Re)(\bar{e}\gamma_\mu Re) \tag{86}$$

In the Standard Model, the muon decays principally (BR$\simeq$ 100%) to $e\bar{\nu}_e\nu_\mu$ via a W-induced four-lepton vertex similiar to (86):

$$\frac{4G_F}{\sqrt{2}}(\bar{\mu}\gamma^\mu L\nu)(\bar{\nu}\gamma_\mu Le) \tag{87}$$

If we neglect the electron mass and the difference in the matrix element arising from having identical fermions in the final state, we can constrain the effective couplings C_{LL}, C_{RR} by requiring

$$(C_{LL,RR})^2 < 8G_F^2\frac{BR(\mu\to 3e)}{BR(\mu\to\nu\bar{\nu}e)} \tag{88}$$

where $BR(\mu\to 3e)$ of course means the experimental upper bound [38] of 10^{-12}.

Had we defined an effective coupling like C_{LL} for the meson-anti-meson mixing, we would have had $\Delta M/2 = C_{LL}\times$ [hadronic matrix element], so we can estimate C_{LL} and C_{RR} from the meson-anti-meson mixing equations. From (69) we have for gauge vector leptoquarks

$$C_{LL}, C_{RR} \simeq \frac{3}{32\pi^2}(\lambda^{1q})^3\lambda^{2q}\frac{m_q^2}{m_{lq}^4} \tag{89}$$

where we have assumed the same quark flavour on both internal fermion lines, multiplied by three for colour, and neglected the second term of (70) for simplicity. This does not give very interesting constraints on the gauge leptoquarks because the amplitude is "GIM" suppressed, giving a decay rate proportional to $\lambda^8 m_q^4/m_{lq}^8$. However, it does give bounds on couplings of electrons and muons to quarks of all flavours (except t). We have, from (88)

$$\sqrt{\lambda^{2q}(\lambda^{1q})^3} < .6\left(\frac{m_{lq}}{100\text{ GeV}}\right)^2\left(\frac{1\text{ GeV}}{m_q}\right)\quad\text{(gauge vectors)} \tag{90}$$

where q is a quark flavour other than top.

For non-gauge vectors we have, in the same approximation used for meson-anti-meson mixing:

$$C_{LL}, C_{RR} = \frac{3(\lambda^{1q})^3\lambda^{2q}}{32\pi^2 m_{lq}^2} \tag{91}$$

This gives a more interesting bound than (90):

$$\sqrt{\lambda^{2q}(\lambda^{1q})^3} < 6\times 10^{-3}\left(\frac{m_{lq}}{100\text{ GeV}}\right)\quad(\text{non}-\text{gauge vectors}) \tag{92}$$

where again $q\neq t$. We do not list the non-gauge results in the tables.

Scalar leptoquarks give

$$C_{LL}, C_{RR} = \frac{3(\lambda^{1q})^3\lambda^{2q}}{128\pi^2 m_{lq}^2} \tag{93}$$

for light quarks, which implies ($q \neq t$)

$$\sqrt{\lambda^{2q}(\lambda^{1q})^3} < 10^{-2}\left(\frac{m_{lq}}{100\ \mathrm{GeV}}\right) \quad \text{(scalars)}. \tag{94}$$

These approximations for C_{LL} and C_{RR} only apply if the internal line fermions are light. For meson-anti-meson mixing induced by leptoquarks, this is a good approximation because the internal line fermions are leptons. However, one could in principle get constraints on leptoquark couplings to the top quark from $\mu \to e\bar{e}e$ box diagrams involving top quarks and leptoquarks. The exact expression for the intergral in (76) and (81) is [69]

$$\frac{1}{64\pi^2 m_{lq}^2}\frac{1}{x_t - x_q}\left[\frac{1}{1-x_q} - \frac{1}{1-x_t} + \frac{x_q^2}{(1-x_q)^2}\ln x_q^2 - \frac{x_t^2}{(1-x_t)^2}\ln x_t^2\right] \tag{95}$$

where $x_t = m_t^2/m_{lq}^2$, $x_q = m_q^2/m_{lq}^2$, and q is any up-type quark. Any bounds will clearly depend sensitively on the top and leptoquark masses, so we neglect them here.

6.2 $\mu \to e\gamma$

The lepton flavour violating decay $\mu \to e\gamma$ could be mediated by leptoquark triangle diagrams (see figures 3 and 4). This decay will again be "GIM" suppressed for gauge leptoquarks, but in this case will be of order $\lambda^4 m_q^4/m_{lq}^8$ rather than $O(\lambda^8 m_q^4/m_{lq}^8)$ as in the $\mu \to e\bar{e}e$ case. There are also fewer powers of λ in the scalar and non-gauge vector rates.

The decay rate of an initial fermion f_i into a photon and a final fermion f_f is [71]

$$\Gamma(f_i \to f_f\gamma) = \frac{m_i}{8\pi}\left(1 - \frac{m_f}{m_i}\right)^2\left(1 - (\frac{m_f}{m_i})^2\right)\left[F^V(0)^2 + F^A(0)^2\right] \tag{96}$$

where F^V and F^A are the vector and axial vector coefficients of the magnetic moment term in the matrix element:

$$\mathcal{M}^\mu[f_i(p_i) \to f_f(p_f) + \gamma(q)] = -i\bar{u}_f(p_f)\left\{\ldots + \frac{\sigma^{\mu\nu}q_\nu}{m_f + m_i}[F^V(q^2) + F^A(q^2)\gamma_5] + \ldots\right\}u_i(p_i) \tag{97}$$

and $\sigma^{\mu\nu} = \frac{-i}{2}[\gamma^\mu, \gamma^\nu]$.

The λ_L^2 and λ_R^2 terms in the scalar leptoquark contribution to $(g-2)_e$ have been calculated in detail in [76, 75, 15]. Djouadi, Kohler, Spira and Tuta [76], and Morris [74] also calculate the contribution proportional to $\lambda_L\lambda_R$. These should be closely related to F_V and F_A; if one set $m_i = m_f = m_e$ in the final result for F_V, one ought to get $e(g-2)_e = F_V$. Unfortunately, in this limit our approximate result is smaller than the exact one in [76] by a factor of 1/3, and has a different coefficient in front of the leptoquark charge. In spite of this disturbing discrepancy, we will use our formula because it gives weaker bounds on the couplings, and because it is three times the result in [75] (three for colour), whose results can be checked against Standard Model calculations.(Davies and He [15] seem to multiply Leveille's result by 9 in their published paper, which would agree with [76]. This is apparently a typographical error, and should have been a three [77].) Note that we have also neglected QCD corrections to the diagrams.

If we assume $m_q \ll m_{lq}$ (this assumption was not made in [76, 75], so their result is applicable to all quarks, in particular the top), we can approximate

$$\frac{|F_+^V|}{m_i + m_f} = \frac{|F_-^A|}{m_i + m_f} \simeq \frac{\lambda_L^2}{(8\pi)^2} \frac{m_i \pm m_f}{m_{lq}^2} e(\frac{Q_{lq}}{2} - Q_q) \quad \text{(scalars)} \tag{98}$$

for couplings of the same chirality at either end of the scalar leptoquark propagator. (The same bound applies to λ_R^2.) $Q_q(Q_{lq})$ is the quark (leptoquark) electric charge coming out of the incident muon vertex, "e" is the electromagnetic coupling constant, and λ^{lq} is an orthogonal matrix in our approximation that all coupling constants are real. The $+$ and $-$ on the RHS of (98) are respectively for F^V and F^A (hence the subscipts on F^V and F^A).

The $\lambda_L \lambda_R$ contribution (from scalar leptoquarks) can be larger than the λ_L^2, λ_R^2 terms, because it is proportional to $(m_i m_q / m_{lq}^2) \ln[m_{lq}^2/m_q^2]$. We will nonetheless neglect it because it only applies to those leptoquarks which have couplings of both chiralities. These terms are present in the calculation of [74, 76].

Requiring that radiative muon decay, as calculated from (96) and (98) take place more slowly than the present experimental upper bound $\Gamma(\mu \to e\gamma) < 1.5 \times 10^{-29}$ GeV, gives

$$\lambda_L^{2q} \lambda_L^{1q}, \lambda_R^{2q} \lambda_R^{1q} < 8 \times 10^{-5} \left(\frac{m_{lq}}{100\text{GeV}} \right)^2 \quad \text{(scalars)} \tag{99}$$

where q is a quark other than the top, and we have assumed here (but not in the tables) that $\frac{Q_{lq}}{2} - Q_q \simeq 1$. For the $SU(2)$ doublet and triplet leptoquarks, this formula is an underestimate, because it assumes that only one leptoquark is being exchanged (rather than two or three of different charges). For simplicity, we estimate the rate as being due to the member of the mutiplet that gives the largest $(\frac{Q_{lq}}{2} - Q_q)$. For $\tilde{S}_{1/2}$, $(\frac{Q_{lq}}{2} - Q_q) = 0$ (remember that Q_q and Q_{lq} are defined as the charges coming out of the first vertex), so we have no bound on the tables. There will certainly be a contribution to F_V from $\tilde{S}_{1/2}$ leptoquarks, but it will be suppressed with respect to (98) by powers of m_q/m_{lq} or m_μ/m_{lq}, so we neglect this. (It is also possible that ou approximate formula is wrong, and $F^V \sim aQ_{lq} + Q_q$ with $a \neq -1/2$, in which case the bound would be of order (99).)

The two triangle diagrams for gauge vector leptoquarks contribute approximately

$$\frac{|F_+^V|}{m_i + m_f} = \frac{|F_-^A|}{m_i + m_f} \simeq \frac{\lambda_L^{fq} \lambda_L^{iq} e}{32\pi^2} \frac{(m_i \pm m_f)}{m_{lq}^2} \times \tag{100}$$

$$\left(Q_q \left[2 - \frac{m_q^2}{m_{lq}^2} (\frac{7}{2} + 3 \ln \frac{m_q^2}{m_{lq}^2}) \right] + Q_{lq} \left[\frac{5}{2} - \frac{13}{4} \frac{m_q^2}{m_{lq}^2} \right] \right) \quad \text{(gauge vectors)} \tag{101}$$

where we have neglected a number of (presumably smaller) terms, and the same formula would apply to couplings λ_R^2. The term of order $[\lambda \lambda^T]_{if} / m_{lq}^2$ vanishes for $i \neq f$ due to the orthogonality of λ^{lq}.

If we set $m_i = m_f = m_\mu$, we can check this expression by comparing it to Leveille's [75] formula for the massive gauge boson contribution to $g-2$. Requiring

$$\Gamma(\mu \to e\gamma) \simeq \frac{\alpha}{(4\pi)^4} \frac{(\lambda_L^{2q} \lambda_L^{1q})^2 m_q^4}{m_{lq}^8} m_\mu^5 \left[Q_q \left(\frac{7}{2} + 3 \ln \frac{m_q^2}{m_{lq}^2}) \right) + \frac{13}{4} Q_{lq} \right]^2 < 1.5 \times 10^{-29} \text{ GeV} \tag{102}$$

gives

$$\lambda_L^{2q}\lambda_L^{1q}, \lambda_R^{2q}\lambda_R^{1q} < \frac{.2}{\left[Q_q\left(\frac{7}{2}+3\ln(\frac{m_q^2}{m_{lq}^2})\right)+\frac{13}{4}Q_{lq}\right]}\left(\frac{m_{lq}}{100 \text{ GeV}}\right)^4\left(\frac{1 \text{ GeV}}{m_q}\right)^2 \quad \text{(gauge vectors)} \tag{103}$$

for gauge vector couplings. This applies to all quark flavours except top. Due to the number of approximations in this calculation, this constraint can only be considered a rough estimate.

It would not be difficult to modify the Standard Model amplitude for $b \to s\gamma$ to descibe a leptoquark and a quark of arbitrary charge in the loop. Such a "modified Standard Model" expression could be used to constrain gauge vector leptoquark couplings to the top quark. However, as in the case of meson anti-meson mixing, the formula is fairly complicated, and does not provide clear constraints on the coupling constant when both the top and the leptoquark masses are unknown.

Some of the vector leptoquarks ($V_o^\mu, V_{1/2}^\mu$) have two different interactions with Standard Model fermions. The two couplings have opposite chiral projection operators. We argued that for the box diagrams, the bounds on the product $\lambda_L\lambda_R$ would be of the same magnitude as the "GIM" suppressed bounds on $\lambda_L\lambda_L$ or $\lambda_R\lambda_R$ because the fermion mass must appear in the amplitude to flip the chirality between the gauge boson vertices. Since the effective coupling for the four-fermion vertex has dimension mass^{-2}, it must be of order $\lambda_L^2\lambda_R^2 m_f^2/m_{lq}^4$, where m_f is the mass of the internal line fermions. This is roughly the same as the "GIM" suppressed amplitude constraining λ_L^2 or λ_R^2. However, this is not true for the triangle graphs, where the bounds on $\lambda_L\lambda_R$ are much better than those on λ_L^2 or λ_R^2: the coefficients F_V and F_A are dimensionless, so putting a quark mass in the numerator to flip the chirality simply changes the leading contribution to F_V and F_A from $\lambda_L^2 m_\mu^2/m_{lq}^2$ to $\lambda_L\lambda_R m_\mu m_q/m_{lq}^2$. Since there is no reason to expect $\sum_q \lambda_L^{\mu q}\lambda_R^{eq} = 0$ for vector leptoquarks, we have [78]

$$\frac{|F_+^V|}{m_i+m_f} = \frac{|F_-^A|}{m_i+m_f} \simeq \frac{3e}{16\pi^2}\frac{m_q}{m_{lq}^2}(\lambda_L^{iq}\lambda_R^{fq} \pm \lambda_L^{fq}\lambda_R^{iq})(Q_{lq}-Q_q) \quad \text{(vectors)} \tag{104}$$

where e is the electromagnetic coupling constant, and i and f respectively label the initial and final leptons. Taking $(Q_{lq}-Q_q) \sim 1$ (we do not assume this in the tables), this gives

$$\lambda_L^{2q}\lambda_R^{1q}, \lambda_R^{2q}\lambda_L^{1q} < 10^{-6}\left(\frac{m_{lq}}{100 \text{ GeV}}\right)^2\left(\frac{1 \text{ GeV}}{m_q}\right) \quad \text{(vectors)} \tag{105}$$

from the upper bound on the decay $\mu \to e\gamma$.

For non-gauge vectors, the leptoquark propagator is again of the form (75), and the second term will give cutoff dependant contributions to the decay rate. If the leptoquark mass is well below the scale at which new physics appears, these terms could be large. We will nonetheless neglect them, as we did for the boxes. This should make our bounds conservative. So using only the $g_{\mu\nu}$ part of the vector propagator, and assuming that there are no cancellations between triangles containing different quark flavours, we get the leading order terms of (101):

$$\frac{|F_+^V|}{m_i+m_f} = \frac{|F_-^A|}{m_i+m_f} \simeq \frac{\lambda_L^{1q}\lambda_L^{2q}e}{32\pi^2}\left(\frac{m_i \pm m_f}{m_{lq}^2}\right)(2Q_q + \frac{5}{2}Q_{lq}) \quad (\text{non}-\text{gauge vectors}). \tag{106}$$

This gives

$$\lambda^{2q}\lambda^{1q} < 2\times 10^{-5}\left(\frac{m_{lq}}{100\ \mathrm{GeV}}\right)^2 \qquad (\mathrm{non-gauge\ vectors}) \tag{107}$$

where we have again taken $(2Q_q + \frac{5}{2}Q_{lq}) \approx 2$.

7 τ decays

7.1 Semi-leptonic decays

The tau is the only lepton heavy enough to decay to a meson and another lepton. The Standard Model lepton family number conservation only allows decays to $\nu_\tau + X$, and $\tau \to \pi\nu$ is observed at the expected rate. The leptoquark couplings $\lambda_{LS_o}, \lambda_{LS_1}, \lambda_{LV_o}$ and λ_{LV_1}, which couple a tau, a neutrino, and an up and down type quark, must therefore satisfy

$$\frac{e^2}{m_{lq}^2} < \frac{4G_F}{\sqrt{2}} \quad . \tag{108}$$

Leptoquarks could also mediate $\tau \to eM, \mu M$, where M is a neutral meson lighter than the tau. If we assume that

$$< 0|\bar{u^i}\gamma^\mu\gamma^5 u^i|M^o > \simeq < 0|\bar{d^i}\gamma^\mu\gamma^5 d^i|M^o > \simeq < 0|\bar{u^i}\gamma^\mu\gamma^5 d^i|M^- > \tag{109}$$

(for instance, $< 0|\bar{u}\gamma^\mu\gamma_5 u|\pi^o > \sim < 0|\bar{d}\gamma^\mu\gamma_5 u|\pi^+ >$), and neglect the mass of the decay lepton, then we do not need to calculate the tau decay rate to a lepton (ℓ) and a meson(M). For a four-vertex of the form

$$\frac{e^{3k}e^{ij}}{m_{lq}^2}(\bar{\tau}\gamma^\mu P\ell^i)(\bar{q}^j\gamma_\mu P q^k) \tag{110}$$

we can simply require

$$\left(\frac{e^{3k}e^{ij}}{m_{lq}^2}\right)^2 < 8G_F^2|V|^2\frac{BR(\tau^- \to M^o\ell^-)}{BR(\tau^- \to M^-\nu)} \tag{111}$$

where V is the appropriate CKM matrix element for the decay $\tau^- \to M^-\nu$, k and j label the flavours of the constituent quarks in M, and $BR(\tau^- \to M^o\ell^-)$ is the upper bound on this process [73]. Using $\ell^i = e, \mu$ and $M^o = \pi^o, K^o$, we get the constraints listed in the tables. Note that unlike the pseudoscalar meson decays, there is no chiral suppression so we would not get better bounds on the couplings of the effective pseudoscalar vertices. We therefore do not compute constraints on the product of left- and right-handed couplings.

7.2 Leptonic decays

Leptoquarks will contribute to the decays $\tau \to \ell\bar{\ell}\ell$ and $\tau \to \ell\gamma$ ($\ell = \mu, e$) in the same way as they did for muons. The bounds however, will be weaker, because the rare decays of the tau are not as tightly constrained as those of the muon. Using equations (93), (91) and BR($\tau \to 3\ell$) $\lesssim 1.5\times 10^{-5}$ [73], we find that for scalar and non-gauge vector leptoquarks

$$\lambda_L^{3q}(\lambda_L^{lq})^3, \lambda_R^{3q}(\lambda_R^{lq})^3 < .2\left(\frac{m_{lq}}{100\ \mathrm{GeV}}\right)^2 \quad (\mathrm{non-gauge\ vectors}) \tag{112}$$

$$\lambda_L^{3q}(\lambda_L^{lq})^3, \lambda_R^{3q}(\lambda_R^{lq})^3 < .3\left(\frac{m_{lq}}{100\text{ GeV}}\right)^2 \quad \text{(scalar)} \tag{113}$$

where q is any quark flavour other than top, and ℓ can be μ and/or e. The bounds on gauge vector leptoquarks are to weak to be meaningful.

The experimental upper bounds on the decays $\tau \to \ell\gamma$ are $\Gamma(\tau \to \mu\gamma) < 9.2 \times 10^{-18}$ GeV [72] and $\Gamma(\tau \to e\gamma) < 2.6 \times 10^{-16}$ GeV [73]. The radiative muon decay bounds can be easily modified to constrain leptoquarks coupling to the τ. The constraints on gauge vectors with couplings of one chirality are again to weak to be worth listing, but equations (96) and (104) give bounds on vector couplings of both chiralities ($\lambda_L\lambda_R$), which are in the tables. One can also get bounds on non-gauge vectors from (106) and the upper bounds on $\tau \to \mu\gamma$ and $\tau \to e\gamma$. This gives

$$\lambda_L^{3q}\lambda_L^{2q}, \lambda_R^{3q}\lambda_R^{2q} < \frac{10^{-2}}{Q_q + \frac{5}{4}Q_{lq}}\left(\frac{m_{lq}}{100\text{ GeV}}\right)^2 \quad \text{(non}-\text{gauge vectors)} \tag{114}$$

$$\lambda_L^{3q}\lambda_L^{1q}, \lambda_R^{3q}\lambda_R^{1q} < \frac{.07}{Q_q + \frac{5}{4}Q_{lq}}\left(\frac{m_{lq}}{100\text{ GeV}}\right)^2 \quad \text{(non}-\text{gauge vectors)} \tag{115}$$

Equations (96)and (98) can be used to constrain scalar leptoquarks contributing to radiative tau decays. We list the bounds in the tables.

8 Other Constraints

8.1 Quark-lepton universality

As was pointed out in [2], leptoquarks would contribute at tree level to neutron β-decay, but only via box diagrams to $\mu \to e\nu\bar{\nu}$. We know from (89), (91) and (93) that the effective leptoquark induced coupling constant for the 4-lepton vertex is much smaller than that for the 2-quark 2-lepton vertex:

$$C_{LL,RR} \ll \frac{\lambda^2}{m_{lq}^2} \tag{116}$$

We can therefore neglect the leptoquark contribution to muon decay, and obtain constraints on the $ude\nu$ couplings from requiring that the Fermi constant in β decay not differ signifcantly from the muon decay measurement. If the effective Fermi constant for β decay $\equiv G_\beta$ was

$$\frac{4G_\beta}{\sqrt{2}} = \frac{4G_F}{\sqrt{2}} + \frac{e_L^2}{m_{lq}^2} \tag{117}$$

then the rate for $n \to pe\bar{\nu}$ would be increased by a multiplicative factor

$$1 + \frac{e_L^2}{\sqrt{2}G_F m_{lq}^2} + \frac{e_L^4}{8G_F^2 m_{lq}^4} \tag{118}$$

The CKM matrix element V_{ud} is determined to be 0.9744 $\pm$.0010 [38] by comparing G_β to G_F, and this value is consistent with unitarity. We can therfore require that the leptoquarks contribute less than the error in V_{ud}:

$$\frac{\sqrt{2}e_L^2}{4m_{lq}^2} < \frac{\delta V_{ud}}{V_{ud}}G_F \tag{119}$$

where we take $\delta V_{ud} = 0.9754-$ [the minimum value given in the Particle Data Book] [38]. This gives

$$e_L^2 < 10^{-3} \left(\frac{m_{lq}}{100\mathrm{GeV}}\right)^2 \tag{120}$$

for $ude\nu$ axial vector couplings. We neglect the contributions from effective scalar $\pm$ pseudoscalar quark matrix elements ($\sim \lambda_L\lambda_R$), because they are strongly constrained by the ratio R_π. (They have also been carefully considered in [82].)

8.2 g - 2

Since the magnetic moment of the electron and the muon are two of the most accurately measured quantities in physics, one always hopes to get good constraints on new particles from them. Constraints on leptoquarks from $g-2$ have been calculated in [15] and [76]. It is easy to see from equation (97) that

$$\frac{(g-2)_e}{2} = \frac{m_e}{e}\frac{F_V}{m_i + m_f} \ . \tag{121}$$

with $m_i = m_f = m_e$. Using $F_V/(m_i + m_f)$ from equations (98), (101), (104), and (106), we can constrain some of the couplings by requiring that the leptoquark contribution to $g-2$ for the muon and the electron be less than the difference between the theoretical predictions and the experimental measurements. Although $[g-2]_e$ is more accurately measured than $[g-2]_\mu$ ($\Delta[g-2]_e/2 = 6\times 10^{-10}, \Delta[g-2]_\mu/2 = 2\times 10^{-8}$), the leptoquark contributions are proportional to the lepton mass, or mass squared, so the relative smallness of m_e suppresses most leptoquark contributions to $[g-2]_e$. As can be seen from the tables, these bounds are considerably weaker than those from radiative decays, but constrain different combinations of generation indices. We have neglected couplings to the top, because we assumed $m_q \ll m_{lq}$ in calculating F_V. As previously mentioned, this approximation was not made by Davies and He [15], or Djouadi, Kohler, Spira and Tutas [76], who give a full analytic expression for the scalar leptoquark contribution to $g-2$, and a reference [79] for the non-gauge vector contribution where the divergent $k^\mu k^\nu/m_{lq}^2$ term in the propagator is included.

From equation (101), we can get weak constraints on the gauge vector leptoquarks that contribute to $g-2$. Since the initial and final lepton are of the same type in this diagram, there is no "GIM" suppression, and the leading order terms in (101) give the bound

$$\lambda_L^{2q}\lambda_L^{2q}, \lambda_R^{2q}\lambda_R^{2q} < \frac{2}{Q_q + \frac{5}{4}Q_{lq}}\left(\frac{m_{lq}}{100 \text{ GeV}}\right)^2 \tag{122}$$

where q is any quark other than top. The bounds on $\lambda_L\lambda_R$ for gauge vectors are stronger than this (for heavy quarks in the loop). We get, from $(g-2)_e$ and $(g-2)_\mu$:

$$\lambda_R^{2q}\lambda_L^{2q} < \frac{.1}{Q_{lq} - Q_q}\left(\frac{m_{lq}}{100 \text{ GeV}}\right)^2 \tag{123}$$

$$\lambda_L^{1q}\lambda_R^{1q} < \frac{.6}{Q_{lq} - Q_q}\left(\frac{m_{lq}}{100 \text{ GeV}}\right)^2 \quad . \tag{124}$$

The bounds on scalar leptoquark couplings of the same chirality (λ_L^2, λ_R^2) from $(g-2)_\mu$ are similiar to (122). As in the vector case, we could probably get stronger bounds on $\lambda_L\lambda_R$, at least for heavy quarks on the internal lines. We neglect this, because there are better constraints from elsewhere (see table 15), and bounds on the combination $\lambda_L\lambda_R$ are less interesting in the first place.

8.3 Neutral current couplings

The many different experimental determinations of $\sin^2\theta_W$ seen to agree quite well. Since leptoquarks mediate two-lepton, two-quark interactions like the weak gauge bosons of the Standard Model, they could contribute at tree level in some, but not all, of these experiments. There will therefore be bounds on leptoquark couplings from requiring that they not disrupt the agreement between various determinations of $\sin^2\theta_W$. These will be particularily interesting, because they apply to leptoquarks coupling to first generation fermions– precisely the leptoquarks that could be seen at HERA.

Leptoquarks should not have a significant effect on the LEP measurements, because LEP runs on the Z mass. However, they should contribute on an equal footing with the electroweak gauge bosons in atomic parity violation experiments, and in deep inelastic scattering of neutrinos off nucleons. It turns out that the best bounds on the largest number of leptoquark coupling constants come from atomic parity violation experiments on cesium[38], and these constraints have been considered by Langacker [83]. The atomic weak charge $Q_W = -2[C_{1u}(2Z+N)+C_{1d}(Z+2N)]$ of the cesium atom is measured to be $Q_W = -71.04 \pm 1.58 \pm .88$ (the second error is theoretical). C_{1u} and C_{1d} are coefficients in the effective four-fermion parity-violating Lagrangian

$$\frac{G}{\sqrt{2}}(\bar{l}\gamma^\mu\gamma_5 l)(C_{1u}\bar{u}\gamma_\mu u + C_{1d}\bar{d}\gamma_\mu d) \tag{125}$$

where at tree level (see [38] for the one-loop Standard Model corrections) $C_{1q} = -I_3 + 2Q_{em}\sin^2\theta_W + \Delta C_{1q}$, and ΔC_{1q} is the leptoquark contribution:

$$\Delta C_{1q} = \frac{e^2}{m_{lq}^2}\frac{\sqrt{2}}{4G_F} \ . \tag{126}$$

The Standard Model expectation for Q_W is $-73.21 \pm .08 \pm .03$, so requiring that the leptoquark contribution to Q_W be less than the difference between the experimental and Standard Model values, gives

$$|\Delta C_{1u}| = |\Delta C_{1d}| < .003 \tag{127}$$

for leptoquarks that induce $\Delta C_{1u} = \Delta C_{1d}$, and

$$|\Delta C_{1u}|, |\Delta C_{1d}| < .006 \tag{128}$$

for leptoquarks where $\Delta C_{1u} = 0$ or $\Delta C_{1d} = 0$. These bounds are slightly weaker than those in [83]. They also neglect the sign of the corrections, which is conservative, but not altogether reasonable because the theoretical prediction is one and half σ to one side of the experimental result. From (127), (128) and tables 3 and 4, one gets that for $\lambda_{LV_1}, \lambda_{RV_{1/2}}, \lambda_{RS_{1/2}}$ and λ_{LS_1}

$$e^2 < .001\left(\frac{m_{lq}^2}{100\mathrm{GeV}}\right)^2 \tag{129}$$

and for all the other couplings that only contribute to ΔC_{1u} or to ΔC_{1d}:

$$e^2 < .002\left(\frac{m_{lq}^2}{100\mathrm{GeV}}\right)^2 \ . \tag{130}$$

We have assumed for these constraints that $\lambda_{RS_o} \neq \lambda_{LS_o}$, $\lambda_{RS_{1/2}} \neq \lambda_{LS_{1/2}}$, $\lambda_{RV_o} \neq \lambda_{LV_o}$, and $\lambda_{LV_{1/2}} \neq \lambda_{RV_{1/2}}$, because the parity violating coupling is $\lambda_L - \lambda_R$. This is a reasonable assumption, as noted in [83], because there are strong bounds on the product of these couplings $\lambda_L\lambda_R$ from the ratio $(\pi \to e\nu)/(\pi \to \mu\nu)$ (see section 4.1).

8.4 Neutrino Oscillation Experiments

Four of the leptoquark couplings induce four-fermion interactions that couple a neutrino, a charged lepton and an up and down type quark (see tables 3 and 4). The couplings $\lambda_{LS_o}, \lambda_{LS_1}, \lambda_{LV_o}$ and λ_{LV_1} can therefore be constrained by neutrino oscillation experiments that look for charged leptons of a different flavour from the original neutrino beam.

If the neutrino masses are small, and one neglects the mixing of ν_{l_a} and ν_{l_b} with the third neutrino, then the probability that a neutrino ν_{l_a} with energy E will oscillate into a ν_{l_b} over a distance x is

$$| < \nu_{l_a} | \nu_{l_b} > |^2 = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 x}{4E} \right) \tag{131}$$

where θ is the angle mixing ν_{l_a} and ν_{l_b}, and $\Delta m^2 = |m^2_{\nu_a} - m^2_{\nu_b}|$. If $\sin^2(\Delta m^2 x/4E)$ oscillates rapidly enough, (large Δm^2), it can be averaged to 1/2. The "large Δm^2" bounds on the mixing angle $\sin^2 2\theta$ are therefore bounds on the probability for a ν_{l_a} to turn into a ν_{l_b}, or equivalently on the ratio of l_a to l_b produced by the neutrino beam. This can be used to constrain the leptoquark induced rate for $\bar{\nu}_{l_a} + N \to \bar{l}_b + X$ ($\bar{\nu}_{l_a} + u \to \bar{l}_b + [d, s, b]$) and $\nu_{l_a} + N \to l_b + X$ ($\nu_{l_a} + u \to l_b + [u, c]$):

$$\frac{e_L^2}{m_{lq}^2} < 2G_F |\sin 2\theta| \tag{132}$$

where $\sin^2 2\theta$ is from the "large mass" upper bound from accelerator experiments [38].

The scalar leptoquarks that interact with neutrinos have fermion number 2, so couple an incoming ν to an incoming d, or an incoming $\bar{\nu}$ to an outgoing d. The vectors, on the other hand, have fermion number 0, and couple an incoming ν to an outgoing u or an incoming $\bar{\nu}$ to an incoming u. This is important because we assume that the outgoing quark can be any flavour other than top (we neglect the phase space suppression due to the mass of the b), so to get the flavour indices right on the couplings constants, we need to know if the neutrino couples to the leptoquark and the outgoing quark, or the leptoquark and the incoming quark.

The bounds are listed in the tables.

8.5 Asymmetries at colliders

The constraints on leptoquarks from the forward-backward asymmetry (A_{FB}) at existing $e\bar{e}$ colliders (excluding LEP1) were considered by Hewett and Rizzo [37]. (Leptoquarks are not constrained by A_{FB} at LEP1 because it runs on the Z mass.) Possible constraints from future colliders were considered in [85]. Hewett and Rizzo required that A_{FB} and the total cross-section for $e\bar{e} \to q\bar{q}$ not differ from the Standard Model values by more than 5 or 10% at $\sqrt{s} = 40$ GeV. They claim that a greater contribution would disrupt the agreement between Standard Model predictions and the measured rates for b and c quark production at PEP and PETRA.

Their bounds apply to the couplings $(eu)(eu)$ and $(ed)(ed)$ ((11)(11) in the tables), for scalar leptoquarks. A similar constraint could be calculated for vector leptoquarks, but since the measurement of $\sin^2 \theta_W$ in parity violation experiments gives more stringent limits, this is unneccessary. We list Hewett and Rizzo's bounds in the tables as

$$\lambda^2 < .02 \left(\frac{m_{lq}^2}{100\mathrm{GeV}} \right)^2 \quad . \tag{133}$$

Note that this analytic bound is just an approximation to the limit graphed in their paper.

In principle, some leptoquarks could contribute strongly to A_{LR} at tree level, because they only couple to electrons of one chirality. However, measuring A_{LR} requires polarized electron beams, so we can not get bounds from this at present (SLC has polarized beams, but, like LEP1, is running on the Z mass).

8.6 Muon beams

As was noted by Heusch [5], one should in principle be able to get interesting bounds on leptoquarks from muon beam experiments. Leptoquarks contributing to $\mu N \to \mu X$ can only be constrained by measurements of asymmetries in this process, because the rate is dominated by photon exchange. However, if one could effectively search for $\mu N \to \ell X$ [84], where ℓ is an electron or a tau, one could constrain couplings with the family structure $\lambda^{\mu q}\lambda^{\ell q'}$, where q and q' can be any type of quark (other than top) if one allows the muon to scatter off sea quarks in the nucleon.

We can get weak constraints on leptoquarks mediating the reaction $\mu q \to \mu q'$ from the measurement [86] of the asymmetry

$$B = \frac{d\sigma(\mu_R^-) - d\sigma(\mu_L^+)}{d\sigma(\mu_R^-) + d\sigma(\mu_L^+)} \tag{134}$$

where $d\sigma(\mu_R^-)$ is the cross-section for $\mu N \to \mu X$ for an incident right-handed muon. If one assumes that only valence quarks contribute to this process, one can show that the interference between Z and γ exchange gives [86, 87]

$$B = -\frac{Q^2}{2\pi\alpha}\frac{G_F}{\sqrt{2}}(c_A^\mu - pc_V^\nu)(c_A^d - 2c_A^u)\frac{6}{5}g(y) \tag{135}$$

where Q^2 is the energy transfer, p is the polarization (= .81 in this experiment), $g(y)$ is a function of the ratio of outgoing to incident muon energies ($y \equiv 1 - E_f/E_i$), and c_V^f, c_A^f are the usual vector and axial vector couplings, normalized such that $c_A^\mu = -1/2$. This formula can easily be modified to descibe the interference between leptoquark and photon exchange by substituting $e_L^2/m_{lq}^2 \leftrightarrow 4\sqrt{2}G_F$, and setting the various c's to ± 1 or 0 as required. We then require the leptoquark-photon intererence contribution to the asymmetry (135) to be less than the error in B. This gives the bounds listed in the tables.

9 Discussion

We have attempted to make this catalogue of constraints as comprehensive as possible. However, it is inevitable that we will have omitted some (we have, for instance, neglected bounds from CP violation). One could also do the calculations more carefully than we have done, and probably improve many of the constraints, as we have tried to err on the side of caution.

In the tables, we list the constraints on gauge boson vector leptoquarks. In some cases, the bounds on non-gauge vectors are much stronger, and we have tried to estimate these in the text.

The data in tables 5 to 14 is difficult to interpret, so we have collected the best bound on each product of couplings, and the experiment it came from, in table 15. The couplings $\lambda_{LS_o}, ...\lambda_{LV_1}$ from equations (1) and (2) are really 3×3 matrices in generation space. We therefore list the matrices horizontally and the indices of the two matrix elements whose product is constrained vertically. So the top right-hand entry of table 15 is the best bound on $\lambda^{11}_{LS_o}\lambda^{11}_{LS_o}$. As before, the lepton index comes first, and the numbers in the table are multiplied by $(m_{lq}/100 \text{ GeV})^2$. We have assumed, for this table, that all neutrinos are light ($m_\nu \ll m_e$). ALthough we calculated these limits for leptoquarks, many of the bounds would apply to other massive "beyond the Standard Model" bosons. In particular, the rare meson decay bounds apply to any particle that induces '$V \pm A$ lepton current' $\times$ ' $V \pm A$ quark current' four-fermion operators.

Ideally we would like to have individual bounds on the squares of each element of every matrix. Instead, we find that the squares of each matrix element are in general unconstrained. (By this we mean that for a leptoquark of mass $m_{lq} \sim 100$ GeV, λ^2 could be of order 1.) This is hardly surprising, since leptoquark-mediated interactions whose matrix elements involve the combination $\lambda^{lq}\lambda^{lq}$ do not change lepton generation or quark flavour; with two exceptions, they are therefore competing with electromagnetic amplitudes for the same process. For instance, leptoquarks could mediate the decay of $q\bar{q}$ mesons, but since the photon can also, there are no interesting bounds on leptoquarks from these decays. The two exceptions are parity violation experiments, which is where the bounds on $(\lambda^{11})^2$ come from, and if one of the leptons is a neutrino. $\lambda_{LS_o}, \lambda_{LS_1}, \lambda_{LV_o}$ and λ_{LV_1} are the only couplings to involve a neutrino, a charged lepton and an up and down type quark, and are sometimes better constrained in consequence.

Most of out constraints come from processes that are suppressed or forbidden in the Standard Model by lepton family number conservation, or the absence of flavour-changing neutral currents among the quarks. Such limits tend to involve the product, rather than the square, of couplings: $\lambda^{ij}\lambda^{mn} <$ something, with $(ij) \neq (mn)$. An upper bound on the product of two couplings is not terribly useful, because either coupling could be very large or very small. The other problem associated with bounds from flavour changing processes is that usually $i \neq j$ and/or $m \neq n$ in bounds of the form $\lambda^{ij}\lambda^{mn} <$ something.

If the leptoquark couplings have a generation structure similiar to the Standard Model, we might expect couplings λ^{ij} with $i \neq j$ to be one or two orders of magnitude smaller than the $i = j$ value, by analogy with the CKM matrix. It is unfortunate that the couplings that we might expect to be largest are the ones we have the fewest constraints on.

We have few bounds on leptoquark couplings to the top quark, because these can only be obtained from loop diagrams (meson-anti-meson mixing and μ and τ decay to three leptons or a lepton and a photon), and the quark mass was neglected in the triangle loop calculations. This could be corrected, but would still not lead to simple numerical bounds, because they would depend on the unknown leptoquark and top masses. The interactions which are unconstrained because they involve a top quark are indicated by stars in table 15.

It is worth noting that some of the interactions couple a charged lepton to an up-type quark, some to a down, and some to both. The limits on the coupling constants of interactions involving down-type quarks tend to be better than those for ups, because Ks are better constrained than Ds, and Bs exist.

HERA expects to be able to see leptoquarks with $m_{lq} \lesssim 300$ GeV and $\lambda^2 \gtrsim 10^{-4}$ as peaks in the $e^\pm p \to e^\pm X$ cross-section as a function of x [2], so if the upper bound on $\lambda_\alpha^{11}\lambda_\alpha^{mn}$ from a rare decay is greater than $10^{-5}(m_{lq}/100 \text{ GeV})^2$, then the leptoquark '$\alpha$' can be produced and detected via these couplings at HERA. As is clear from table 15, this means that it is consistent with the bounds computed here for HERA to see any of the leptoquarks. Four of the constraints listed here are strong enough to each rule out a particular production and decay channel, but since the leptoquark could easily decay to a different quark-lepton final state, this is not very significant. For instance, the bounds from muon conversion on titanium ($e^2 < 7 \times 10^{-7}(m_{lq}/100 \text{ GeV})^2$) suggest that no leptoquark produced in the collision of an electron (or $\bar{e}$) and a first generation quark will be detected decaying to a muon and a first generation quark (a second or third generation quark would, however, be quite possible). Similiarly, the absence of kaon decays to $\bar{\mu}\mu, \pi\mu\bar{e}$ or $e\bar{\mu}$ implies that HERA will not see the leptoquarks $V_o^\mu, V_{1/2}^\mu, \vec{V}_1^\mu, \tilde{S}_o, \tilde{S}_{1/2}$, and $\vec{\tilde{S}}_1$ decaying to a strange quark and an electron or muon (but again, some other flavour of quark would not be ruled out). If the leptoquark Yukawa couplings were generation *independant*, these constraints would apply to all initial and final quark-lepton combinations, and HERA would not see leptoquarks. However, if one makes the far more reasonable assumption that the couplings are generation dependent (very true of the Standard Model fermion-fermion-boson couplings), then our bounds from rare processes do not seriously infringe on HERA's chances of seeing leptoquarks. The experiments listed above do constrain $e^2/m_{lq}^2 < 10^{-9}$ GeV^{-2} ($e^2 < 10^{-5}$ in the tables), but they do not apply to the couplings for $e^\pm p \to e^\pm X$ (where X consists of u and d quarks). So leptoquarks can easily have masses and couplings that are accessible to HERA and consistent with the upper bounds on rare processes.

In figure 5 we have plotted, following [2] and using quark distributions from [88], the differential cross-section for $e^- p \to e^- X$ in the presence of the scalar leptoquark S_o with a mass of 200 GeV, and couplings $\lambda_{LS_o}^{11} = .008, \lambda_{LS_o}^{ij} = 0$ for $i,j \neq 1$, and $\lambda_{RS_o} = 0$. This choice of mass and coupling is consistent with the upper bound $(\lambda_{S_o}^{11})^2 < 2 \times 10^{-3}(m_{lq}/100 \text{ GeV})^2$ from quark-lepton universality. One could make similiar plots for other leptoquarks, and second or third generation final state quarks and leptons. It is clear that if ZEUS and H1 do not see leptoquarks, they will impose strong constraints on all couplings of the form $\lambda^{11}\lambda^{mn}$.

10 Conclusion

We have calculated generation dependent upper bounds on the coupling constants of all renormalizable, B and L conserving interactions consistent with the $SU(3)\times SU(2)\times U(1)$ symmetry of the Standard Model, involving a lepton, a quark, and a scalar or vector leptoquark. The constraints come from rare meson decays, meson-anti-meson mixing, lepton decays, and a miscellany of electroweak tests. We list the bounds on the square of each coupling constant from each experiment in tables 5 through 14; since the constraints depend on the leptoquark mass, and only apply for certain generation indices, we quote numerical bounds based on $m_{lq} = 100$ GeV, and put the lepton and quark generation indices in parentheses (so that the top right hand corner of table 5: $(11)(n1) < 10^{-3}$ means that $\lambda_{LS_o}^{ed}\lambda_{LS_o}^{\nu_n u} < 10^{-3}(m_{lq}/100\text{GeV})^2$). In table 15, we list the best bound on each coupling constant for every combination of generation indices.

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References

[1] for a review of leptoquarks at HERA, see: R.J. Cashmore et al., *Phys. Rep.* **122** (1985) 275
"*Physics at HERA*", Proceedings of the Workshop, Hamburg, 1991, W. Buchmuller, G. Ingelman, editors.

[2] W. Buchmuller, R. Ruckl, and D. Wyler, *Phys. Lett.* **B191** (1987) 442.

[3] M. Leurer, *Phys. Rev.* **D46** (1992) 3757.

[4] J.A.Grifolis, S. Peris, *Phys. Lett* **B201** (1988) 287.
M.A. Doncheski, J.L. Hewett, *Zeit. Phys* **C56** (1992) 209.

[5] C.A. Heusch, lecture at the "Rencontre de physique de la vallée d'Aoste", February 1989.

[6] S. Dimopoulos, J. Ellis, *Nucl. Phys* **B182** (1981) 505.

[7] O. Shenkar, *Nucl. Phys.* **B206** (1982) 253.

[8] O. Shenkar, *Nucl. Phys.* **B204** (1982) 375.

[9] R. Mohapatra, G. Segré, L. Wolfenstein, *Phys. Lett* **B145** (1984) 433.

[10] I. Bigi, G. Köpp, P.M. Zerwas, *Phys. Lett.* **166B** (1986) 238.

[11] W. Buchmuller and D. Wyler, *Phys. Lett.* **B177** (1986) 377.

[12] W. Buchmuller and D. Wyler, *Nucl. Phys.* **B268** (1986) 621.

[13] B.A. Campbell, J. Ellis, K. Enqvist, M.K. Gaillard, and D.V. Nanopoulos, *Int. J. Mod. Phys.* **A2** (1987) 831.

[14] J. Hewett, T. Rizzo, *Phys. Rep.* **183** (1989) 193.

[15] A.J.Davies, X. He, *Phys. Rev.* **D43** (1991) 225.

[16] X. He, B.H.J. McKellar, S. Pakvasa, *Phys. Lett.* **B283** (1992) 348.

[17] C.Q. Geng, *Zeit. Phys.* **C48** (1990) 279.

[18] S.M. Barr, E.M. Freire, *Phys. Rev.* **D41** (1990) 2129.

[19] G.G. Ross, *Grand Unified Theories*, Benjamin/Cummings, 1984.

[20] J.C. Pati, A. Salam, *Phys. Rev.* **D8** (1973) 1240; *Phys.Rev. Lett.* **31** (1973) 661; *Phys. Rev.* **D10** (1974) 275.

[21] P. Langacker, *Phys. Rep.* 72 (1981) 187.

[22] the first SU(5) model was proposed by H. Georgi, S.L. Glashow, *Phys. Rev. Lett.* **32** (1974) 438.

[23] H. Murayama, T. Yanagida, TU 370 (1991).

[24] for a review, see G.G. Ross, *Grand Unified Theories*, Benjamin-Cummings, 1985
E. Farhi, L. Susskind, *Phys. Rep.* **74** (1981) 277
E. Eichenten et al., *Rev. Mod. Phys.* **56** (1984) 579.

[25] for a review, see: I. Bars, *Proc. of the 1984 Summer Study on the SSC*, editors R.Donaldson, J. Morphin (APF New York, 1985) 38.
W. Buchmüller, *Acta Phys. Austr. Suppl.* **XXVII** (1985) 517.
B. Schrempp, *Proc. of the XXIII Int. Conf. on High Energy Physics*, Berkeley,
R. Peccei, in *Proceedings of the 2nd Lake Louise Winter Institute*, J.M. Cameron et al., eds., World Scientific, Singapore (1987).

[26] L.F. Abbott, E. Farhi, *Phys. Lett.* **101B** (1981) 69; *Nucl. Phys.* **B189** (1981) 547.

[27] W. Bardeen, V. Visnjić, *Nucl. Phys.* **B194** (1982) 151.
W. Buchmuller, R. Peccei, T. Yanagida, *Phys. Lett.* **124B** (1983) 67.
R. Barbieri, A. Masiero, G. Veneziano, *Phys. Lett.* **128B** (1983) 179.

[28] B. Schrempp, F. Schrempp, *Nucl. Phys.* **B231** (1984) 109.

[29] J.C. Pati, *Phys. Lett.* **B228** (1989) 228.
see also references in [25] for other composite models.

[30] G. 't Hooft, in *Recent Developements in Gauge Theories* ed. G 't Hooft et al., Plenum, N.Y. (1980).

[31] Y. Frishman et al., *Nucl. Phys.* **B177** (1981) 157.

[32] S. Coleman, B. Grossman, *Nucl. Phys.* **B203** (1982) 205.

[33] S. Dimopoulos, S. Raby, L. Susskind, *Nucl. Phys.* **B169** (1980) 373.

[34] S. Dimopoulos, S. Raby, L. Susskind, *Nucl. Phys.* **B173** (1980) 208.

[35] M. Peskin, in *Proceedings of the 1985 Lepton-Photon Symposium.*

[36] K.M. Case, S.G. Gasierowicz, *Phys. Rev.* **125** (1962) 1055. S. Weinberg, E. Witten, *Phys. Lett.* **96B** (1980) 159.

[37] J.L. Hewett, T.G. Rizzo, *Phys. Rev.* **D36** (1987) 3367.

[38] Particle Data Book, *Phys. Rev.* **D45**, #11 (1992).

[39] W. Buchmuller,*Phys. Lett.* **145B** (1984) 151.
B. Schrempp, F. Schrempp, *Phys. Lett.* **153B** (1985) 101.

[40] AMY Collaboration, G.N. Kim et al., *Phys. Lett.* **B240** (1990) 243.

[41] ALEPH Collaboration, CERN PPE 91/149.
L3 Collaboration, *Phys. Lett.* **B261** (1992) 169,
OPAL Collaboration, *Phys. Lett.* **B263** (1992) 123.

[42] M. A. Doncheski et al., *Phys. Rev.* **D40** (1989) 2301.

[43] T.D Papadopoulos,, for the Delphi Collaboration, in *Proceedings of the Fermilab Meeting, DPF '92*, World Scientific.

[44] UA2 Collaboration, *Phys. Lett.* **B274** (1992) 507.

[45] CDF Collaboration, Abe et al., Fermilab-Pub-93/070-E, submitted to *Phys Rev* **D**.

[46] D0 Collaboration, talk presented at the 1993 Lepton Photon Symposium.

[47] J. Hewett, S. Pakvasa, Pomeral, T. Rizzo, work in progress.

[48] ZEUS Collaboration, *Phys. Lett.* **B306** (1993) 173.
H1 Collaboration, *Nucl. Phys.* **B396** (1993) 3.
K. McLean for the ZEUS Collaboration, at the "Conference of the European Physical Society", Marseille, 1993.

[49] D. I. Britton et al., *Phys. Rev. Lett.* **68** (1992) 3000.
G. Czapek et al., *Phys. Rev. Lett.* **70** (1993) 17.
We get the numbers quoted in the text by averaging the results and the errors.

[50] S. Berman, *Phys. Rev. Lett.* **1** (1958) 468.
T. Kinoshita, *Phys. Rev. Lett.* **2** (1957) 477.
W.J. Marciano, A. Sirlin, *Phys. Rev. Lett.* **36** (1976) 1425. ($1.233 \pm .004 \times 10^{-4}$).
T. Goldman and W. Wilson, *Phys. Rev.* **D15** (1977) 709, ($1.239 \pm .001 \times 10^{-4}$).
W. Marciano, see [49], ($1.2345 \pm .0010 \times 10^{-4}$).

[51] A. Deshpande et al., *Phys. Rev. Lett.* **71** (1993) 27. ($6.9 \pm 2.3 \pm .6 \times 10^{-8}$)
K.S. McFarland et al., *Phys. Rev. Lett.* **71** (1993) 31. ($7.6 + 3.9 - 2.8 \pm .5 \times 10^{-8}$)

[52] G.Martinelli in *Proceeedings of the XXVth Rencontre de Moriond: Electroweak Interactions and Unified Theories*, Editions Frontières, 1991.

[53] M. Leurer, Weizmann Institute preprint, WIS-93/26/March-PH.

[54] for a review, see M. Wise, in *Proceedings of the 6th Lake Louise Winter Institute* (1991), eds. B.A.Campbell et al., World Scientific, Singapore.

[55] ALEPH Collaboration, contributed paper at the XXVI International Conference on High Energy Physics, Dallas, Texas (1992).

[56] R. Fulton et al., CLEO Collaboration, *Phys. Rev. Lett.* **64** (1990) 16.
H. Albrecht et al., ARGUS Collaboration, *Phys. Lett.* **B255** (1991) 297.

[57] CLEO Collaboration, *Phys. Rev.* **D35** (1987) 3533.

[58] UA1 Collaboration, *Phys. Lett.* **B262** (1991) 163.

[59] N. Isgur, M. Wise *Phys. Rev.* **D42** (1990) 2388.

[60] N. Isgur, D. Scora, B. Grinstein, M. Wise, *Phys. Rev.* **D39** (1991) 799.

[61] J.C. Anjos et al., *Phys. Rev. Lett.* **67** (1991) 1507.

[62] Mark III Collaboration, J. Adler et al., *Phys. Rev. Lett.* **62** (1989) 1821.

[63] CLEO Collaboration, CLNS-93-1212.

[64] H. Dreiner, *Mod. Phys. Lett* **A3** (1988) 867.

[65] K.A. Peterson, *Phys. Lett.* **B 282** (1992) 207.
K.A. Peterson, Ph.D. thesis, University of Alberta, 1991.

[66] S. Ahmad et al., *Phys. Rev. Lett.* **59** (1987) 970.
S. Ahmad et al., *Phys. Rev.* **D38** (1988) 2102.

[67] O. Shenkar, *Phys. Rev.* **D 20** (1979) 1608.

[68] R.N. Cahn, H. Harari, *Nucl. Phys.* **B176** (1980), 135.

[69] T.Cheng, L. Li, Gauge Theory of Elementary Particle Physics, Oxford University Press, (1989) chapter 12.

[70] M.K. Gaillard, B.W. Lee, *Phys. Rev.* **D10** (1974) 897.

[71] B.W. Lee, R.E. Shrock, *Phys. Rev.* **D16** (1977) 1444.

[72] CLEO collaboration, *Phys. Rev. Lett.* **70** (1993) 138.

[73] ARGUS Collaboration, *Zeit. Phys.* **C55** (1992) 179.

[74] D.A. Morris, *Phys. Rev.* **D 37** (1988) 2012.

[75] J.P. Leveille, *Nucl. Phys.* **B137** (1978) 63.

[76] A. Djouadi, T. Kohler, M. Spira, J. Tutas, *Zeit. Phys.* **C46** (1990) 679.

[77] X. He, private communication.

[78] W.J. Marciano, A.I. Sanda, *Phys. Lett.* **67B** (1977) 303.

[79] M. Spira, diploma thesis, PWTH Aachen, 1989.

[80] B. Grinstein, R. Springer, M.B. Wise, *Phys. Lett.* **B 202** (1988) 138; *Nucl. Phys* **B339** (1990) 269.
R. Grigjanis, P.J. O'Donnell, M. Sutherland, H. Navelet, *Phys. Lett.* **B 223** (1989) 239.

[81] B.A. Campbell, P.J. O'Donnell, *Phys. Rev.* **D 25** (1982) 1989.
S.P. Chia, *Phys. Lett.* **240B** (1990) 465.
T. Inami, C.S. Lim, *Prog. Theor. Phys.* **65** (1981) 297.

[82] P. Herczag, in *Fundamental Symmetries in Nuclei and Particles*, World Scientific, 1989.

[83] P. Langacker, *Phys. Lett.* **256B** (1991) 277.

[84] C.A. Heusch, in **Progress in Electroweak Interactions**, J. Tran Thanh Van, ed., (1986).

[85] H. Dreiner et al., *Mod. Phys. Lett* **A3** (1988) 443.

[86] A. Argento et al., *Phys. Lett.* **B120** (1983) 245.

[87] S.M. Berman, J.R. Primack, *Phys. Rev.* **D 9** (1974) 2171.

[88] J.G. Morfin, W.K. Tung, *Zeit. Phys.* **C52** (1991) 13.