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Brookhaven

Recent Advances in GPDs

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Outline

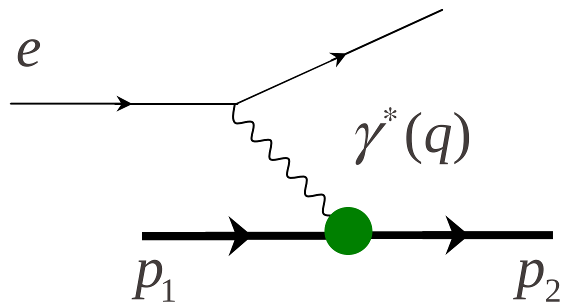
- Folklore
- Recent results on:
  - Evolution of GPDs
  - Higher order effects in exclusive processes
  - Parametrizations
  - Lattice for spin crisis
  - Hard meson(s) production
- AdS/QCD for GPDs?
- Conclusions

Topics not touched: nuclear GPDs , large angle processes, dynamical models, nondiagonal GPDs, power corrections, experiment

Folklore

Traditional probes of nucleon structure

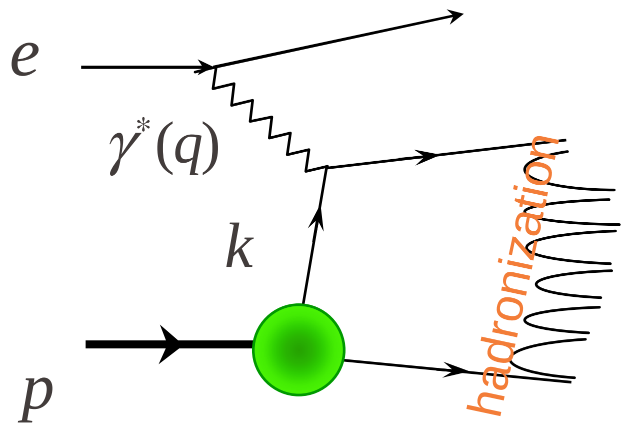
- Elastic electron-proton scattering $ep \to e'p'$:



$$\langle p_2 | \overline{\psi}(0) \gamma^\mu \psi(0) | p_1 \rangle$$

local operator

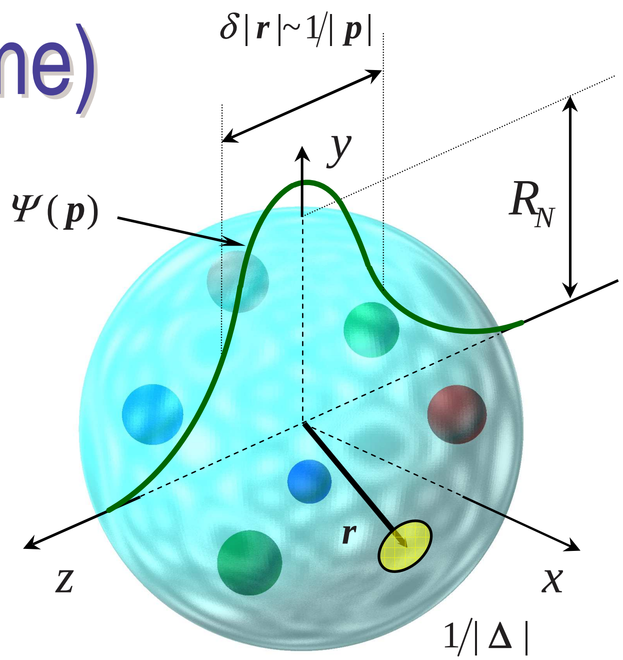
- Inelastic electron-proton scattering $ep \to e'X$:



$$\langle p | \overline{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \int dx \, \mathrm{e}^{-ixp^+z^-} \, q(x)$$

light-cone operator

Physics of form factors (Breit frame)



Localized proton as a wave packet:

$$\left|\vec{R}\right\rangle = \int \frac{d^3\vec{p}}{(2\pi)^3} e^{i\vec{p}\cdot\vec{R}}\,\Psi(\vec{p})\left|\vec{p}\right\rangle$$

Charge distribution in the wave packet:

$$\rho(\vec{r}) = \left\langle \vec{R}=0 \middle| j_0(\vec{r}) \middle| \vec{R}=0 \right\rangle$$

- Size of the wave-packet << system size: $\delta|\vec{r}| \ll R_N$
- Resolution scale >> size of the wave packet (one does not want to see the wave packet): $\delta|\vec{r}| \ll 1/|\vec{\Delta}|$
- Size of the wave packet >> Compton wave length (to be insensitive to wave nature of the proton): $\delta|\vec{r}| \gg 1/M_N$

$$1/R_N \ll |\vec{\Delta}| \ll |\vec{p}| \ll M_N$$

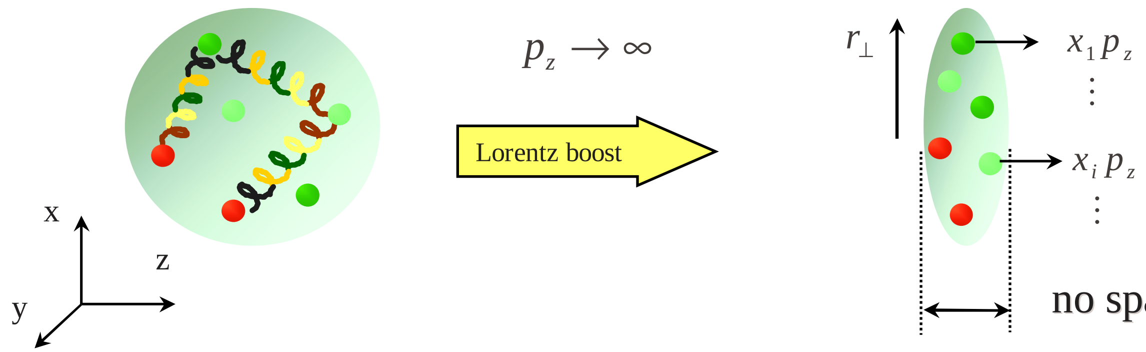
$$\int d^3\vec{x}\, e^{i\vec{q}\cdot\vec{r}} \rho(\vec{r}) \approx \langle \vec{q}/2 | j_0(0) | -\vec{q}/2 \rangle = 2M_N G_E(Q^2)$$

Electric form factor in the Breit frame is (with reservations) a Fourier transform of the charge distribution.

Physics in the infinite momentum frame

proton at rest

momentum frame of
a fast moving proton

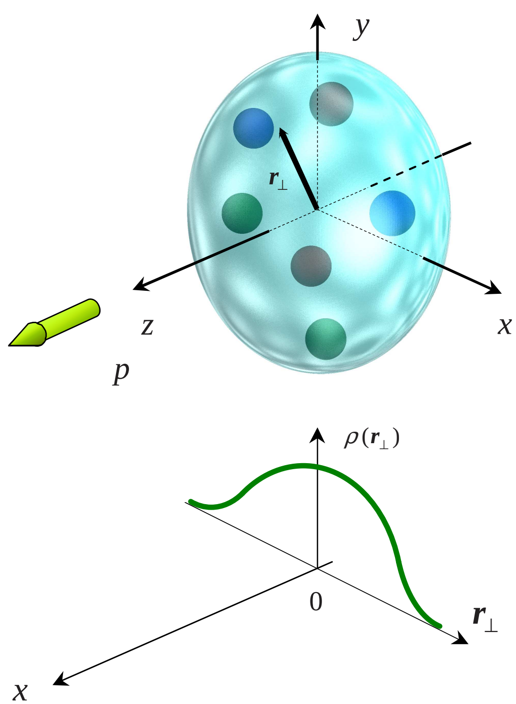


no spatial extent

Form factors:

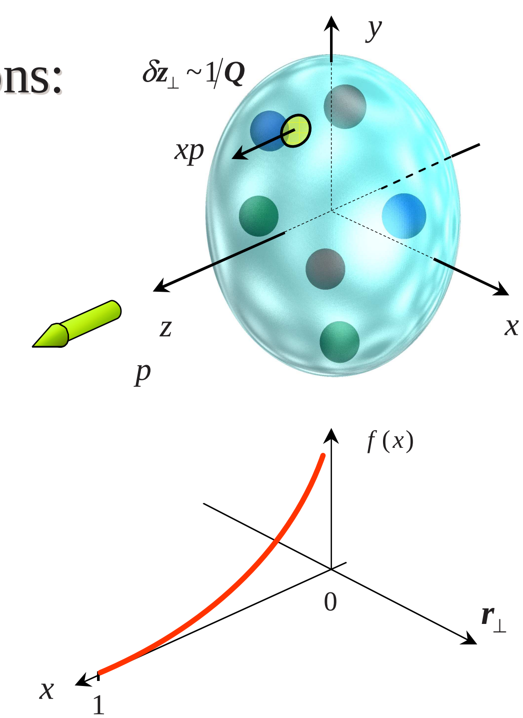
Distribution of quarks in transverse plane irrespective of their longitudinal motion.

No corrections!



Parton distributions:

Density of partons of a given longitudinal momentum $x = k_{||}/p$ measured with transverse resolution ~$1/Q$



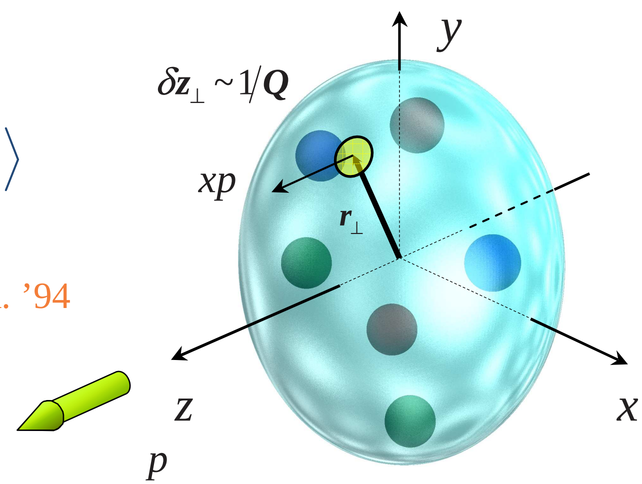
Impact parameter parton distributions

Localized wave packet in transverse plane:

Soper ’77
Burkardt ’00

$$\left| p^+, \boldsymbol{R}_\perp \right\rangle = \int \frac{d^2 \boldsymbol{p}_\perp}{(2\pi)^2} \boldsymbol{e}^{i\vec{p}_\perp \cdot \vec{R}_\perp} \Psi(\boldsymbol{p}_\perp) \left| p^+, \boldsymbol{p}_\perp \right\rangle$$

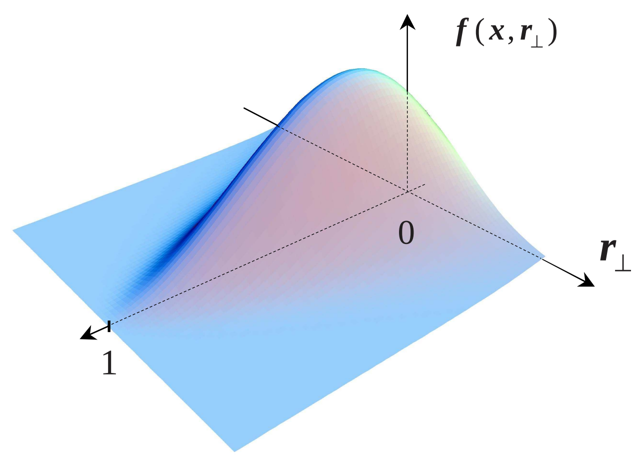
The probability of a parton to possess the momentum fractions x at transverse position $\boldsymbol{r}_\perp$



$$f(x, \boldsymbol{r}_\perp) = \int dz^- \, \mathrm{e}^{ixp^+ z^-} \left\langle p^+, 0_\perp \right| \overline{\psi}(z^-, \boldsymbol{r}_\perp) \gamma^+ \overline{\psi}(0, \boldsymbol{r}_\perp) \left| p^+, 0_\perp \right\rangle$$

Generalized parton distributions simultaneously carry information on both longitudinal and transverse distribution of partons in a fast moving nucleon

Mueller, Robaschik et al. ’94
Ji ’96
Radyushkin ’96



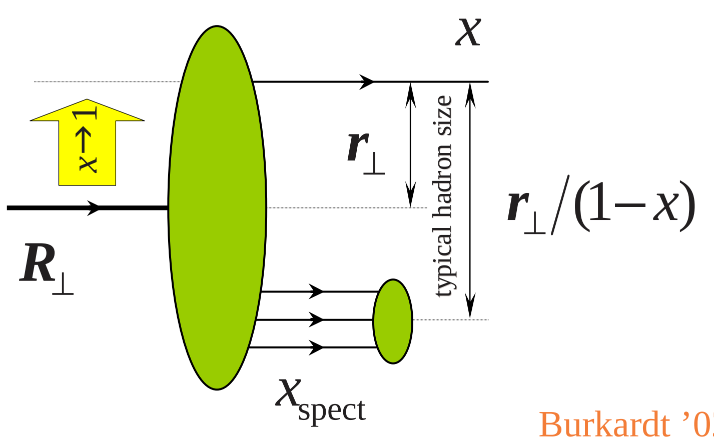
$$f(x, \boldsymbol{r}_\perp) = \int d^2 \Delta_\perp \, \mathrm{e}^{-i\vec{r}_\perp \cdot \vec{\Delta}_\perp} H(x, \eta = 0, \Delta_\perp^2)$$

GPDs carry information on the angular momentum of partons.

Q: What is the physical significance of skewness η?

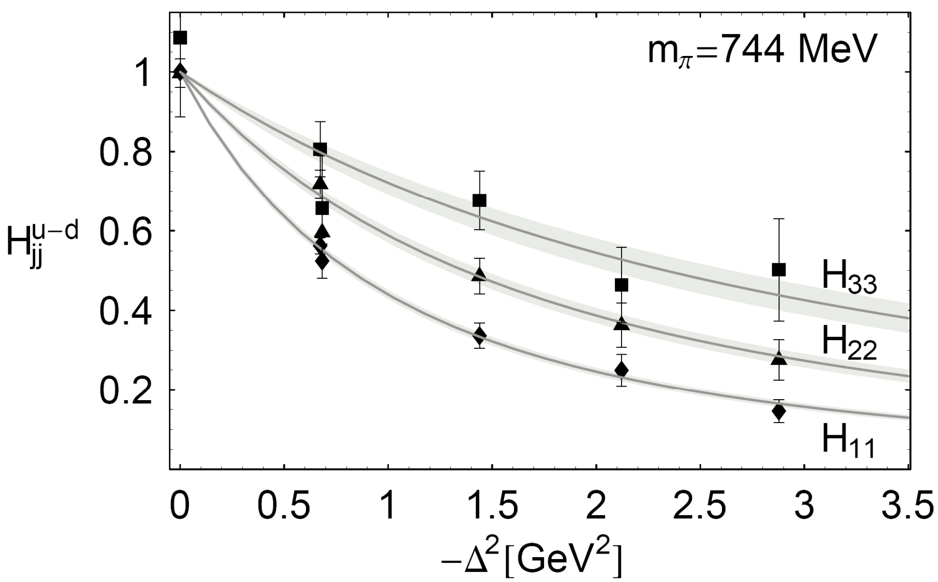
Longitudinal-transverse interplay in GPDs

Skewless GPDs:



Burkardt '02

Support from lattice:



Negele et al.'03

Confinement:

$R_{hadron} \sim \boldsymbol{r}_\perp/(1-x)$

$\langle \boldsymbol{r}_\perp^2 \rangle \sim (1-x)^2$

$\boldsymbol{r}_\perp$-dependence narrows down

Δ^2-dependence becomes flat

Combining small and large-x:

$$\langle r_\perp^2 \rangle = \alpha'(1-x)\log\frac{1}{x} + B(1-x)^2 + \ldots$$

Diehl, Feldman, Jacob, Kroll, '04

Wigner function in quantum mechanics

$$W(p,r) = \int dD\ \mathrm{e}^{-ipD/\hbar}\, \psi^*\left(r - \tfrac{1}{2}D\right)\psi\left(r + \tfrac{1}{2}D\right)$$

Wigner ’32

Contains full information about the single particle wave function.

Properties:

- Real
- It can and most often does go negative: a hallmark of interference!
- Projections lead to probabilities:

$$\int dp\, W(p,r) = \int dq\, \mathrm{e}^{iqr/\hbar} F(q), \quad \int dr\, W(p,r) = n(p)$$

- Operator expectation values:

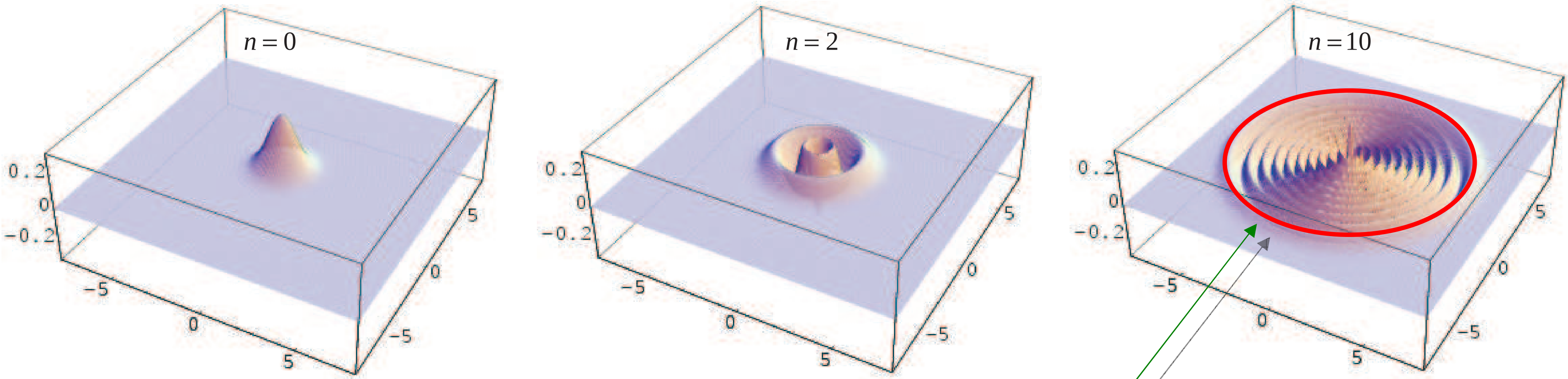
Weyl-ordered

$$\langle A(p,r)\rangle = \int dp\, dr\ W(p,r) A(p,r)$$

The quantum-mechanical uncertainty principle restricts the amount of localization that a Wigner distribution might have. This yields a “fuzzy” phase-space description of the system compared to the “sharp” determination of its momentum and coordinates separately. Wigner distribution provides an appealing opportunity to characterize a quantum state using the classical concept of the phase space.

Wigner function of 1D harmonic oscillator

$$H(p,x) = \frac{p^2}{2m} + m\frac{\omega^2 x^2}{2}$$



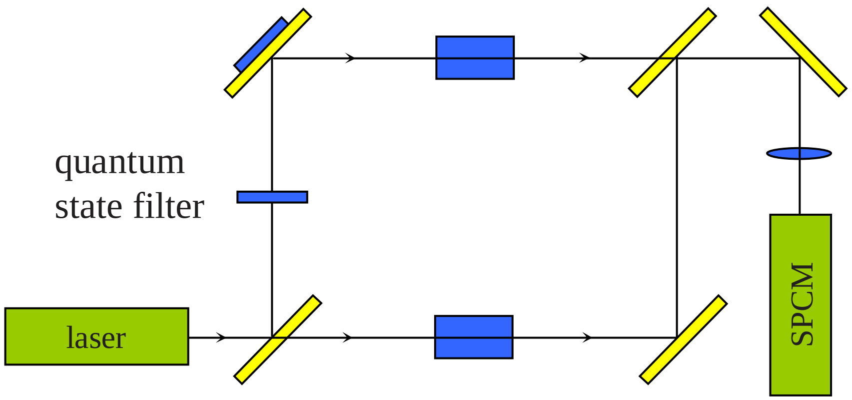
$$\lim_{\hbar\to 0, n\to\infty} W_n(p,x) \sim \delta\big(H(p,x) - E\big)$$

WKB Wigner distribution resides on classical trajectories in phase space:

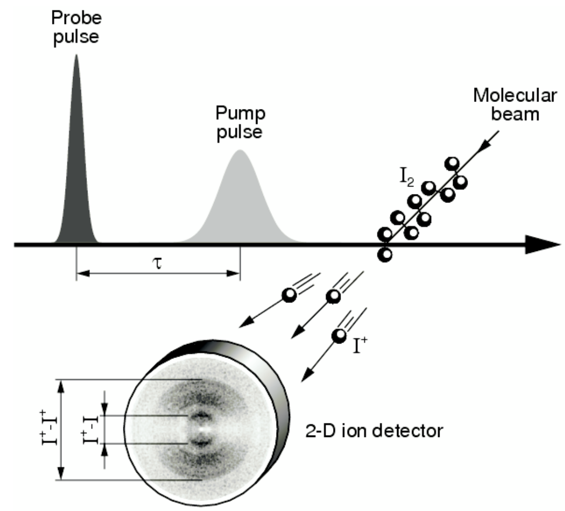
$$\psi = C(x)\mathrm{e}^{iS/\hbar} \quad \Rightarrow \quad W(p,x) = |C|^2\,\delta\!\left(p - \frac{\partial S}{\partial x}\right)$$

Measurement of QM Wigner distributions

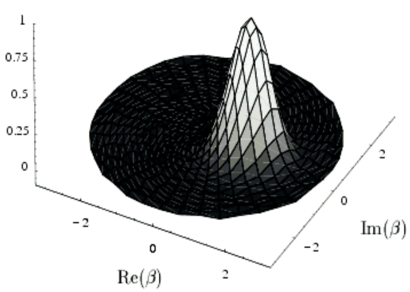
- Mach-Zender interferometry of quantum state of light:



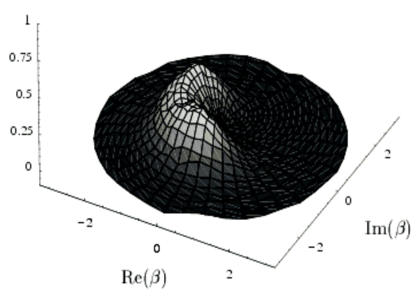
- Quantum state tomography of dissociated molecules:



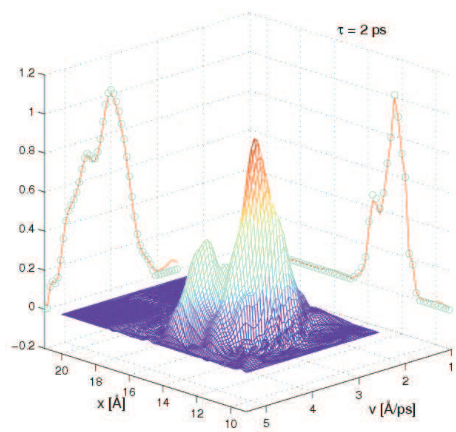
weak coherent state

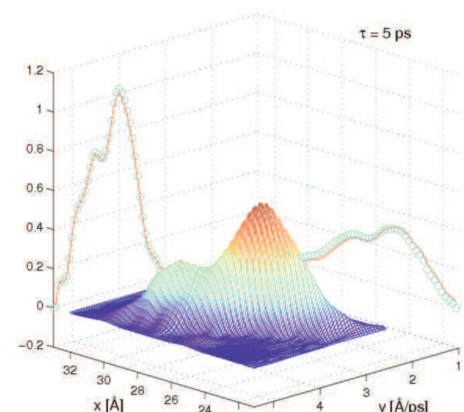


phase-diffused coherent state



Banaszek et al.'99





Skovsen et al.'03

Wigner distributions of the nucleon

Introduce the Wigner operator

AB, Ji, Yuan '03

$$\mathcal{W}\left(\vec{r},k_+,k_\perp\right)\equiv\int d\eta\_d^2\eta_\perp e^{i\left(\eta_-,k_+-\vec{\eta}_\perp\cdot\vec{k}_\perp\right)}\overline{\psi}\left(\vec{r}-\eta/2\right)\Gamma\psi\left(\vec{r}+\eta/2\right)$$

Define quark quasi-probability distribution in the proton (in the Breit frame)

$$W[\vec{r},k_+,k_\perp]\equiv\int\frac{d^3\vec{q}}{(2\pi)^3}\left\langle\vec{q}/2\right|\mathcal{W}\left(\vec{r},k_+,k_\perp\right)\left|-\vec{q}/2\right\rangle$$

- Generalized parton distributions

$$f\,(\vec{r},k_+)=\int\frac{d^2\vec{k}_\perp}{(2\pi)^2}W\,[\vec{r},k_+,k_\perp]$$

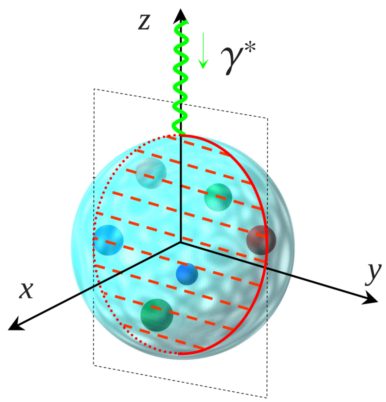
$$=\int\frac{d^3\vec{q}}{(2\pi)^3}F\left(x,\eta,\Delta^2\right)$$

(skewness: $\eta=q_z/2E=q_z\Big/2\sqrt{M_N^2+\vec{q}^{\,2}/4}$)

- Unintegrated parton distributions

$$q\,(x,\vec{k}_\perp)=\int\frac{d^3\vec{r}}{(2\pi)^2}W\,[\vec{r},k_+,k_\perp]$$

Viewing nucleon through momentum “filters”



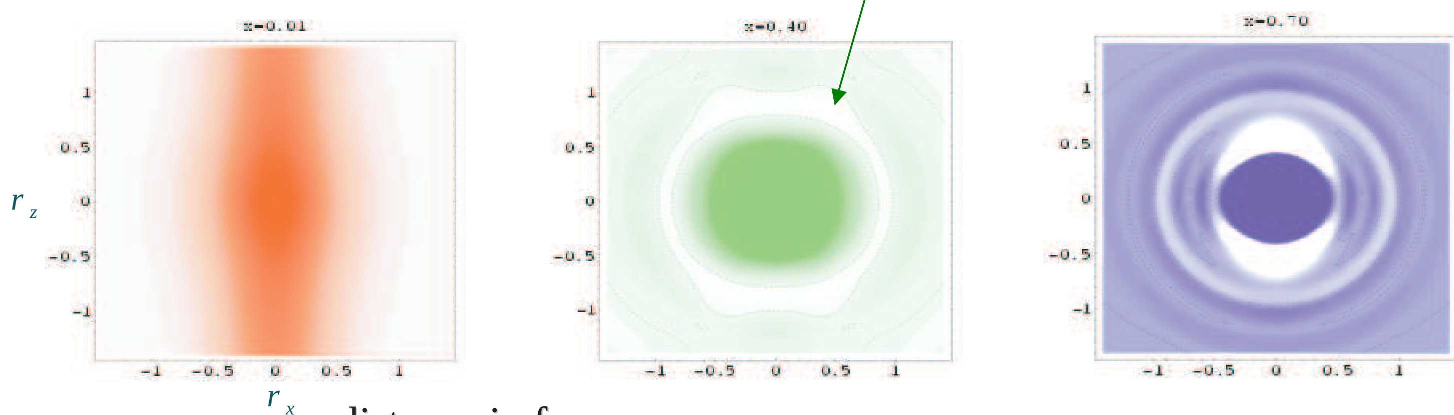
patches of “negative probability”

Feynman momentum:

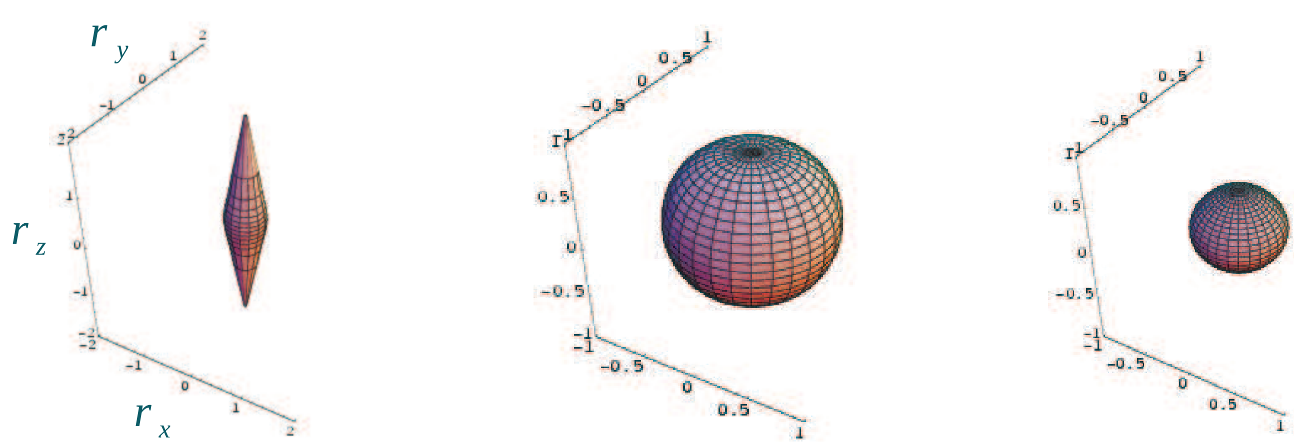
Low

Moderate

High



distance in fm



Limitations on the interpretation

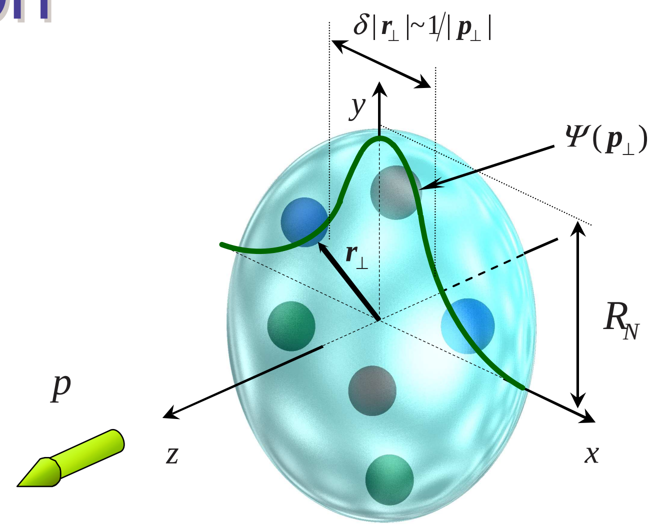
- Transverse dynamics:

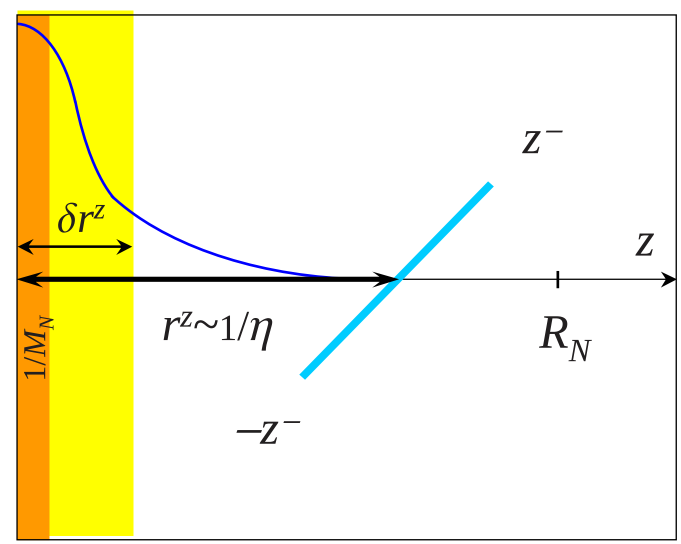
  $1/R_N << |\vec{\Delta}| << |\vec{p}| << M_N$

- Longitudinal dynamics:

  - The longitudinal position of partons is set by skewness:
    $r^z \sim 1/\Delta^z \sim 1/(\eta E)$
  - Typical longitudinal momentum in the wave packet:
    $\delta r^z \sim 1/p^z$
  - The nonlocality of the probe:

    $z^- \sim 1/(xE)$





Constraints:

$$|\Delta^z| << |p^z| << M_N$$

$$|z^-| << r^z \quad \Rightarrow \quad x << \eta$$

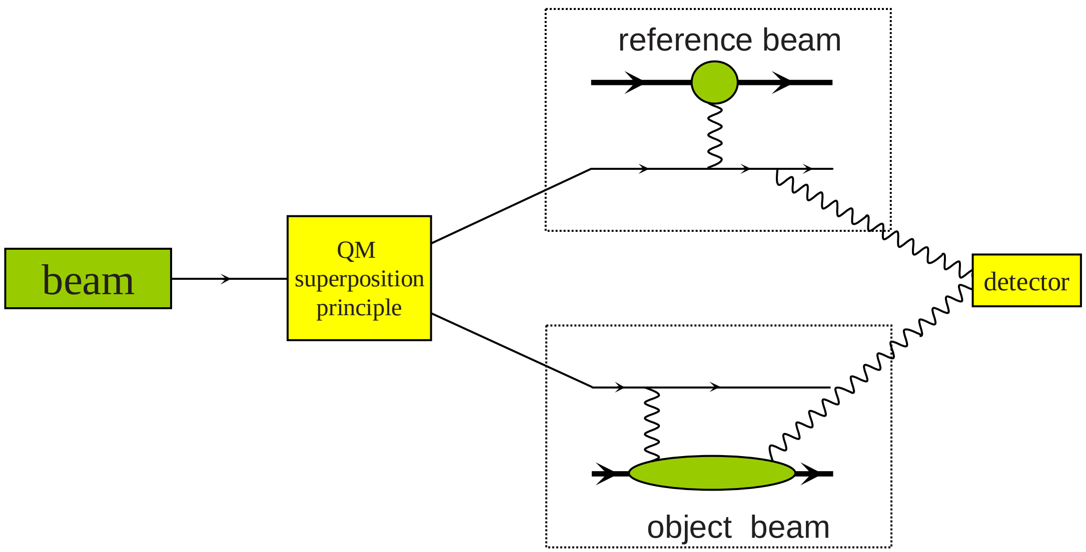
The classical interpretation of GPDs as Wigner quasiprobabilities is valid in deep DGLAP domain!

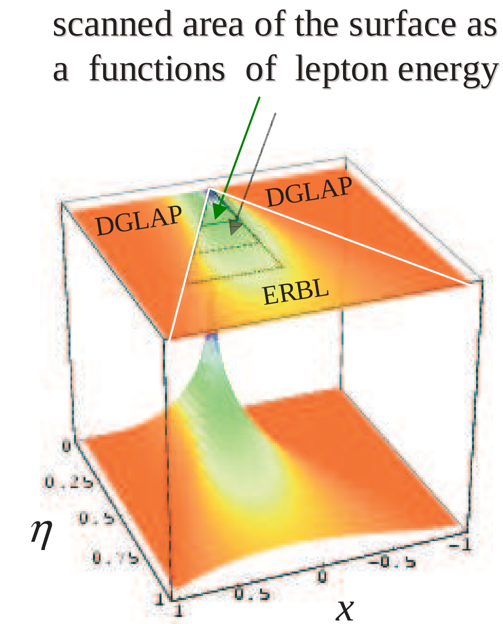
Measurement of nucleon Wigner distribution

Exclusive processes sensitive to GPDs:

- Compton-induced processes: $eN \to e'N'\gamma, \quad \gamma N \to N'l^+l^-, \quad eN \to e'N'l^+l^-, \quad \nu N \to lN'\gamma$
- Hard re-scattering processes: $eN \to e'N'M_l, \quad eN \to e'N'M_h, \quad \gamma N \to N'M_l, \quad \pi N \to N'l^+l^-$
- Diffractive processes: $\gamma N \to (2\,\text{jets})N', \quad \pi N \to (2\,\text{jets})N', \quad \nu_\mu N \to \mu D_s N', \quad lN \to l'\pi\pi N'$

Leptoproduction of a real photon (cf. Mach-Zender interferometry):





$ep \to e'p'l^+l^-$

Parton's Wigner distributions determine 3D structure of hadrons and are measurable via GPDs.

Recent developments

Evolution with Mellin-Barnes representation of GPDs

$$\frac{d}{d\log Q^2} H(x,\eta,\Delta^2;Q^2) = \int_{-1}^{1} \frac{dy}{\eta} K\left(\frac{x}{\eta},\frac{y}{\eta}\right) H(y,\eta,\Delta^2;Q^2)$$

Conformal moments of GPDs do not mix at one-loop order

known to two loops

$$H_n(\eta,\Delta^2;Q^2) = \eta^n \int_{-1}^{1} dx\, C_n^{3/2}(x/\eta) H(x,\eta,\Delta^2;Q^2)$$

- Old-fashioned way: expand in orthogonal polynomials $P_n(x)$

$$H(x,\eta,\Delta^2;Q^2) = \sum_{n=0}^{\infty} P_n(x) \sum_{k=0}^{n} c_{nk} H_k(\eta,\Delta^2;Q^2)$$

Advantage: trivially generalizable to all loops.
Disadvantage: slow, divergent for low x.

- Other "old" methods: effective forward distributions, numerical integration. Vinnikov '06
- The "new" way: invert moments with Melin transform

expressed in terms of Legendre functions

$$H(x,\eta,\Delta^2;Q^2) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dn\, \frac{p_n(x,\eta)}{\sin(\pi n)} E_n(Q^2,Q_0^2) H_n(\eta,\Delta^2;Q_0^2)$$

Advantage: fast and stable evolution, even at small x.
Disadvantage: no obvious generalization beyond leading order.

Mueller, Schaefer '05
Kirch, Manashov, Schaefer' 05

Mellin-Barnes representation of amplitudes

Leading order DVCS amplitude:

$$A(\xi,\Delta^2;Q^2)=\int_{-1}^{1}dx\,\frac{H(x,\xi,\Delta^2;Q^2)}{\xi-x-i0}$$

Mellin-Barnes

$$A(\xi,\Delta^2;Q^2)=\frac{1}{2i}\int_{c-i\infty}^{c+i\infty}dn\;\xi^{-n}\,\frac{2^n\Gamma\left(\frac{3}{2}+n\right)}{\Gamma\left(\frac{3}{2}\right)\Gamma(2+n)}\left[i-\tan\frac{\pi n}{2}\right]H_n(\xi,\Delta^2;Q^2)$$

conformal scheme+higher loops

forward anomalous dimensions

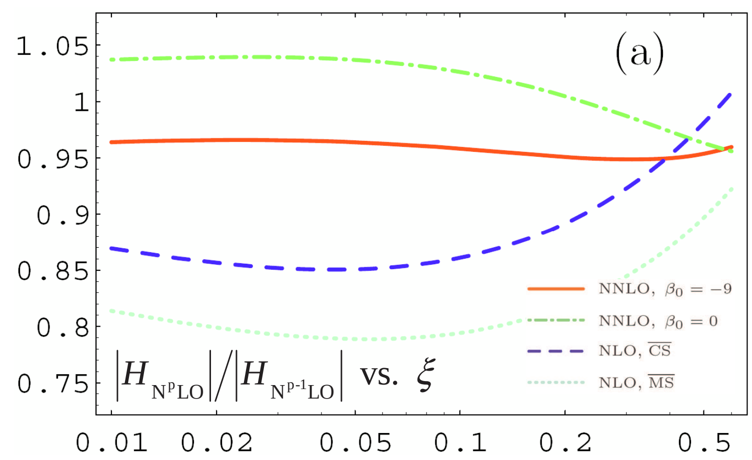
$$A(\xi,\Delta^2;Q^2)=\frac{1}{2i}\int_{c-i\infty}^{c+i\infty}dn\;\xi^{-n}\,\frac{2^n\Gamma\left(\frac{3}{2}+n+\frac{1}{2}\gamma_n(\alpha_s)\right)}{\Gamma\left(\frac{3}{2}\right)\Gamma\left(2+n+\frac{1}{2}\gamma_n(\alpha_s)\right)}\left[i-\tan\frac{\pi n}{2}\right]c_n^{\mathrm{DIS}}(\alpha_s)H_n(\xi,\Delta^2;Q^2)$$

forward coefficient functions

Advantages:

- exact, concise result at NLO
- stable numerics
- an estimate on higher loops in CS

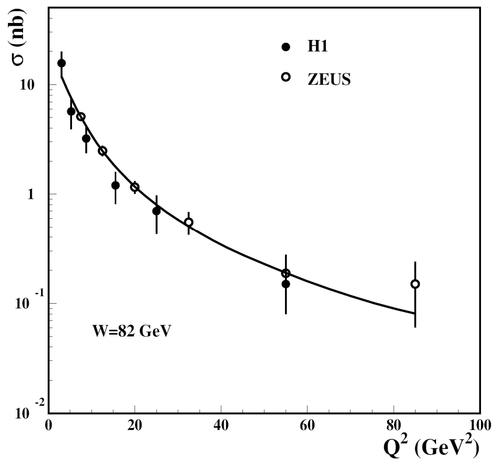
Mueller '06
Kumericki, Mueller, Passek-Kumericki, Schaefer '06

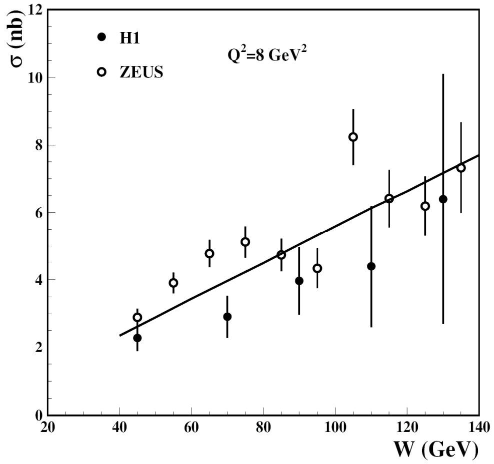


Parametrization of GPDs

- Dual parametrization of GPDs $$H_q(x,\eta,\Delta^2) = (1-\tfrac{x^2}{\eta^2})\sum_{n,l} B_{nl}(\Delta^2)\, C_n^{3/2}(\tfrac{x}{\eta})P_l(\tfrac{1}{\eta})$$

Guzey, Polyakov '05

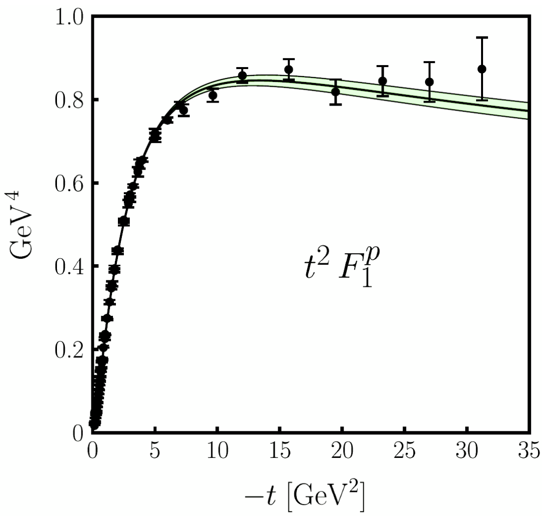




- Global fits (zero skewness, so far)

$$H_q(x,0,\Delta^2) = q(x)\exp\left\{\Delta^2 \left\langle r_\perp^2 \right\rangle / 4\right\}$$

$$\left\langle r_\perp^2 \right\rangle = \alpha'(1-x)\log\frac{1}{x} + B(1-x)^2 + \ldots$$



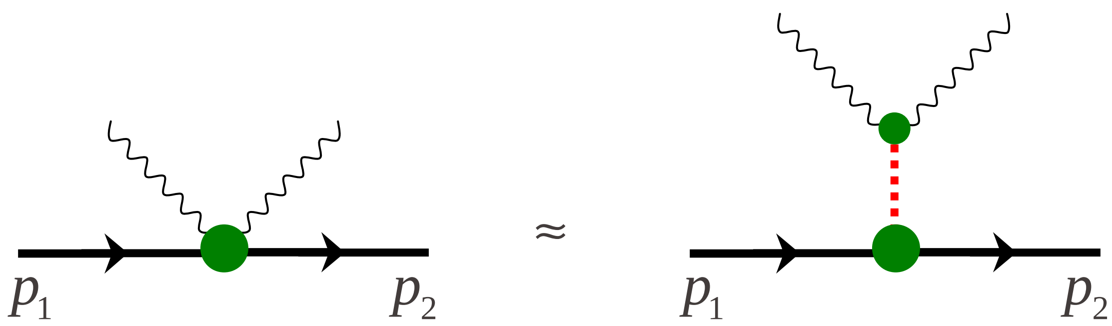
form factors, pdfs with input for longitudinal-transverse interplay

Diehl, Feldman, Jacob, Kroll, '04
Guidal, Polyakov, Radyushkin, Vanderhaeghen'04

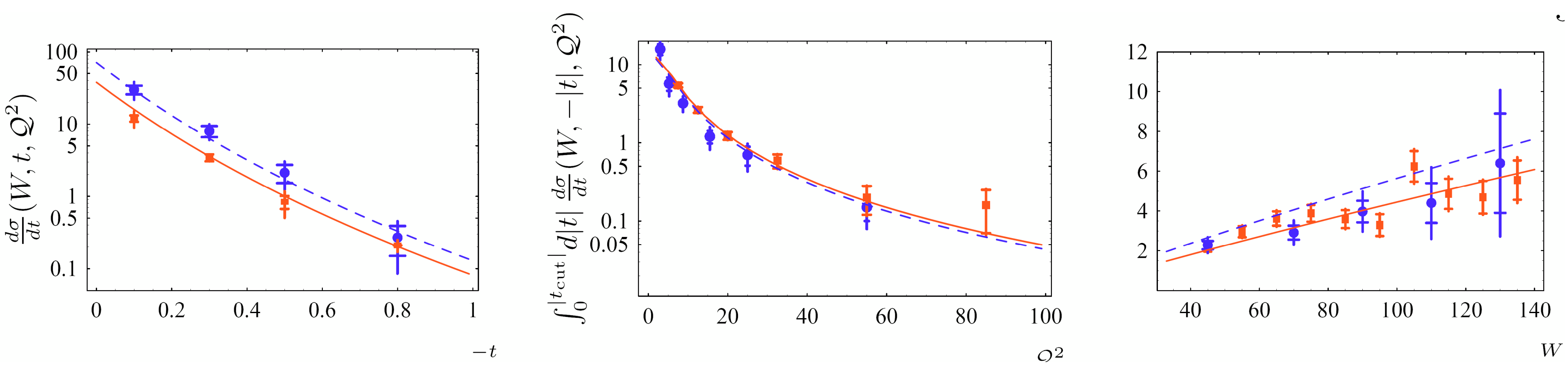
Parametrization of GPDs (cont'd)

- Modeling conformal moments: small-x

Mueller '06



$$H_n(\eta,\Delta^2) \sim \frac{1}{n-\alpha(\Delta^2)}$$



Proton spin crisis ... still?

Quark orbital angular momentum:

$$\left\langle L_q^v \right\rangle = \tfrac{1}{2}\int dx \left(x\, H_q^v(x,\eta,0) + x\, E_q^v(x,\eta,0) - \Delta q(x) \right)$$

- Form factor data:

$$\left\langle L_u^v - L_d^v \right\rangle = -\tfrac{1}{2}(0.77 \div 0.92)$$

$$\left\langle L_u^v + L_d^v \right\rangle = -\tfrac{1}{2}(0.11 \div 0.22)$$

Diehl, Feldman, Jacob, Kroll, ’04

- QCDSF Lattice:

$$\left\langle L_u^v - L_d^v \right\rangle = -\tfrac{1}{2}(0.9 \pm 0.12)$$

$$\left\langle L_u^v + L_d^v \right\rangle = +\tfrac{1}{2}(0.06 \pm 0.14)$$

Schierholtz et al. ’05

- LHPC Lattice:

$$\left\langle L_u^v - L_d^v \right\rangle = -\tfrac{1}{2}(0.25 \pm 0.05)$$

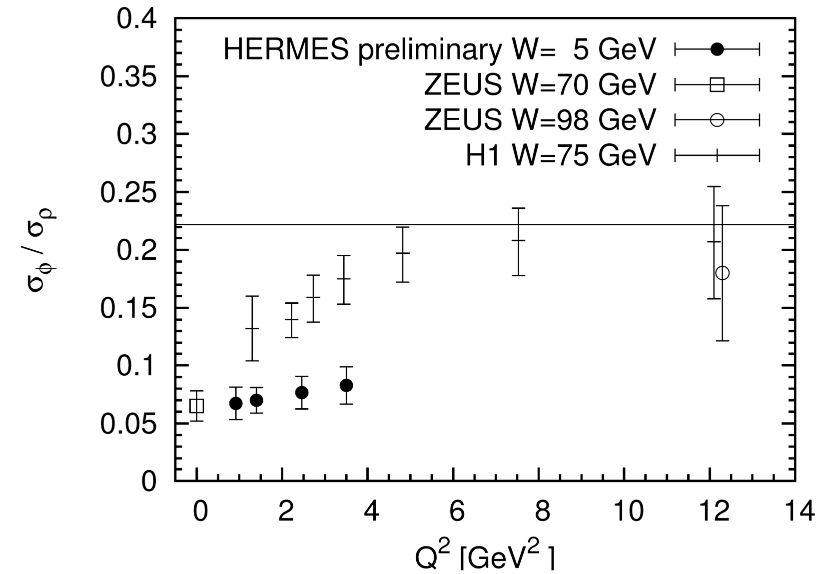
$$\left\langle L_u^v + L_d^v \right\rangle = -\tfrac{1}{2}(0.10 \pm 0.05)$$

Negele et al. ’04

Exclusive hard meson production

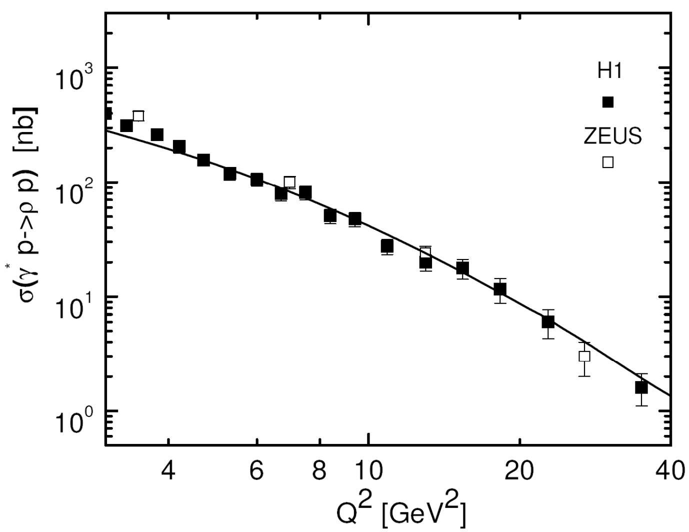
- Significant gluon contribution even in the valence region

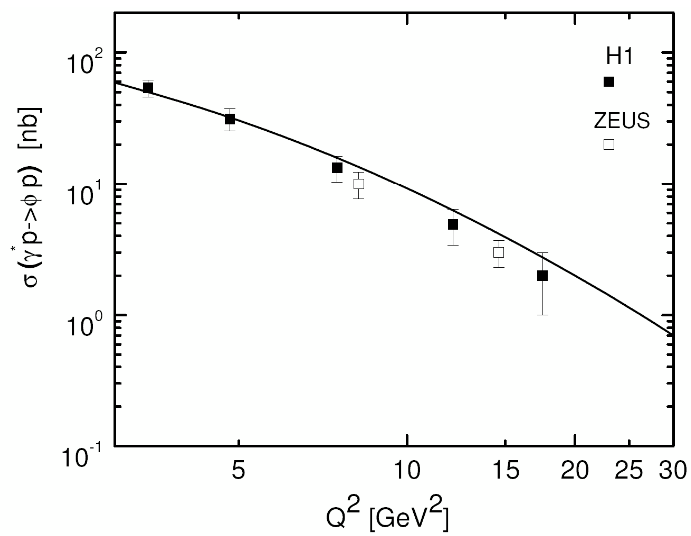
Diehl, Vinnikov '04



- k_T-smearing

Goloskokov, Kroll '05

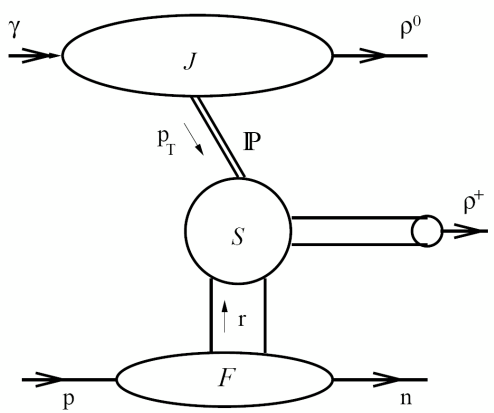


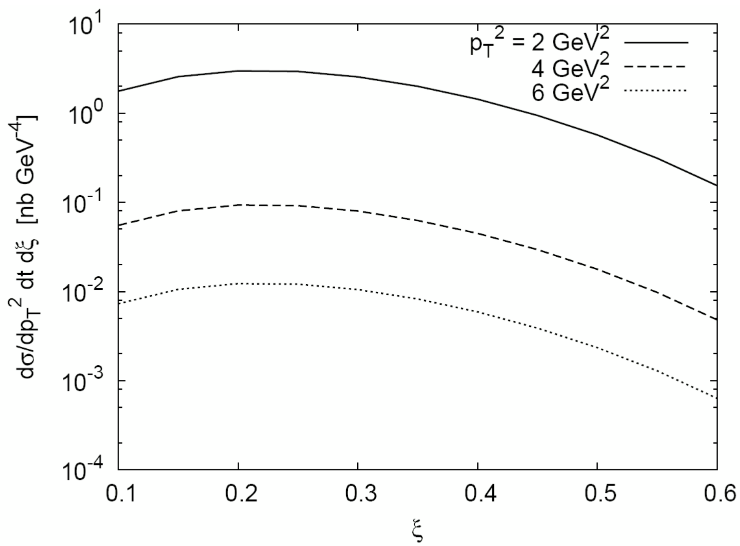


Exclusive production of meson pairs and hybrids

Enberg, Pire, Szymanowski '05

- Sensitivity to transversity GPDs in $\gamma^* p \to \rho^0 \rho^+ n$





- Electroproduction of hybrids (study of the scale-setting procedure):

$\gamma^* p \to \pi_1(1400)\, p$

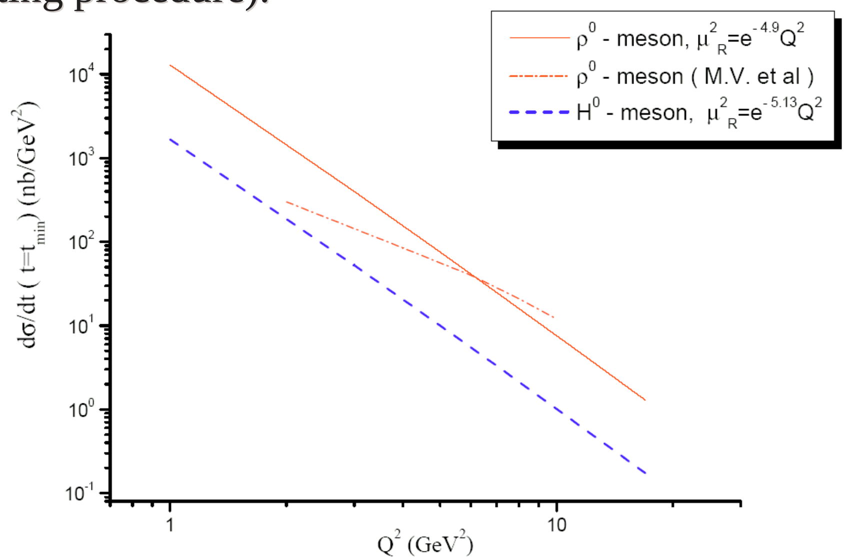
Naively, parton content:

$J^{PC} = 1^{+-} \sim \bar{\psi}\gamma_\mu G_{\mu\nu}\psi$

Nonvanishing twist-two component:

$\bar{\psi}\gamma_{\{\mu}D_{\nu\}}\psi$

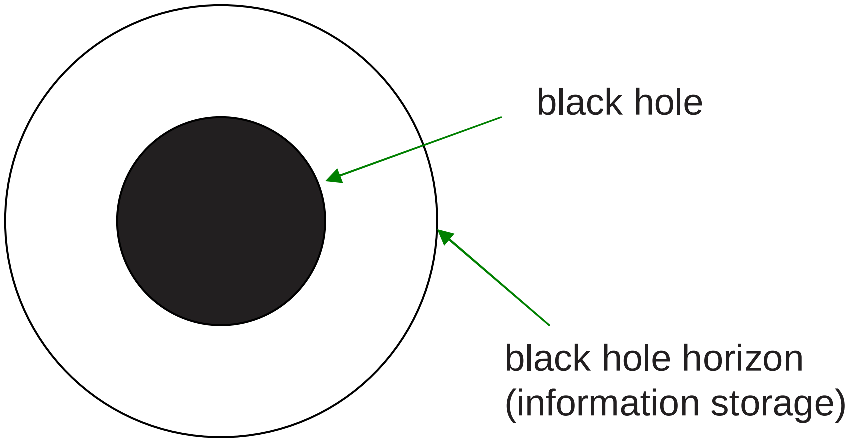
Not suppressed (by powers of hard scale) compared to electroproduction of nonexotics!

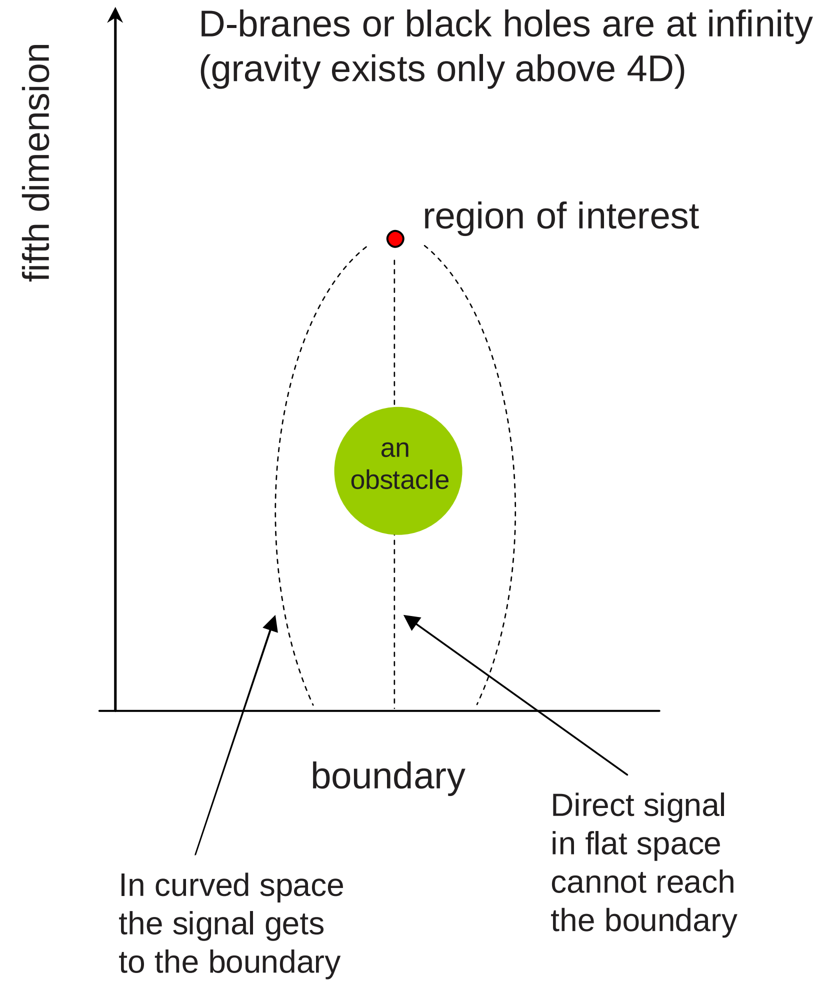


Anikin, Pire, Szymanowski, Teryaev, Wallon '05

AdS/QCD for GPDs?

Holographic principle





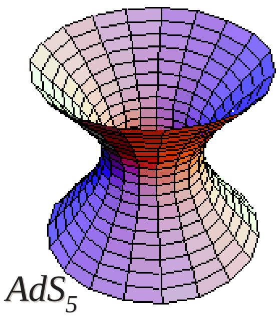
't Hooft and Susskind proposed in the early 1990s that in some description of nature, all of information about the physical state of a system with gravity in a region is stored in terms of a suitable set of variables defined on the boundary (outside the black hole horizon in case of black holes) of the region. The 'boundary' theory is supposed to be an 'ordinary' theory, without gravity. The idea behind the name 'holographic' is that the boundary theory captures a 'hologram' of the contents in the interior, recording the detailed contents of the interior in a subtle fashion in terms of boundary variables.

So in curved space information about the interior of the 5D space is stored on the 4D boundary

Gauge/string correspondence

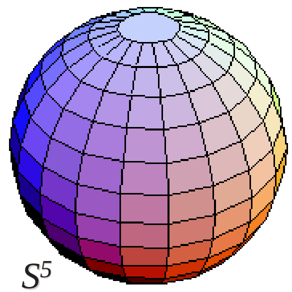
Maldacena'97

$\mathcal{N}$=4 SYM is dual to type IIB string theory on curved superspace:



AdS_5

$\times$

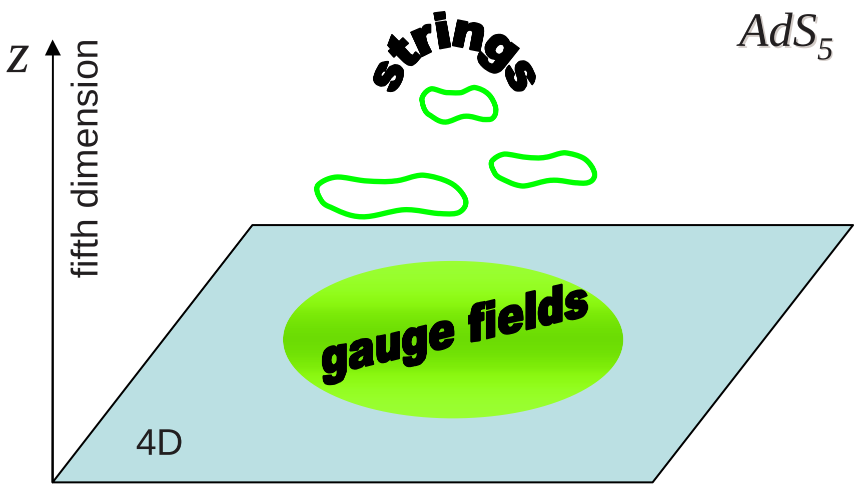


S^5

$$\times\,\mathrm{fermi} = \frac{\mathrm{PSU}(2,2\,|\,4)}{\mathrm{Sp}(1,1)\times\mathrm{Sp}(2)}$$

Strings in 4d behave as if they are living in the 5D space, the fifth (Liouville) dimension being a result of quantum fluctuations.

Polyakov'89



AdS_5 metric:

$$ds^2 = \frac{dx_\mu dx^\mu + dz^2}{z^2}$$

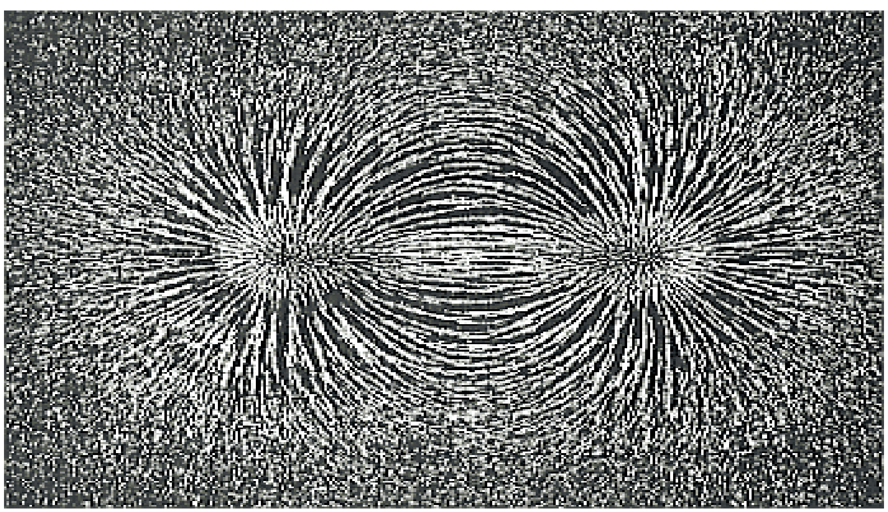
Dictionary:

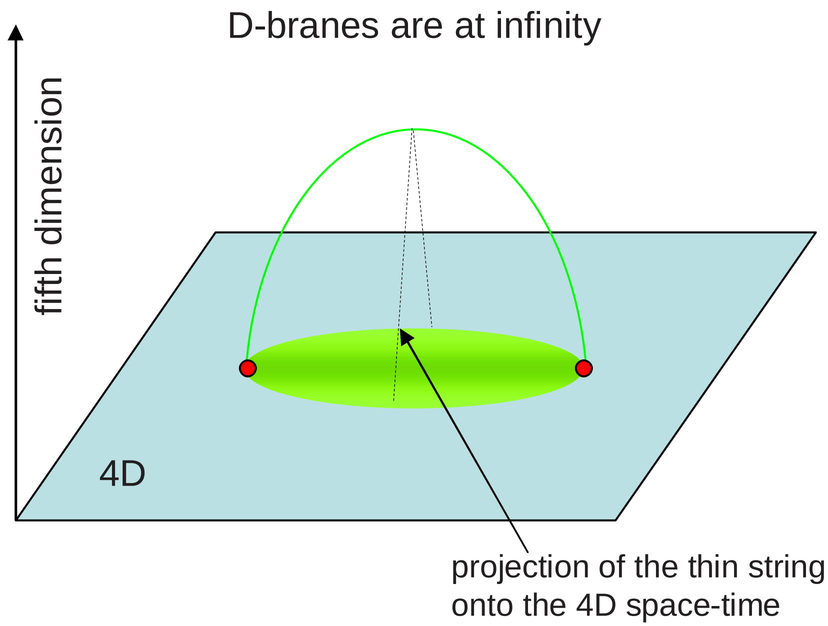
$$\sqrt{\lambda} = R/\alpha', \quad g_s = \lambda/(4\pi N_c)$$

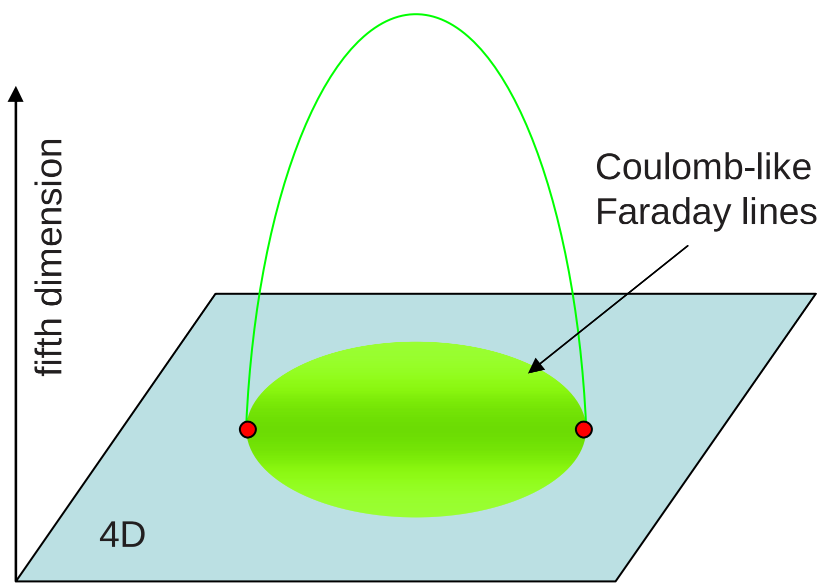
Conformal gauge theory

String moves in 5D space with the usual 4D Minkowski being its boundary. The fifth coordinate goes into the interior of the curved space-time. In this warped 5D space there is a very strong gravitational field pulling objects away from the boundary. Time flows more slowly further from the boundary. As well, the objects with fixed size in the 5D space appear to have different projections on 4D: the string away from the boundary have a larger size than those closer to 4D boundary.

In the lack of dimensional transmutation in conformal gauge theories, they do not enjoy quark confinement.

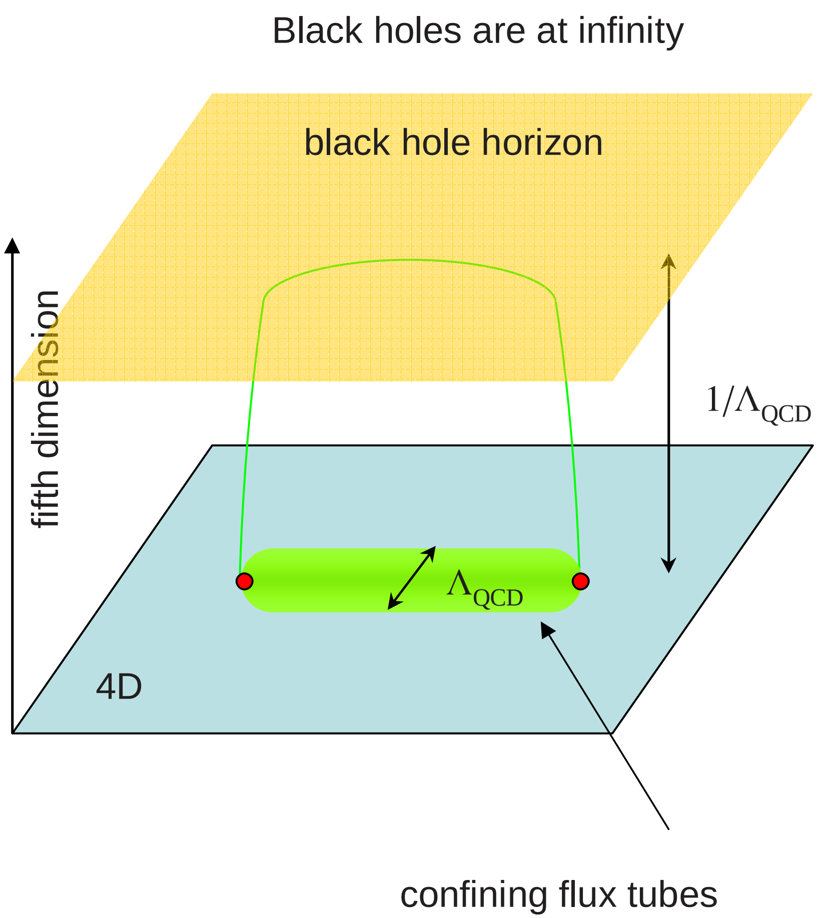


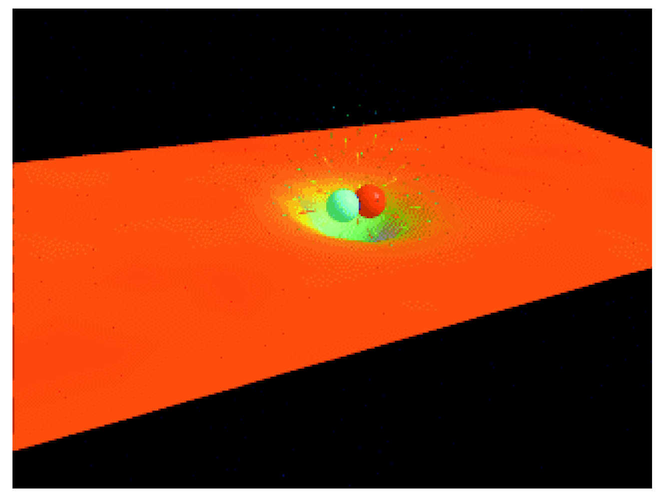




Confining gauge theory

In the presence of a confining black-hole background, string does not penetrate into the bulk at arbitrary distances. Therefore, the projection on the boundary has an intrinsic width proportional to the distance from the boundary to the IR brane.





animation by Leinweber

Operator-state correspondence

normalizable supergravity modes ⟺ low-spin hadrons

non-normalizable supergarvity modes ⟺ low-spin probes ("electromagnetic current", graviton)

E.g.,

$$S=\int d^5x\sqrt{\det g}\left[-|DX|^2+3|X|^2-\frac{1}{4g_5^2}(F_L^2+F_R^2)\right]$$

Confinement is implemented with a hard IR wall: 0<z<Λ_{QCD}.

Erlich, Katz, Son, Stephanov '05

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

??? ⟺ spin-*N* twist-two operators

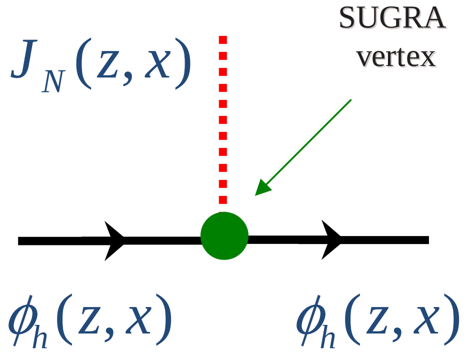
rotating quasiclassical string ⟺ high-spin twist-two operators (or Wilson loops with cusps)

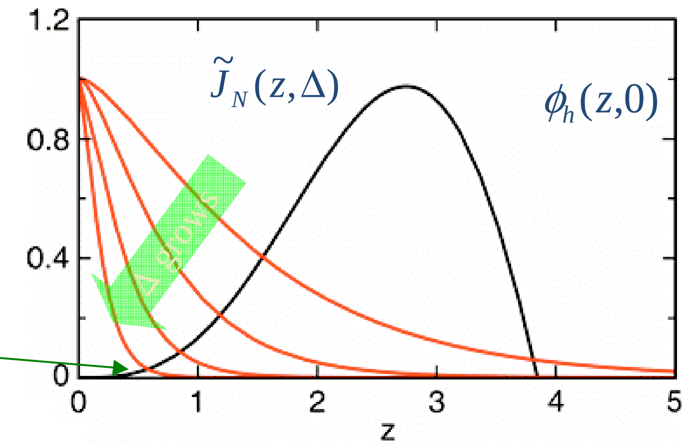
What can be done ...

Study of correlations between the *x*- and transverse momentum dependence.

- Slope of spin-*N* (=1,2) form factors with increasing *N*:

$$F_N(\Delta^2) = \int d^4x\, e^{i\Delta\cdot x} \int_0^{\Lambda^{-1}_{QCD}} \frac{dz}{z^3}\, \phi_h(z,x) J_N(z,x) \phi_h(z,x)$$

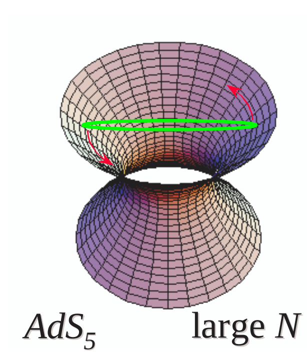




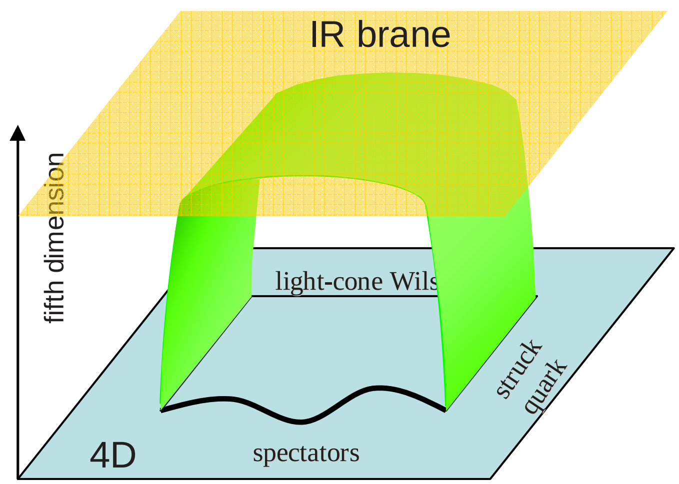
spin-0, 1/2, 3/2 SUGRA modes $\phi_h(z,0) \sim z^{-d}$

- GPDs at large-*x*: rapidly rotating string in AdS

  or area of the Wilson loop in AdS



or



Summary

- Strong correlations between momentum fraction and transverse momentum dependence
- Model-independent quantitative description of (moments of) GPDs (via lattice)
- Successful combined multiparameter analyses to fit skewless GPDs
- Better knowledge of higher order effects in exclusive processes
- Faster leading order evolution routines (based on inverse Mellin transform)
- The final word is yet to be said by experiments