KOPIO

Technical Note #126

The BSM reach of KOPIO

Laurence Littenberg

Physics Department
Brookhaven National Laboratory
Upton, N.Y. 11973 USA

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1. Introduction

The ability of KOPIO to falsify non-Standard Model predictions of $B(K_L \to \pi^0 \nu\bar{\nu})$ becomes greater as the experiment's sensitivity increases. This note studies the power of the experiment to limit the branching ratio at various confidence levels and with varying assumptions.

2. Assumptions and Ingrediants

The question being answered is the following: "if the Standard Model (SM) is correct, what branching ratio can be eliminated at a given confidence level at a given sensitivity?" In general, one can eliminate a branching ratio above a certain level and also a branching ratio below a certain level. However the lower level will initially be 0 and may never become finite. There's always a band encompassing the SM level that can't be eliminated.

In this note, I use only simple statistics, no maximum likelihood methods are exercised. However an attempt to mock up the effect of such a method is made, after the basic exposition.

One needs the following input:

- S, the number of SM signal events for the entire experiment
- B, the number of background events for the entire experiment
- σ the standard deviation of the measured number of events
- N_σ, the number of standard deviations by which one wants to eliminate branching ratios
- ϵ_S, the relative systematic error on the signal
- ϵ_B, the relative systematic error on the background.
- b, the ratio of the predicted BR to the SM BR
- f, the fraction of the total sensitivity of the experiment

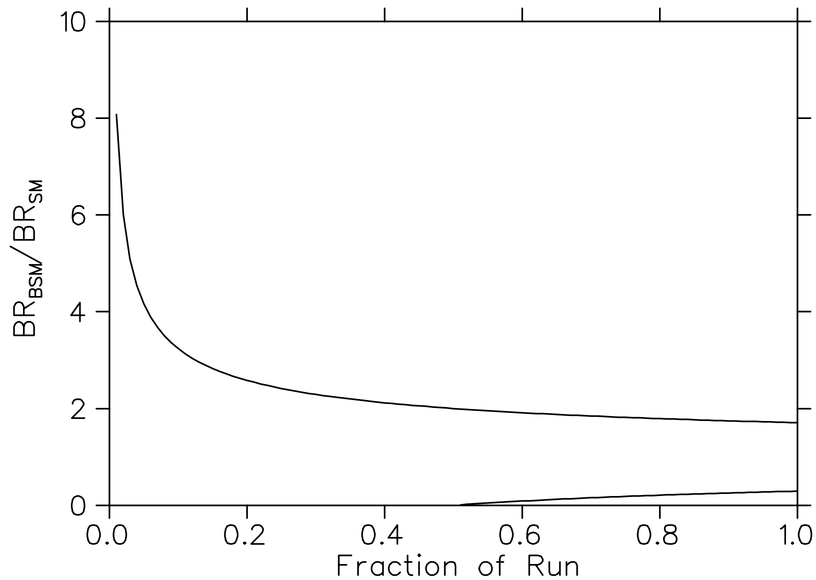

Figure 4.1: Five σ limits on $B(K_L \to \pi^0 \nu\bar{\nu})$ from KOPIO vs fraction of KOPIO run. Assumes SM is correct.

3. Formulae

For a given sensitivity and a given number of events observed, one asks how great a fluctuation is required for the SM to have produced that number. The N_σ for the fraction, f of the experiment, is reached when

$$bSf = Sf \pm N_\sigma \sigma \tag{3.1}$$

In the absence of systematic errors, $\sigma = \sqrt{fS + fB}$. If systematic errors are present, the expression for σ becomes more complicated:

$$\sigma = \left(fS + fB + (\epsilon_S fS)^2 + (\epsilon_B fB)^2\right)^{\frac{1}{2}} \tag{3.2}$$

Plugging Eq. (3.2) into Eq. (3.1), one can obtains for b:

$$b = 1 \pm N_\sigma \left(fS + fB + (\epsilon_S fS)^2 + (\epsilon_B fB)^2\right)^{\frac{1}{2}} / fS \tag{3.3}$$

4. A simple example

To illustrate the general features of the solutions of Eq. (3.3), we take a simple example in which there are no systematic errors, $S = 200$ and $B = 600$. We ask for the 5σ limits for the whole experiment, i.e. $f = 1$. Then Eq. (3.3) becomes $b = 1 \pm 5\sqrt{800}/200$, whose solutions are $b = 1.71, -0.29$. Thus, if $B_{SM}(K_L \to \pi^0 \nu\bar{\nu}) = 3 \times 10^{-11}$, one can eliminate branching ratios greater than 5.12×10^{-11} and less than 0.88×10^{-11} at 5σ.

Fig. 4.1 shows the result as a function of f. The region below the top curve or between the top and bottom curve where the latter exists cannot be ruled out at the 5σ level. Although this is a very simplifed case, it shows several features that are generic:

1. The progress in improving the limit is very fast in the first 10-20% of the run.
2. After this the progress slows and becomes almost asymptotic.
3. There's no lower limit until well into the run (applying the formula gives a negative result).

5. What would the Junk method give?

Since I would like to produce a best guess estimate, I need to approximate the result of a maximum likelihood analysis. I do this by increasing S without increasing B according to the following recipe. We know that for the full experiment the Junk method gives a 12% determination of the BR whereas simple statistics would give 14%. In fact the $S = 200$, $B = 600$ assumed here gives 14%. So the question is what value of S, would give 12%, using simple statistics. This can easily be determined to be $S = 262$. Therefore, using this value of S will be my approximation to what the Junk method would give. Fig. 5.1 shows the improvement given by this.

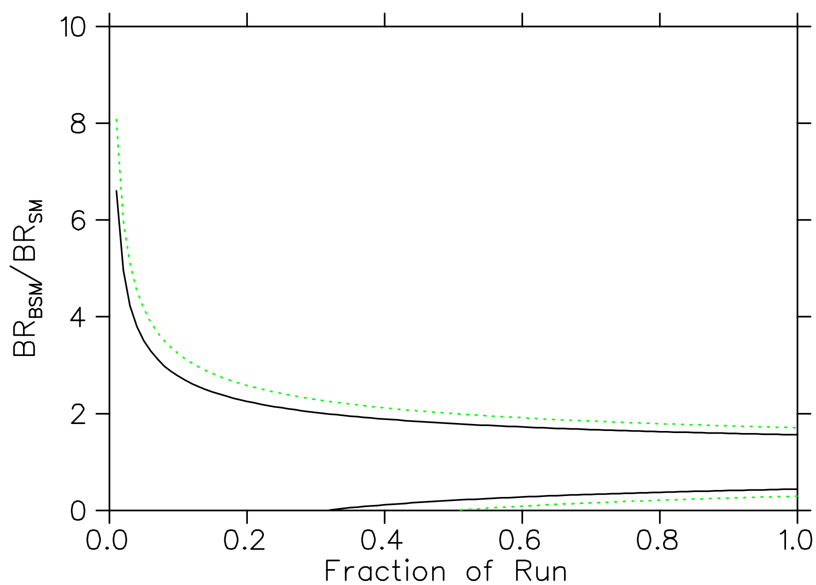

Figure 5.1: Five σ limits on $B(K_L \to \pi^0 \nu \bar{\nu})$ from KOPIO vs fraction of KOPIO run. Assumes SM is correct. The solid curve is the result with $S = 262$ and $B = 600$. The dashed curve is the same as that of Fig. 4.1.

6. Systematic error

Fig. 6.1 shows the effect of including systematic error. For the first part of the run, the systematic error has a very small effect, but as the run progresses the effect becomes more and more important. In the present example, by the end of the experiment, a 10% systematic error raises the limit on new physics by $\sim 35\%$. In what follows, I will assume that we can eventually reduce the systematic error to around 5% on both signal and background.

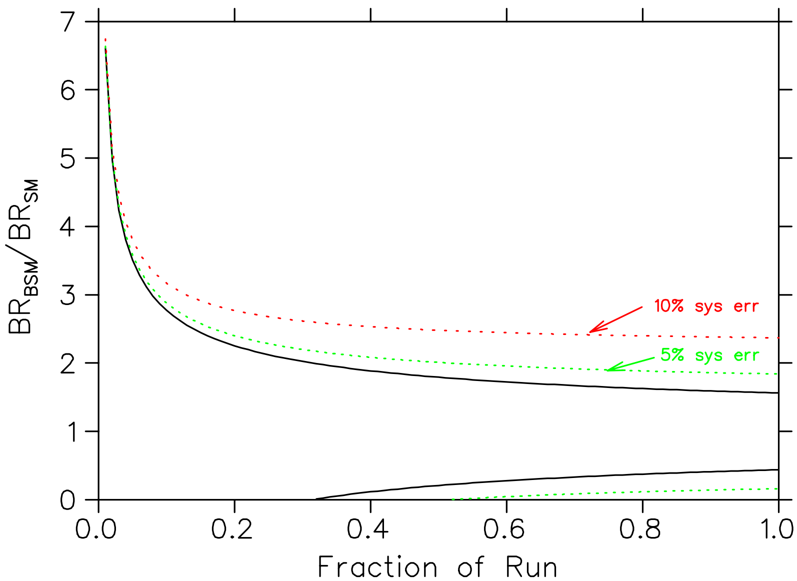

Figure 6.1: Five σ limits on $B(K_L \to \pi^0 \nu \bar{\nu})$ from KOPIO vs fraction of KOPIO run. Assumes SM is correct. The solid curve is the result with $S = 262$ and $B = 600$ with no systematic error. The green dashed curve is the result of including a 5% systematic error on the signal and on the background. The red dashed curve is the result of adding a 10% systematic error.

To deterimine the influence on the result of the various possible factors, I vary each of S, B, ϵ_S and ϵ_B by +10% to see its effect on the second term of Eq. (3.3). These are shown in Table 6.1. Note that the 10% increase in the ϵ's bring them from 0.05 to 0.055.

Table 6.1: Result of increasing parameters by 10%.

input	initial value	result
S	262	-7.7%
B	600	+6.2%
ϵ_S	0.05	+0.9%
ϵ_B	0.05	+4.8%

Not surprisingly it's good to increase the signal and bad to increase any of the other parameters.

7. If not 5σ, then what?

Fig. 7.1 shows the 3σ and 7σ limits compared with the 5σ limits we've been considering so far. The results are not terribly surprising.

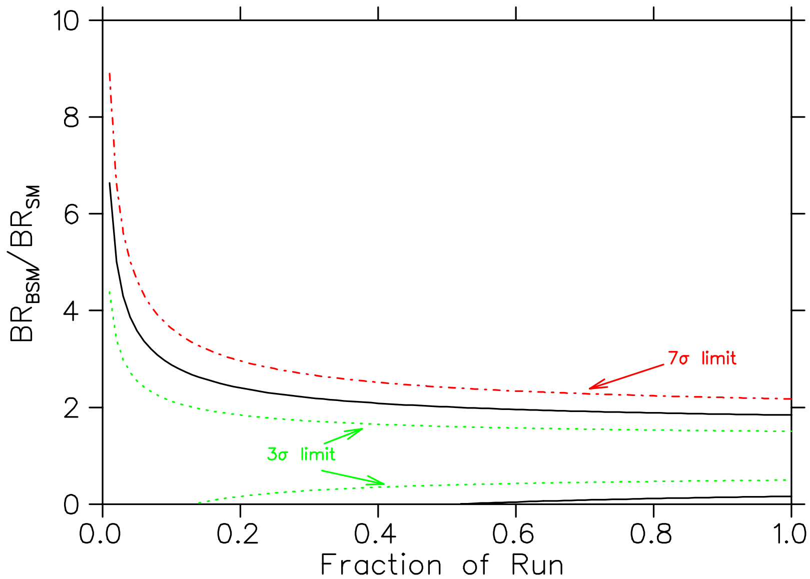

Figure 7.1: Three, five, and seven σ limits on $B(K_L \to \pi^0 \nu \bar{\nu})$ from KOPIO vs fraction of KOPIO run. Assumes SM is correct. Further assume $S = 262$ and $B = 600$ with 5% systematic error on both signal and background. The full black curves are the 5σ limits. The green dashed curves are the 3σ limits. The red dash-dot curve is the 7σ limit.

8. What happens if we see something?

All previous sections dealt with *limits* on new physics in $K_L \to \pi^0 \nu \bar{\nu}$. But that's the booby prize. What if we see a signal? Under the best guess assumptions (approximation to Junk, 5% systematic error), if the SM obtains and the branching ratio is 3×10^{-11}, the relative error on our measurement will go as shown by the black solid curve in Fig. 8.1. The shape indicates that progress will be slow after about a third of the run. However it is not negligible: the relative error improves by about 40% from that point to the end of the run. Note that the relative error for the full run is about 16.8%, not the 12% mentioned earlier. The difference between these two figures is almost entirely due to the 5% systematic error on the background. Clearly it would be useful to beat this error down as far as possible. To study this further one needs to introduce more realistic systematic errors. For example it is likely that there will be an overall scale error, corresponding to the uncertainty in the flux, as well as errors on the acceptance for the signal and for the individual components of the background. Moreover some of the acceptance errors will be correlated between signal and backgrounds.

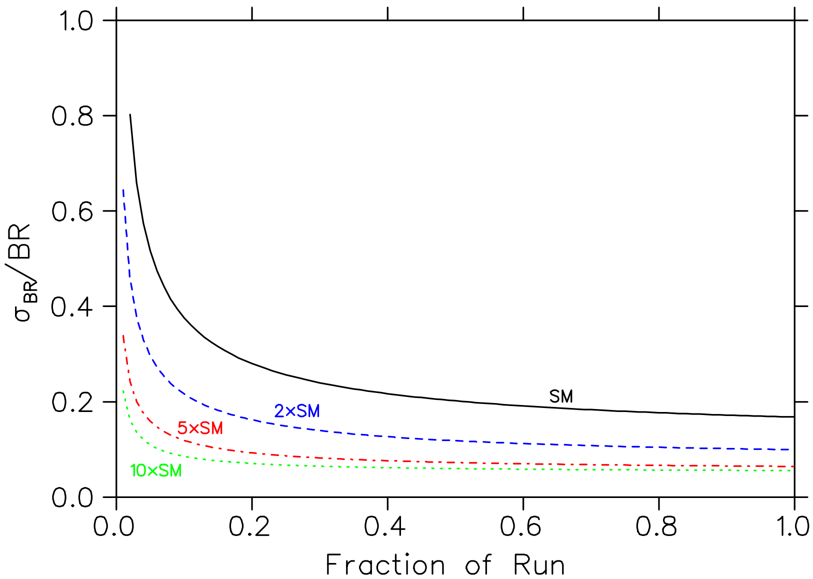

Figure 8.1: Relative error for a SM signal as a function of the fraction of the run for four possible outcomes of the experiment.

I've also shown the resulting relative errors for a signal at 10, 5, and twice the SM branching ratio. If any of these obtain, we won't have to worry about small systematic errors. Even after 20% of the run, it will be clear that a non-SM signal obtains.

9. How low can we go?

David Jaffe asks how low a signal we can see. This depends on the confidence demanded. One simply sets the signal equal to however many σ are desired:

$$S = N_\sigma \left((S+B)/f + (\epsilon_S S)^2 + (\epsilon_B B)^2 \right)^{\frac{1}{2}} \tag{9.1}$$

and then solves the resulting quadratic equation for S. Fig. 9.1 shows the answer for 3σ, 5σ, and 7σ, for the case where systematic errors are ignored. For the full run, at 3σ one can see a signal that is 0.3 of the SM. Fig. 9.2 shows the case for 5% systematic errors on the signal and background. The result is considerably worse. For example for the full run, at 3σ one can see a signal that is 0.47 of the SM.

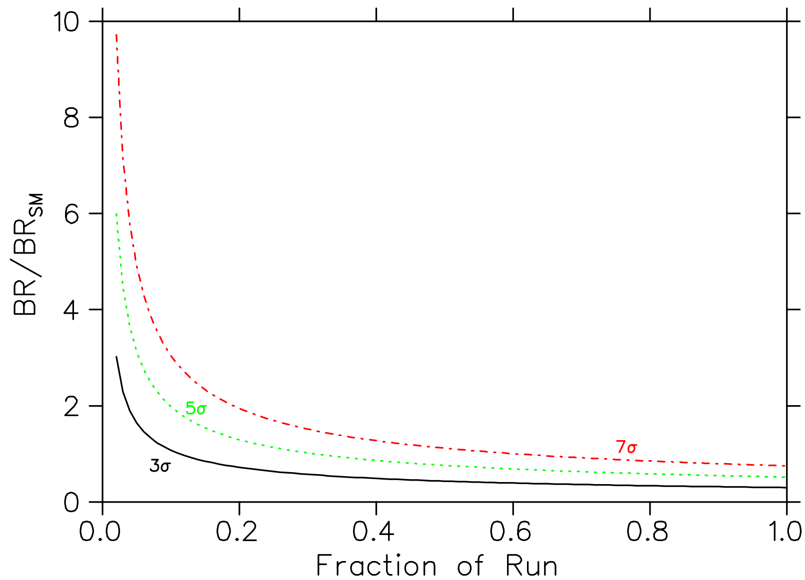

Figure 9.1: Lowest BR that can be observed by KOPIO at 3 (black full line), 5 (green dashed line), and 7σ (red dash-dot line) limit as a function of the fraction of the full KOPIO run. Systematic errors are set to zero.

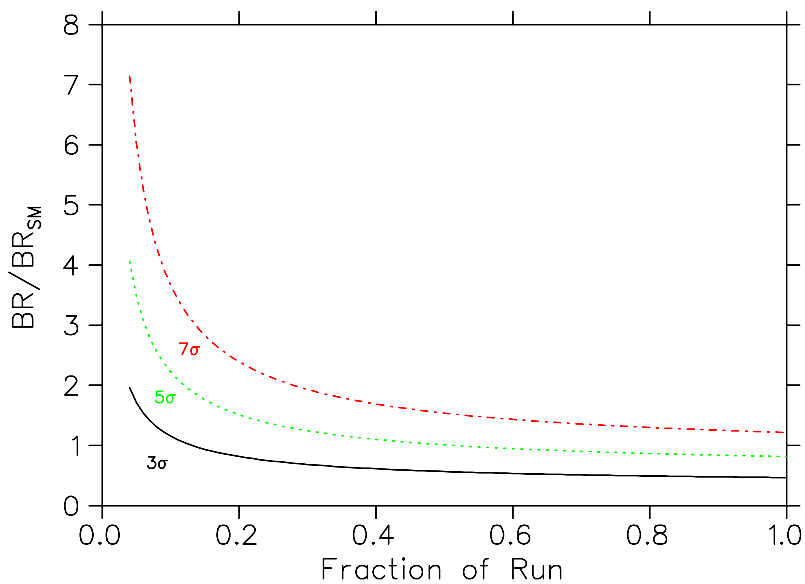

Figure 9.2: Lowest BR that can be observed by KOPIO at 3 (black full line), 5 (green dashed line), and 7σ (red dash-dot line) limit as a function of the fraction of the full KOPIO run. Systematic errors on signal and background are set to 5%.