Collective Flow or Random Walk?

S. Esumi

Physikalisches Institut, Universität Heidelberg, D-69120 Heidelberg, Germany

U. Heinz

Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

N. Xu[†]

P-25, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
e-Mail address: NXu@lbl.gov

Using recently measured transverse momentum spectra from 158 AGeV/c Pb+Pb and 200 AGeV/c S+S collisions at the CERN SPS, we address the question whether the transverse motion of hadrons produced in ultra-relativistic heavy-ion collisions is due to random walk initial state collisions or due to multi-scattering induced collective flow.

The goal of studying ultra-relativistic heavy-ion collisions is to understand hadronic matter under extreme conditions. Transverse momentum distributions are often used to study the properties of hadrons at the stage when interactions among them cease. Comparing the measured distributions with theoretical model predictions, we may gain knowledge of the dynamics at earlier stages of the collision. Very recently, hadron spectra of identified pions, kaons, and protons from the truly heavy-ion collision Pb+Pb have been measured by the NA44 [1–3] and NA49 [4–6] collaborations [1] at CERN. There are two important qualitative features in the data that we wish to point out here:

- For a given collision system, the slope parameter [2] $T_{\rm eff}$ is a function of the particle mass, i.e., the heavier the particle, the higher the $T_{\rm eff}$;

[1] Note that within overlapping regions of acceptance the results from both collaborations are mutually consistent [6].
[2] $T_{\rm eff}$ is extracted by fitting the exponential function $\exp(-m_T/T_{\rm eff})$ to the measured transverse mass distributions.

Preprint submitted to Elsevier Preprint *25 March 1997*

– The value of $T_{\rm eff}$ is larger for heavier collision systems.

Hydrodynamically, matter flows. In other words, particles of different mass all move with similar velocities. Since the particle transverse energy, whose distribution is described by the slope parameter $T_{\rm eff}$, is a combination of thermal motion and transverse collective flow motion, its value then depends on the mass of the particle and the collective flow velocity. Furthermore, because of this velocity dependence, for heavier collision systems, with presumably stronger collective transverse flow, the value of $T_{\rm eff}$ is expected to be larger. The above observations are thus qualitatively consistent with the hypothesis of transverse hydrodynamic flow produced in the heavy-ion collisions [7] and can be quantitatively explained by such a picture [2,3].

It is well known that also in p-A collisions most of the produced particles, from Drell-Yan dileptons and quarkonium resonances to conventional hadrons, show a broadening of their transverse momentum distributions relative to the respective spectra from p-p collisions (see Ref. [8] and references therein). Because the particle density produced in p-A collisions is low and because some of the secondaries have relatively long mean free paths, final-state rescattering among these produced particles cannot explain this behavior. This broadening effect has therefore been interpreted in terms of random walk initial state collisions [9]. The same random walk model has also been compared [9,10] with the transverse momentum spectra from Pb+Pb collisions presented in Ref. [4]. The authors of Ref. [9] conclude that "the results of the random walk model account fully for the observed broadening of the transverse momentum distributions (produced in heavy-ion collisions), so that single-particle spectra require neither collective flow nor a temperature increase."

Since in the hydrodynamic approach collective flow can be directly connected to the equation of state and thus carries information about the earlier stages of collision [11], the importance of identifying and quantitatively measuring the collective flow in heavy-ion collisions cannot be overstressed [7,12–15]. The question of whether the experimental results are caused by collective flow or by random walk initial-state collisions should thus not be taken lightly. In this short report we will discuss a more detailed comparison of the random walk model (RW model) [9] with the recently published pion, kaon, and proton transverse mass ($m_T = \sqrt{p_T^2 + m^2}$) spectra measured in the NA44 experiment [2].

The model of Ref. [9] assumes that each p-p collision in a nuclear collision forms a little thermalized fireball at temperature T, and that multiple initial-state collisions give rise to a Gaussian transverse velocity distribution of these fireballs. The longitudinal velocity distribution is assumed to be boost-invariant. The random transverse velocity distribution of the fireballs gives rise to a p_T-broadening of the thermal particle spectra produced by the decaying fireballs, very much in the same way as the p_T-broadening generated by collective hydrodynamic flow in a locally thermalized fluid. The main difference between the two physical pictures is that in the RW model the transverse velocity of the fireball is not correlated with its position in coordinate space, in contrast to the ordered and directed motion in a hydrodynamic fluid as described by the hydrodynamic flow velocity profile $u^\mu(x)$, and that the width of the random transverse velocity distribution can be calculated within the RW model by extrapolating from p-p and p-A to A-A collisions.

Within the RW model the mid-rapidity transverse mass distribution of hadrons is given by Eq. (19) of Ref. [9]:

$$\left.\frac{dN}{dy\,dm_T^2}\right|_{y=0} = \frac{A\,m_T}{\delta_{AB}} \int d\rho\,\exp\left(-\frac{\rho^2}{\delta_{AB}^2}\right)\,I_0\left(\frac{p_T \sinh\rho}{T}\right)\cosh\rho$$
$$\times \int_{-Y_L}^{Y_L} dY\,\cosh Y\,\exp\left(-\frac{m_T}{T}\cosh Y\cosh\rho\right)\,. \tag{1}$$

A is an arbitrary normalization constant, and Y and ρ are the longitudinal and transverse rapidities of the fireball under consideration. The modified Bessel function I_0 arises from an integration over the azimuthal angle of the transverse flow velocity. For A+B collisions the broadening parameter δ_{AB} is defined as

$$\delta_{AB}^2 = (N_A + N_B - 2)\delta_0^2, \tag{2}$$

with $N_{A,B} \approx \frac{3}{4}(2\pi r_0^2 R_{A,B})n_0$. Parameters $r_0 = 0.8$ fm, $R_A = 1.12\,A^{1/3}$ fm, and $n_0 = 0.17$ fm^{-3} are used in the calculations. The average transverse rapidity kick per collision, $\delta_0 = 0.2$, was determined from p-W collisions [8,9,16]. With this value one finds $\delta_{\rm SS}$=0.26 and $\delta_{\rm PbPb}$=0.44 for S+S and Pb+Pb collisions, respectively.

In Fig. 1 we show the m_T-distributions calculated from Eq. (1) with the above values for the broadening parameter and for an assumed fireball temperature of $T = 150$ MeV and compare them with the data (open circles). While the rough trends of the data are reproduced by the RW model calculation, it clearly fails to correctly give the detailed shape of the transverse mass distributions. For the Pb+Pb collisions, the overall values of the slope parameter from the model are smaller than from the experimental measurement. For the S+S collisions, the situation is slightly better, but the model still misses the larger slope parameter of the proton spectrum. The description of the data by the model is clearly not satisfactory.

We then proceed to address the question of whether different choices of the fireball temperature and the broadening parameter can lead to a more satisfactory description of the data. Changing the broadening parameter is, of course, no longer in the spirit of the RW model, which attempts to predict this parameter for A+A collisions from p-A collisions. It is, however, useful for our conclusions. In Fig. 2 we show as curves, for three choices of the fireball temperature $T = 100, 150$, and 200 MeV, the slope parameters $T_{\rm eff}$ for the calculated m_T-spectra of pions, kaons, protons, and deuterons, as a function of the broadening parameter δ_{AB}. The slope parameters were extracted by fitting the calculated spectra by an exponential in m_T, using the similar fitting range as for the data [2,3], namely $0.2 < m_T - m < 1.0$ GeV for kaons, protons, and deuterons, and $0.2 < m_T - m < 0.8$ GeV for pions. The fit to the measured pion spectra thus excludes the low-m_T contributions from resonance decays that are also not included in the model calculation. As expected, the slope parameter increases as a function of the broadening parameter, and the rate of increase is stronger for heavier mass particles. However, at the predicted value of $\delta_{\rm PbPb} = 0.44$ (see above), the predicted mass splitting of the slope parameters cannot account for the experimental observations from Pb+Pb collisions. The latter are shown in Fig. 2 as the hatched bars centered at $\delta_{\rm PbPb} = 0.44$ (from bottom to top for pions, kaons, and protons). The predicted mass splitting is always too small. [3] Agreement with the data would require values for the broadening parameter that are about a factor 2-3 larger than the values calculated from Eq. (2).

The prediction of the random walk model [9] is based on the additional assumption that, after being formed with the transverse velocity distribution given in Eq. (1), the fireballs decay and hadronize independently, and that the emitted hadrons reach the detector without further rescattering. This is an extreme assumption, which appears unrealistic in collisions between heavy nuclei where the fireballs of Ref. [9] are produced very close to each other and may even overlap in space. The high particle density resulting from these overlapping fireballs will lead to rescattering among the particles inside them and among the particles emitted from different fireballs. The large particle density and pressure gradients from the inside to the outside of the collision region will naturally lead to the build-up of collective flow. [4] In this case, even if the initial state collisions exist, their effect will be washed out and naturally

[3] Of course, a direct comparison between the measured and calculated transverse momentum spectra would be preferable to a comparison of slope parameters. But the latter is more practical for a compact presentation of the results of such a systematic comparison.

[4] In fact, the hydrodynamical flow may already develop at the parton level [17] due to secondary collisions among the partons and strings in the hot and dense region of overlapping fireballs. A quantitative dynamical description of how this collective information evolves through the processes of hadronization and freeze-out and affects the final hadronic observables remains a difficult theoretical challenge [18].

dissolve in the developing collective flow pattern resulting from the frequent final state rescatterings. We believe that our observation, that the strength of the broadening parameter δ_{AB} must be drastically increased beyond the expectation from the p-A extrapolation to achieve even qualitative agreement with the data, supports this picture.

Transverse mass spectra have also been calculated using the hydrodynamical parameterization of Refs. [13,14] and were compared with the same measurements in Fig. 3 of Ref. [2]. With two parameters, the freeze-out temperature T and a transverse velocity v_s that scales the strength of an assumed linearly increasing transverse flow velocity profile, the model results accurately account for all observed spectra, both for S+S and Pb+Pb collisions [2]. Taking $T = 140$ MeV, we find the averaged transverse flow velocity [2,19] to be $\langle v_T \rangle = 0.27c$ and $0.4c$ for S+S and Pb+Pb collisions, respectively. This is in agreement with (3+1)-dimensional hydrodynamical model predictions for both collision systems [20]. When we apply this model to the unpublished NA49 data [5], we find good agreement between data and model calculation. The RWM fails in the low p_T region covered by NA44 while the unpublished NA49 data [5] seem to still be describable by the RWM in the region above $m_T \geq 0.5$GeV, but fails similarly at low p_T [21]. This shows that the RWM has at least the wrong weighting of the transverse velocities: at low m_T the data (in particular those of the heavier hadrons) require a lot more transverse flow than provided by the RWM. The tails of the RWM transverse velocity distribution, however, may be sufficient to describe the heavy ion data at larger m_T, thus providing a smooth transition between dominance of hydrodynamical flow at small m_T and onset of "hard scattering" at larger m_T.

In summary, the observed mass spectra of pions, kaons, and protons emitted at mid-rapidity in central S+S and Pb+Pb collisions cannot be fully reproduced by a superposition of random walk initial-state collisions of the participating nucleons. Such an approach predicts, in both collision systems, a considerably smaller mass splitting of the slope parameters than that observed. By including collective transverse flow, on the other hand, in the framework of hydrodynamical model calculations, one achieves excellent consistency with the experimental data. This study suggests that incorporating collective behavior is essential for our understanding of the experimental transverse momentum distributions.

Acknowledgments: The authors acknowledge exciting discussions with A. Leonidov and H. Satz. This work was supported in part by grants from BMBF, DFG, GSI, and the U.S. Department of Energy.

† Present Address: MS 50A-1148, LBNL, 1 Cyclotron Road, Berkeley, CA 94720, USA.

References

[1] I.G. Bearden *et al.* (NA44 Collaboration), Phys. Lett. **B388**, 431(1996).

[2] I.G. Bearden *et al.* (NA44 Collaboration), Phys. Rev. Lett., in print.

[3] N. Xu *et al.* (NA44 Collaboration), Quark Matter '96, May 20 - 24, 1996, Heidelberg, Germany.

[4] P. Seyboth (NA49 Collaboration), XXXI^e Rencontre de Moriond, March 1996.

[5] P.G. Jones *et al.* (NA49 Collaboration), Quark Matter '96, May 20 - 24, 1996, Heidelberg, Germany.

[6] P. Jacobs *et al.* (NA49 Collaboration), HIPAGS '96.

[7] R. Hagedorn, CERN-TH/3014, January 1981.

[8] H.R. Schmidt and J. Schukraft, J. Phys. **G 19**, 1705 (1993).

[9] A. Leonidov, M. Nardi, and H. Satz, CERN-TH/96-71; BI-TP 96/17; Z. Phys. **C** (1996), in press.

[10] J. Alam and H. Satz, CERN-TH/96-221.

[11] H. Stöcker and W. Gieiner, Phys. Rep. **137**, 277 (1986).

[12] B. Müller, Nucl. Phys. **A590**, 3c (1995).

[13] K.S. Lee, U. Heinz, and E. Schnedermann, Z. Phys. **C48**, 525 (1990).

[14] E. Schnedermann, J. Sollfrank, and U. Heinz, Phys. Rev. **C48**, 2462 (1993).

[15] P. Braun-Munzinger, J. Stachel, J. Wessels, and N. Xu, Phys. Lett. **B344**, 43 (1995); and Phys. Lett. **B356**, 1 (1996).

[16] T. Akesson *et al.* (HELIOS Collaboration), Z. Phys. **C46**, 361 (1990).

[17] K.J. Eskola and K. Kajantie, CERN-TH/96-259, October 1996.

[18] E. Shuryak, private communication, August 1996.

[19] S. Esumi, S. Chapman, H. van Hecke, and N. Xu, submitted to Phys. Rev. **C**, August 1996.

[20] B. Schlei *et al.*, Phys. Lett. **B376**, 212 (1996).

[21] H. Satz, private communications, March 1997.

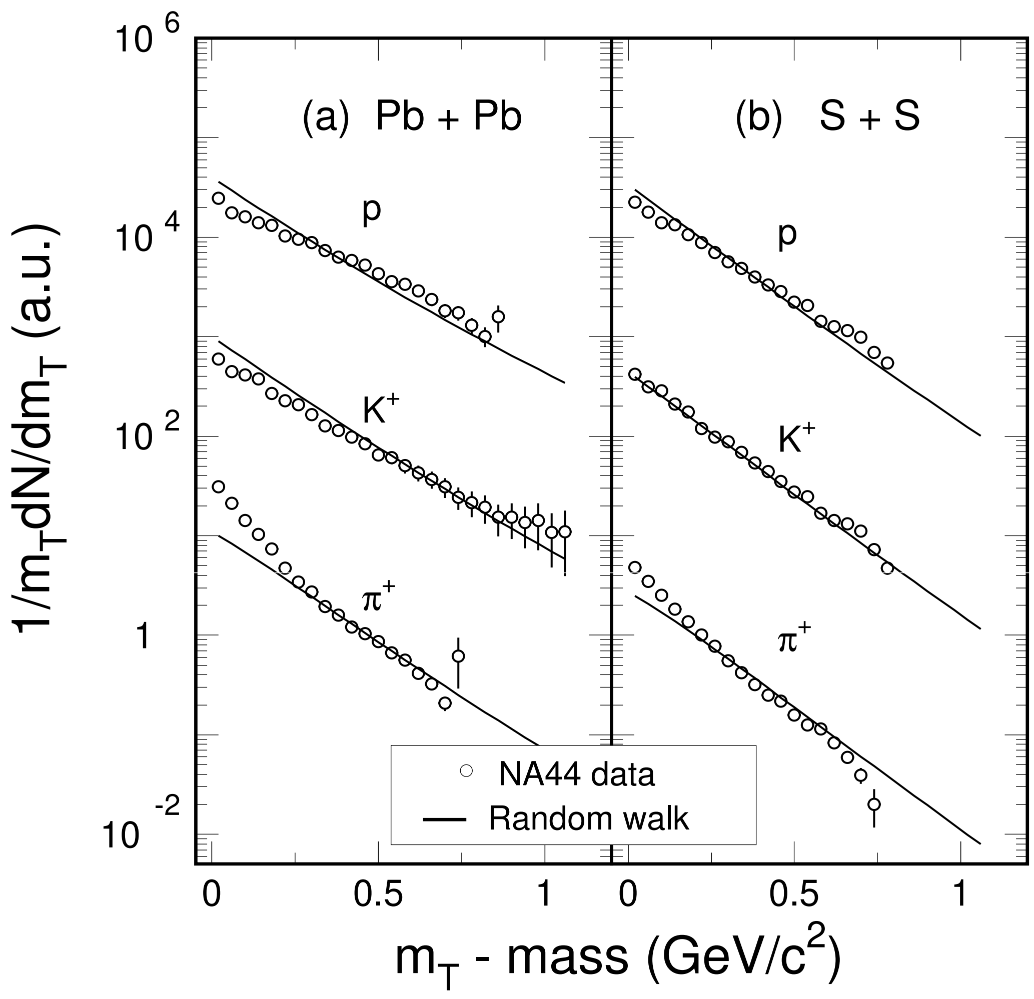

Fig. 1. The invariant cross sections for mid-rapidity pions, kaons, and protons as a function of $m_T - mass$ from (a) 158 AGeV/c Pb+Pb and (b) 200 AGeV/c S+S collisions. Lines represent the predictions of the model discussed in the text.

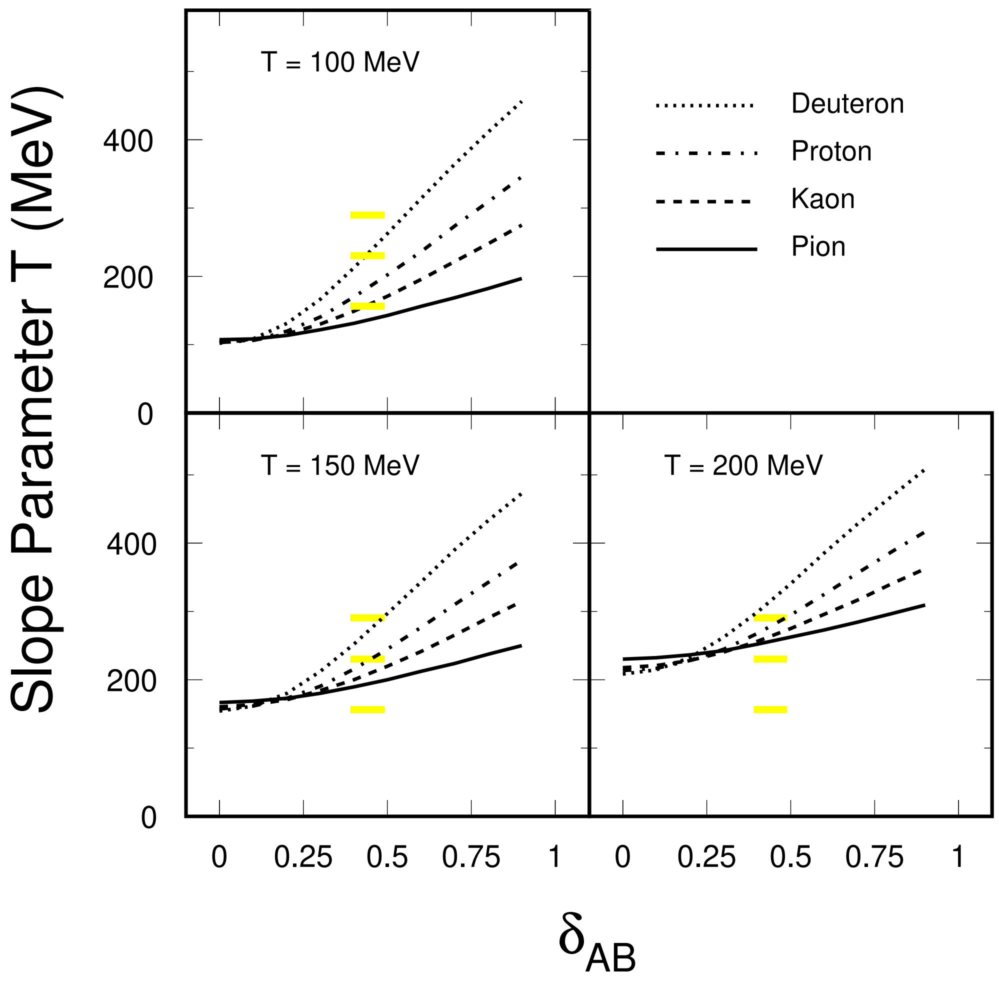

Fig. 2. Slope parameters $T_{\rm eff}$ from the random walk model for three different freeze-out temperatures as a function of the broadening parameter δ_{AB}. The hatched bars represent the measured slope parameters of (from bottom to top) pions, kaons, and protons for central 158 AGeV/c Pb+Pb collisions.