Observation of Isolated Many-Body Resonances in Continuum States

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Hokkaido University
K. K.

1. Resolution of Identity in Complex Scaling Method

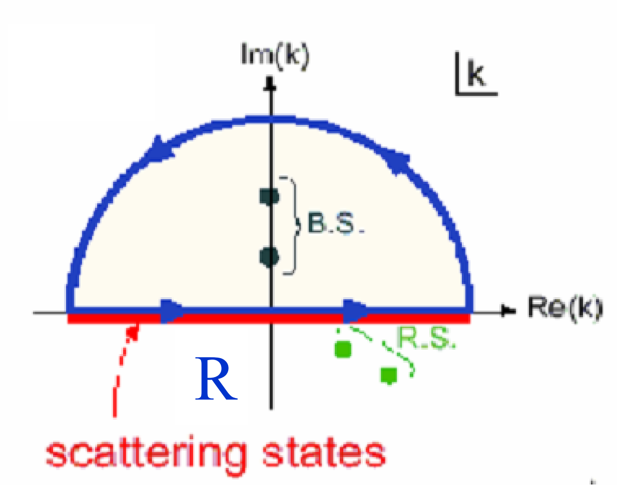
Solutions of Hamiltonian

- **Bound st.** $\xrightarrow[r\to\infty]{} e^{-\kappa r}$
- **Continuum st.**
  - **Resonant st.** $\xrightarrow[r\to\infty]{} e^{+\gamma r}$
  - **Non-Resonant st.**

Completeness Relation (Resolution of Identity)

$$1 = \sum_{n=b} |u_n\rangle\langle\tilde{u}_n| + \frac{1}{\pi}\int_R dk\, |\psi_k\rangle\langle\tilde{\psi}_k|$$

R.G. Newton, J. Math. Phys. **1** (1960), 319

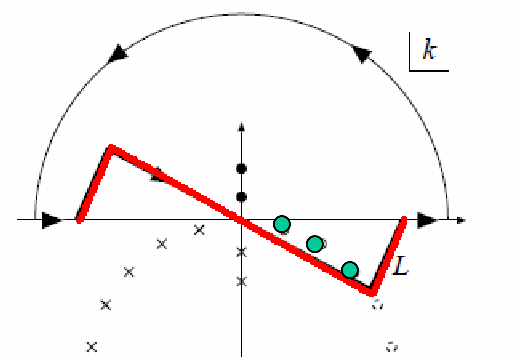


Separation of resonant states from continuum states

$$1=\sum_{n=b}|u_n\rangle\langle\tilde{u}_n|+\sum_{n=r}^{N_r(L)}|u_r\rangle\langle\tilde{u}_r|+\frac{1}{\pi}\int_L dk\,|\psi_k\rangle\langle\tilde{\psi}_k|$$

Resonant states

Deformed continuum states



Deformation of the contour

T. Berggren, Nucl. Phys. A 109, 265 (1968)

Matrix elements of resonant states

$$\langle\tilde{u}_1|\hat{O}|u_2\rangle=\lim_{\alpha\to 0}\int_R dr\, e^{-\alpha r^2}\ \tilde{u}_1^*\hat{O}u_2$$

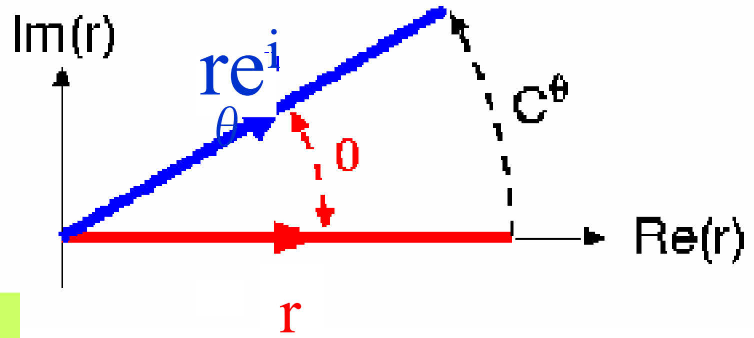
Convergence Factor Method

Ya.B. Zel'dovich, Sov. Phys. JETP **12**, 542 (1961).

N. Hokkyo, Prog. Theor. Phys. **33**, 1116 (1965).

Complex scaling method

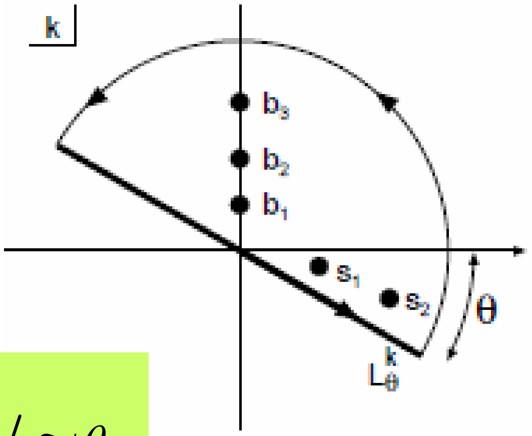
coordinate: $r \to re^{i\theta}$



$$\langle \tilde{u}_1 | \hat{O} | u_2 \rangle = \lim_{\alpha \to 0} \int_R dr\, e^{-\alpha r^2}\, \tilde{u}_1^* \hat{O} u_2$$
$$= \int_{R^\theta} d(re^{i\theta})\, \tilde{u}_1^*(re^{i\theta}) \hat{O}(\theta) u_2(re^{i\theta})$$

B. Gyarmati and T. Vertse, Nucl. Phys. **A160**, 523 (1971).

momentum: $k \to ke^{-i\theta}$



$$1 = \sum_{n=b} |u_n^\theta\rangle\langle\tilde{u}_n^\theta| + \sum_{n=r}^{N_r^\theta} |u_n^\theta\rangle\langle\tilde{u}_n^\theta| + \frac{1}{\pi}\int_{L_\theta^k} dk\, |\psi_k^\theta\rangle\langle\tilde{\psi}_k^\theta|$$

T. Myo, A. Ohnishi and K. Kato. Prog. Theor. Phys. **99**(1998)801]

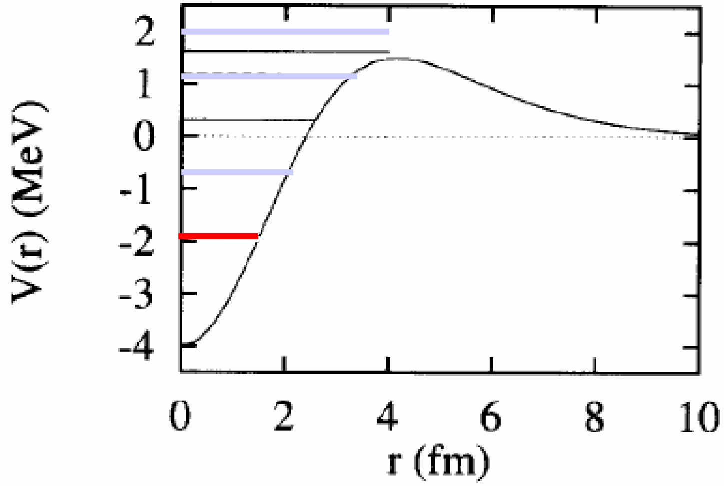


Fig. 1. The employed potential with its low-lying 0^+ (solid lines) and 1^- (dashed lines) states.

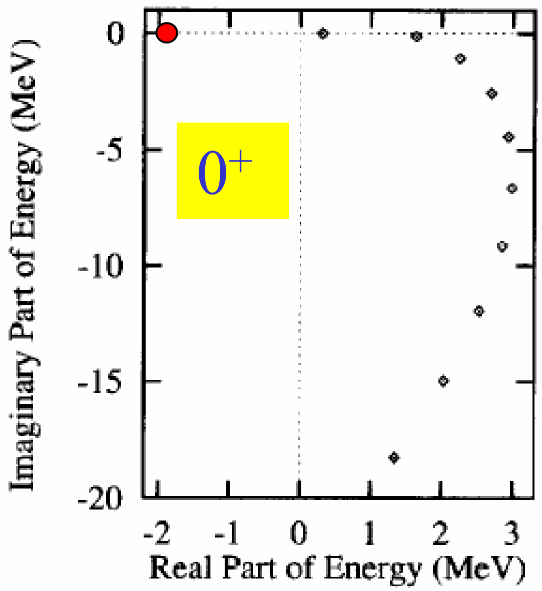


Fig. 2. The eigenvalue distribution on the complex energy plane for 0^+ obtained using the complex scaling method.

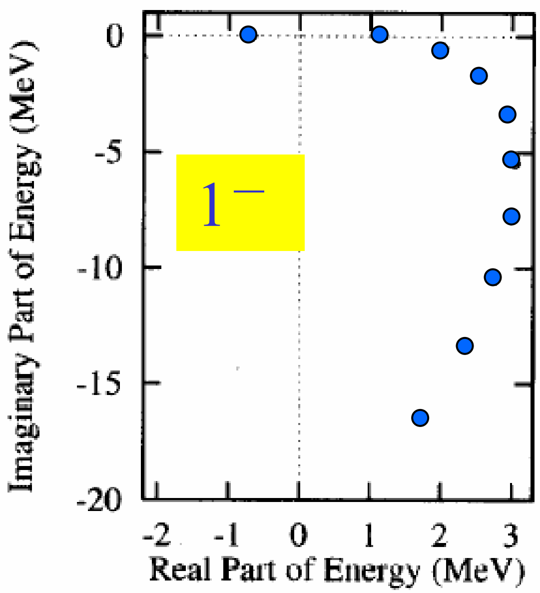
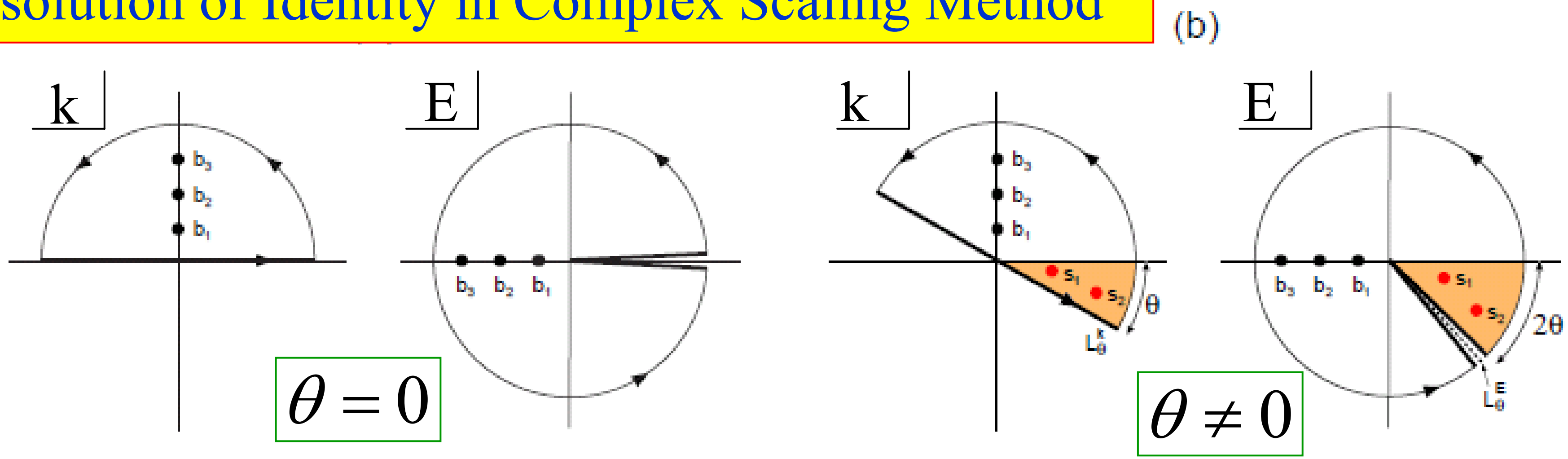


Fig. 3. The eigenvalue distribution on the complex energy plane for 1^- obtained using the complex scaling method.

State	$E(1^-_n)$ (MeV)	$4\pi(E(1^-_n)-E(0^+_1))(\Phi^\theta_{1^-_n}\vert\hat{O}^\theta(E1)\vert\Phi^\theta_{0^+_1})^2$	
1^-_1	-0.67465	1.4939917	← B.S.
1^-_2	$1.1710 - i0.0048642$	$0.0054307 - i0.0001410$	R.S.
1^-_3	$2.0175 - i0.48630$	$0.0007025 + i0.0000021$	
1^-_4	$2.5588 - i1.7378$	$-0.0001465 + i0.0002028$	
1^-_5	$2.9008 - i3.4185$	$0.0000323 - i0.0000881$	
1^-_6	$3.0427 - i5.4629$	$-0.0000187 + i0.0000313$	
1^-_7	$2.9943 - i7.8265$	$0.0000122 - i0.0000071$	
1^-_8	$2.7610 - i10.475$	$0.0000053 - i0.0000011$	
1^-_9	$2.3466 - i13.388$	$0.0000009 + i0.0000019$	
1^-_{10}	$1.7395 - i16.537$	$0.0000004 - i0.0000006$	
Sum		$1.5000002 + i0.0000002$	$S_{exc}=1.5$ e^2fm^2MeV

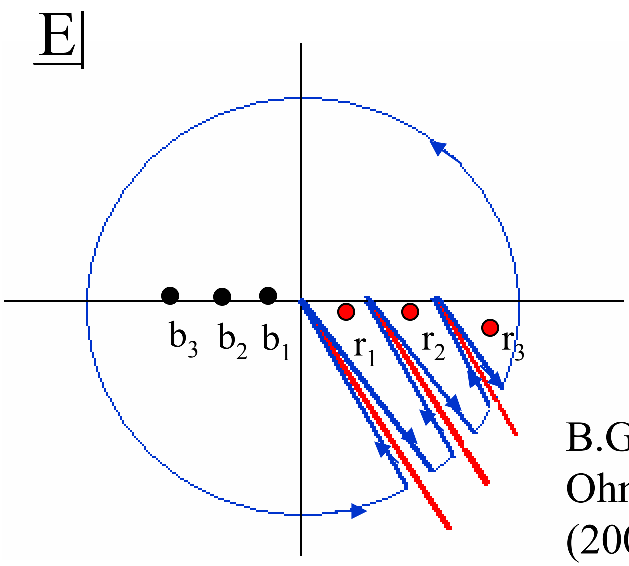
Contributions from B.S. and R.S. to the Sum rule value

Resolution of Identity in Complex Scaling Method



Single Channel system

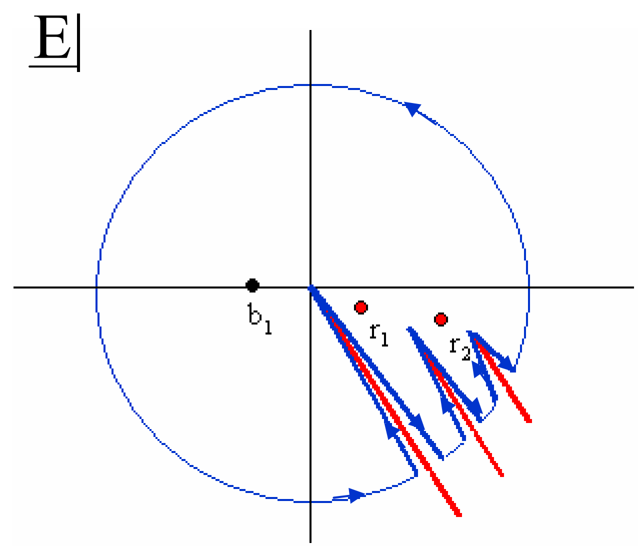
B.Giraud and K.Kato, Ann.of Phys. 308 (2003), 115.



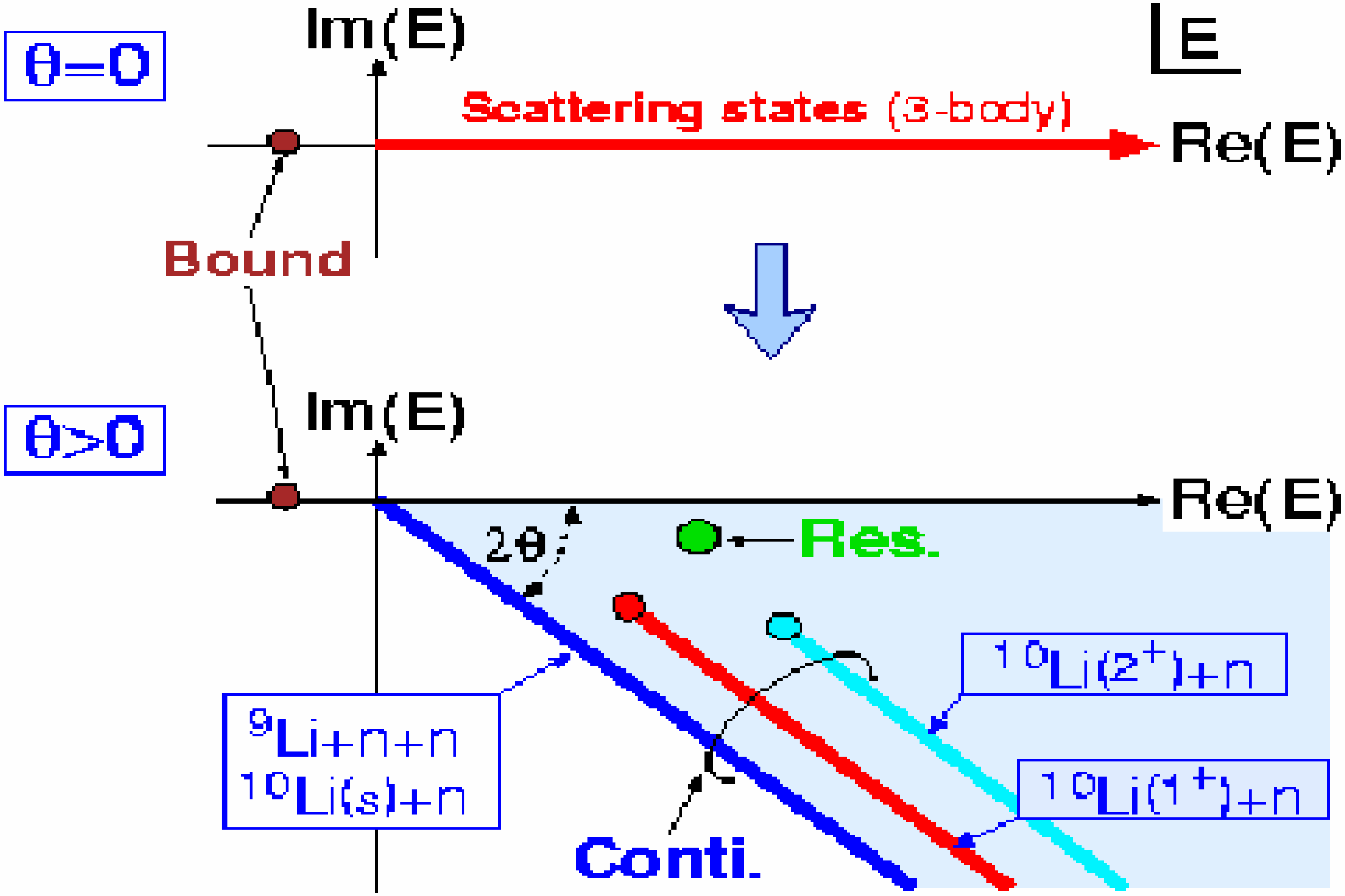
$\theta \neq 0$

B.Giraud, K.Kato and A. Ohnishi, J. of Phys. A37 (2004),11575

Coupled Channel system



Three-body system



T. Myo, A. Ohnishi and K. Kato, Prog. Theor. Phys. **99** (1998), 801.

- Extended completeness relation in CSM

$$1 = \sum_{n=b} |u_n^\theta\rangle\langle\tilde{u}_n^\theta| + \sum_{n=r}^{N_r^\theta} |u_n^\theta\rangle\langle\tilde{u}_n^\theta| + \frac{1}{\pi}\int_{L_\theta^k} dk\, |\psi_k^\theta\rangle\langle\tilde{\psi}_k^\theta| + \frac{1}{\pi}\int_{L_\theta^{k'}} dk'\, |\psi_{k'}^\theta\rangle\langle\tilde{\psi}_{k'}^\theta| + \frac{1}{\pi}\int_{L_\theta^{k''}} dk''\, |\psi_{k''}^\theta\rangle\langle\tilde{\psi}_{k''}^\theta$$

Resonances ^{9}Li+n+n ^{10}Li(1^+)+n ^{10}Li(2^+)+n

2. Strength Functions and Coulomb Breakup Reaction

∘ Strength function

$$\mathcal{S}_\lambda(E) = \sum_\nu \langle \tilde{\Phi}_I | \widehat{O}^\dagger_\lambda | \nu \rangle \langle \tilde{\nu} | \widehat{O}_\lambda | \Phi_I \rangle \, \delta(E - E_\nu) = -\frac{1}{\pi} \mathrm{Im}\,[R(E)]$$

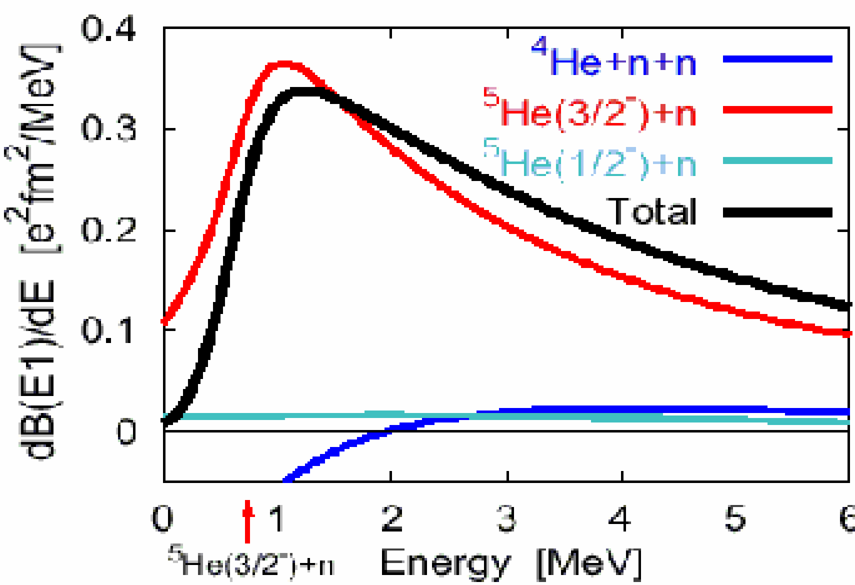
∘ Response function and Green's function

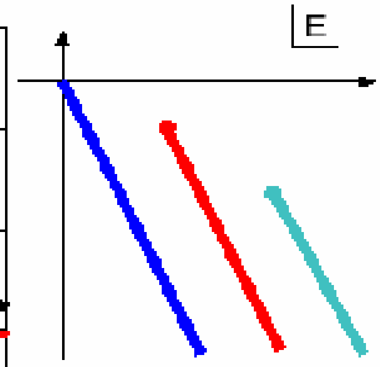
$$R(E) = \int d\boldsymbol{\xi}\, d\boldsymbol{\xi}'\, \tilde{\Phi}^*_I(\boldsymbol{\xi})\, \hat{O}^\dagger_\lambda\, \underline{\mathcal{G}(E, \boldsymbol{\xi}, \boldsymbol{\xi}')}\, \hat{O}_\lambda\, \Phi_I(\boldsymbol{\xi}')$$

$$\mathcal{G}^\theta(E, \boldsymbol{\xi}, \boldsymbol{\xi}') = \left\langle \boldsymbol{\xi} \left| \frac{\mathbf{1}}{E - H(\theta)} \right| \boldsymbol{\xi}' \right\rangle$$

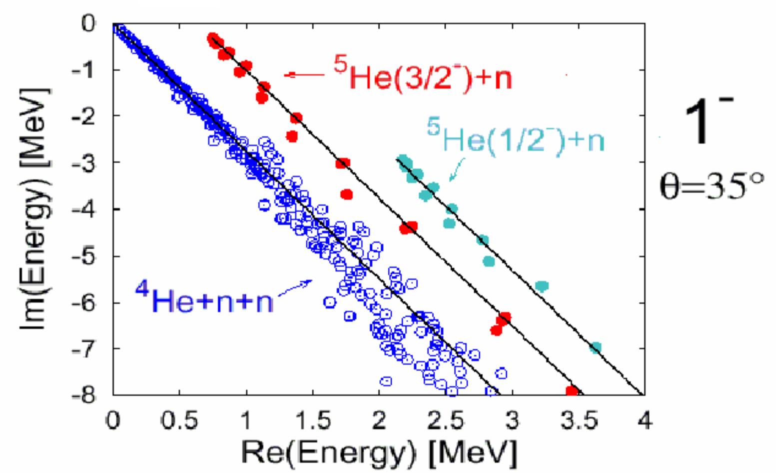
$$= \sum_B \frac{\phi_B(\boldsymbol{\xi}) \tilde{\phi}^*_B(\boldsymbol{\xi}')}{E - E_B} + \sum_R \frac{\phi_R(\boldsymbol{\xi}) \tilde{\phi}^*_R(\boldsymbol{\xi}')}{E - E_R} + \not{\sum_C} \frac{\phi_C(\boldsymbol{\xi}) \tilde{\phi}^*_C(\boldsymbol{\xi}')}{E - E_C}$$

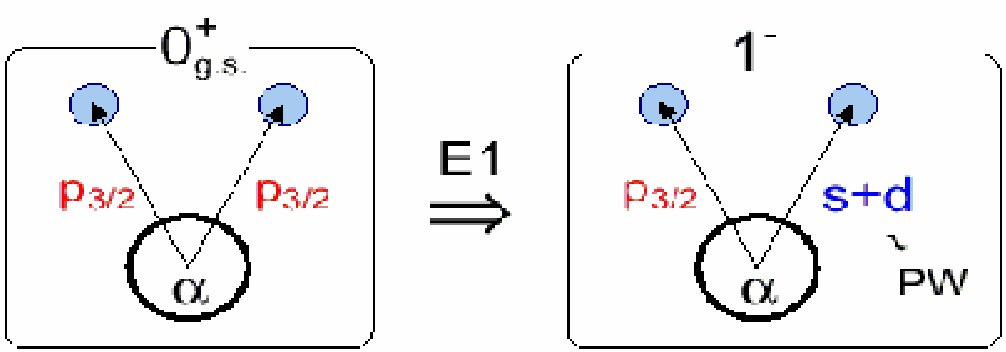
- E1 transition ($0^+ \to 1^-$)





- 1⁻, energies of ^{6}He with CSM

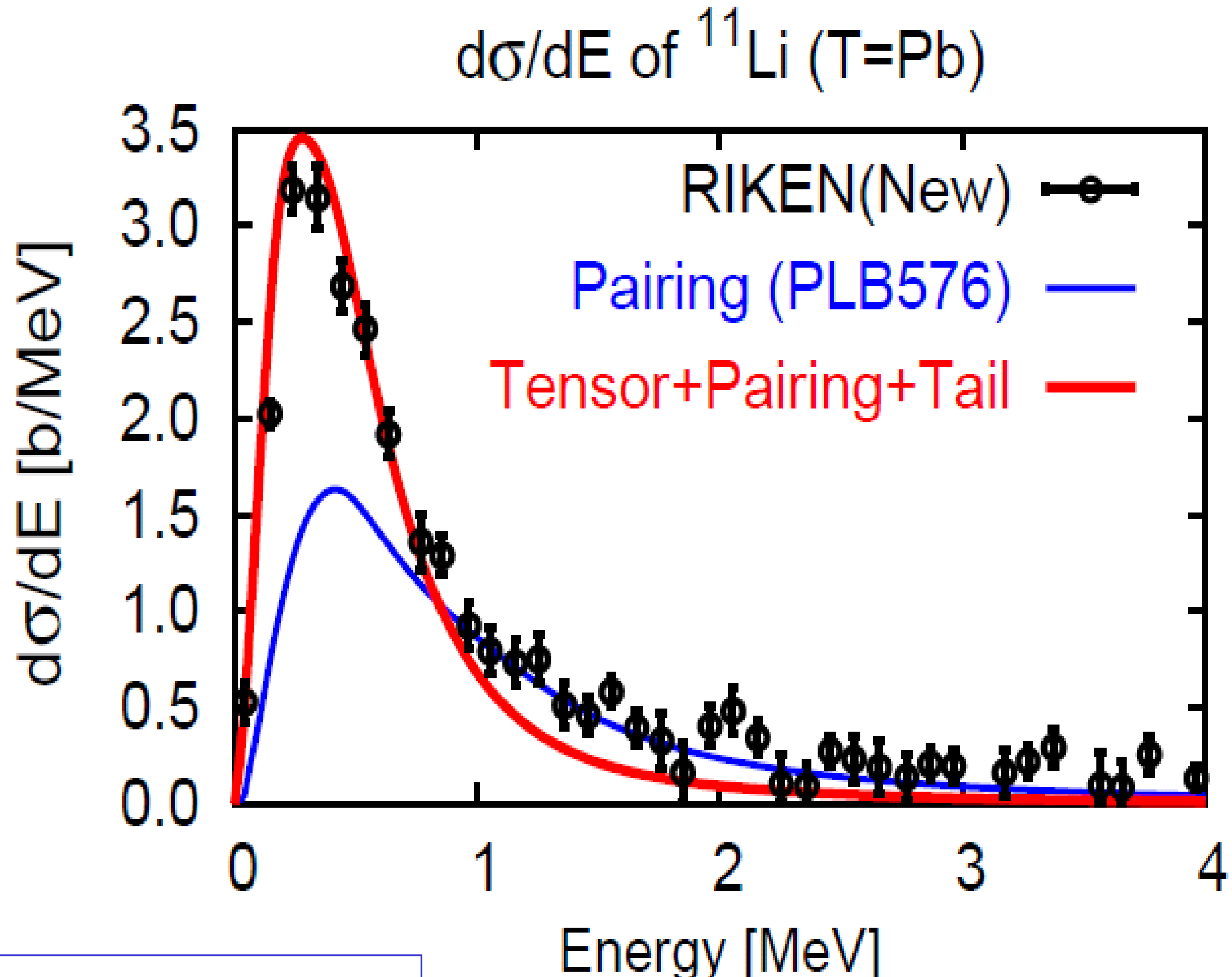




- ^{6}He ⇒ ^{5}He(3/2⁻)+n ⇒ ^{4}He—n—n
- threshold effect of ^{5}He+n
  ⇒ Low energy enhancement

T. Myo, K. Kato, S. Aoyama and K. Ikeda, PRC63(2001), 054313

Comparison to RIKEN New data (Nakamura et al. PRL 96, 252502 (2006)).



coupled channel [^{9}Li+n+n]

+

[^{9}Li*+n+n]

PRC to be published

We can see a good agreement.

3. Continuum Level Density

Definition of LD:

$$\rho(E) = \sum\!\!\!\!\!\!\int \delta(E - E_i)$$

$$H\psi_i = E_i\psi_i$$

$$\rho(E) = -\frac{1}{\pi}\,\mathrm{Im}\left[Tr\left\{ \frac{1}{E - H + i\varepsilon} \right\} \right]$$

A.T.Kruppa, Phys. Lett. B 431 (1998), 237-241

A.T. Kruppa and K. Arai, Phys. Rev. A59 (1999), 2556

K. Arai and A.T. Kruppa, Phys. Rev. C 60 (1999) 064315

$$\rho(E) = -\frac{1}{\pi}\,\mathrm{Im}\left[Tr\left\{ \frac{\mathbf{1}}{E - H + i\varepsilon} \right\} \right]$$

RI in complex scaling

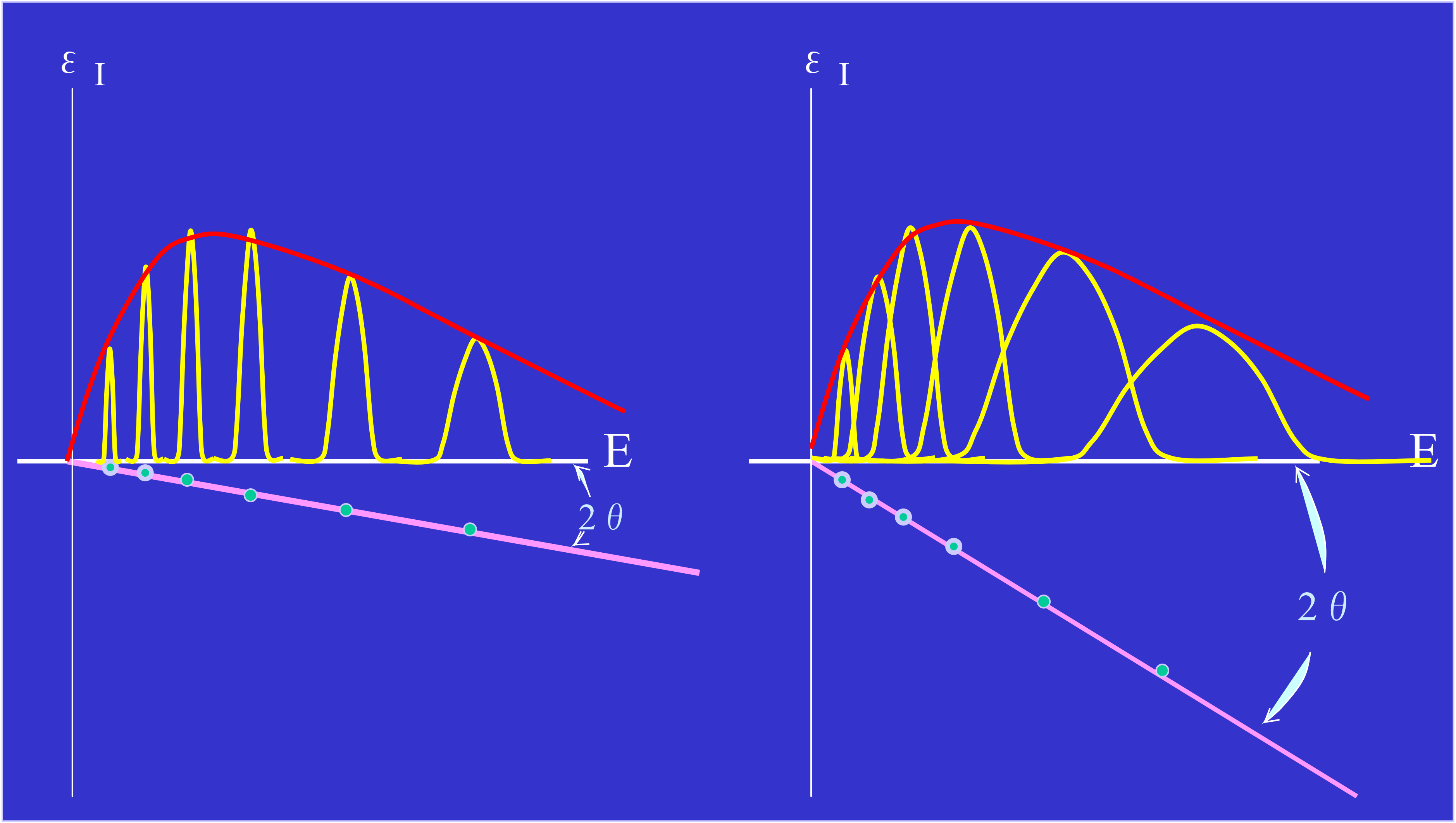
$$= -\frac{1}{\pi}\,\mathrm{Im}\left[\sum_{n_B}^{N_B} \frac{1}{E - E_{n_B}^{B}} + \sum_{n_R}^{N_B^{\theta}} \frac{1}{E - E_{n_R}^{R}} + \int_{L_\theta} dE^C \frac{1}{E - E^C} \right]$$

Resonance: $E_{n_R}^{R} = \mathrm{E}_{n_R}^{R} - i\frac{\Gamma_{n_R}}{2}$

Continuum: $E^C = \varepsilon_R - i\varepsilon_I$

$$= \sum_{n_B}^{N_B} \delta(E - E_{n_B}^{B}) + \frac{1}{\pi}\sum_{n_R}^{N_R^{\theta}} \frac{\Gamma_{n_R}/2}{(E - \mathrm{E}_{n_R}^{R})^2 + \Gamma_{n_R}^2/4} + \frac{1}{\pi}\int_{L_\theta} dE^C \frac{\varepsilon_I}{(E - \varepsilon_R)^2 + \varepsilon_I^2}$$

Descretization


ε
I
E
2 θ
ε
I
E
2 θ

Continuum Level Density:

$$\Delta(E) = \rho(E) - \rho_0(E)$$

$$\Delta(E) = -\frac{1}{\pi}\,\mathrm{Im}\left[Tr\left\{ \frac{1}{E-H+i\varepsilon} - \frac{1}{E-H_0+i\varepsilon} \right\} \right]$$

$$= -\frac{1}{\pi}\,\mathrm{Im}\left[Tr\left\{ G(E) - G_0(E) \right\} \right]$$

Basis function method:

$$\psi = \sum_{n=1}^{N} c_n \phi_n$$

$$\Delta^{\theta}(E) = -\frac{1}{\pi} Im \left[\sum_{B} \frac{1}{E-e_B} + \sum_{R}^{N_\theta} \frac{1}{E-e_R^{\theta}} + \sum_{C} \frac{1}{E-e_C^{\theta}} - \sum_{j}^{M} \frac{1}{E-\epsilon_j^{0}(\theta)} \right]$$

$$= g_{M,B}(E) + g_{M,R}^{\theta}(E) + g_{M,C}^{\theta}(E)$$

Phase shift calculation in the complex scaled basis function method

$$\Delta(E) = \frac{1}{2i\pi} Tr\left[S(E)^{+} \frac{d}{dE} S(E) \right]$$

S.Shlomo, Nucl. Phys. A539 (1992), 17.

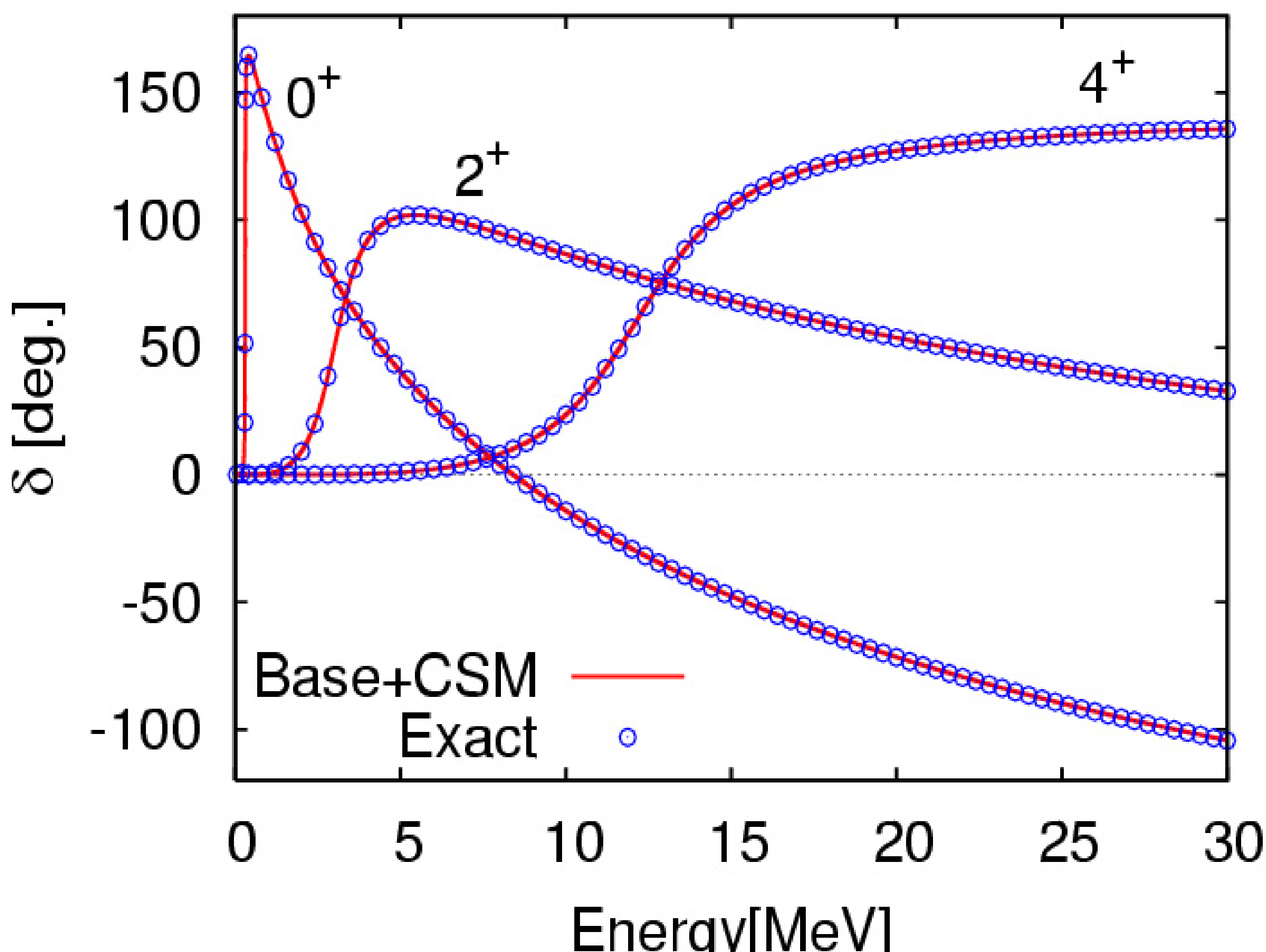
In a single channel case, $S(E) = \exp\{2i\delta(E)\}$

$$\Delta(E) = \frac{1}{\pi} \frac{d\delta(E)}{dE}$$

$$\delta(E) = \pi \int_0^E dE' \Delta(E')$$

Phase shift of $^8Be=\alpha+\alpha$ calculated with discretized app.

Base+CSM: 30 Gaussian basis and θ=20 deg.



3 α Orthogonality Condition Model (OCM)

$$H_0 = \sum_{i=1}^{3} t_i - T_G + \sum_{i=1}^{3} V_{\alpha\alpha}(\xi_i) + V_{3\alpha}(\xi_1, \xi_2, \xi_3) + V_{pauli}$$

- $V_{\alpha\alpha}$ **folding for Nucleon-Nucleon interaction(Nuclear+Coulomb)**
- $V_{3\alpha} = V_{3\alpha}^{J^\pi} \exp[-\mu(\xi_1^2 + \xi_2^2 + \xi_3^2)]$ **K. Wildermuth, Nucl. Phys. 26 (1961) 463**

  $V_{3\alpha}^{0^+} = 31.7$ MeV $(J^\pi = 0^+$, -parity)

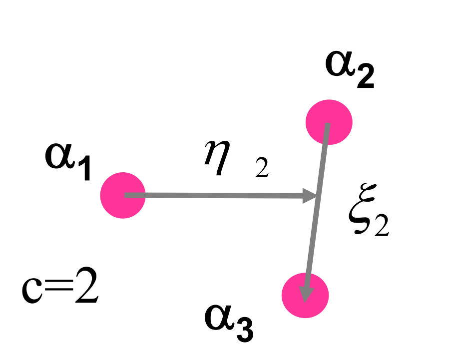
  $V_{3\alpha}^{2^+} = 63.0$ MeV $(J^\pi = 2^+)$

  $V_{3\alpha}^{4^+} = 150.0$ MeV $(J^\pi = 4^+)$

  μ =0.15 fm^{-2}
- $V_{pseud} = \lim_{\lambda \to \infty} \lambda \sum_{pf=0s,1s,0d} |\phi_{pf}(\xi_i)\rangle\langle\phi_{pf}(\xi_i')|, \quad i = 1,\ 2 \text{ or } 3$

  : OCM **[Ref.]: S.Saito, PTP Supple. 62(1977),11**

Phase shifts and Energies of ^{8}Be, and Ground band states of ^{12}C

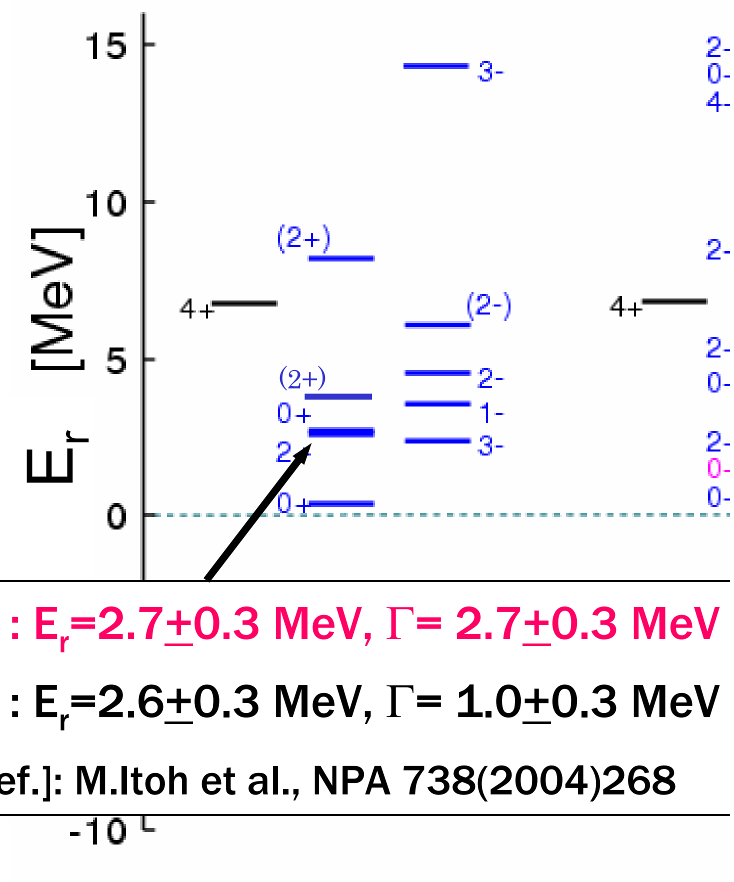


$$H_0\Psi = E\Psi$$

$$\Psi = \sum_{\alpha} \Phi_\alpha = \sum_{\alpha} \sum_{c=1}^{3} \varphi(\eta_c, \xi_c) \ , \quad \varphi(\eta_c, \xi_c) = \sum_i \tilde{C}_{i,c}^{\alpha} \phi_i$$

[Ref.]: M.Kamimura, Phys. Rev. A38(1988),621

Results of applications of CSM and ACCC+CSM to 3αOCM —Energy levels Ex< 15 MeV—



0^+: E_r=2.7±0.3 MeV, Γ= 2.7±0.3 MeV

2^+: E_r=2.6±0.3 MeV, Γ= 1.0±0.3 MeV

[Ref.]: M.Itoh et al., NPA 738(2004)268

3 α Model can reproduce 2
by taking into account

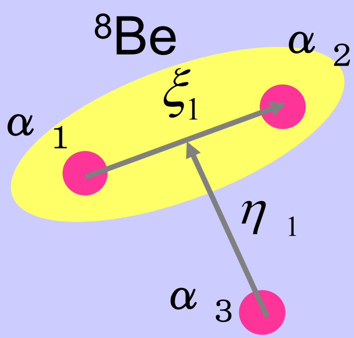
[Ref.]: M.It

J^π	Present result E_r	Γ	Experimental data E_r	Γ
0_1^+	-7.29	–	-7.2747	–
2_1^+	-2.98	–	-2.8358 ± 0.0003	–
4_1^+	6.82	0.24	6.808 ± 0.015	0.258 ± 0.15
0_2^+	0.76	2.4×10^{-3}	0.3795 ± 0.0002	$(8.5 \pm 1.0) \times 10^{-6}$
0_3^+	1.66	1.48	2.8 ± 0.3	2.7 ± 0.3
2_2^+	2.28	1.1	2.6 ± 0.3	1.0 ± 0.3
0_4^+	4.58	1.1		
2_3^+	5.14	1.9	3.885 ± 0.50	0.43 ± 0.08 [1]
2_4^+	8.64	3.9	8.165 ± 0.40	1.5 ± 0.2 [2]
4_2^+	13.1	3.4		
0_5^+	14.3	1.5		
2_5^+	15.1	1.2		
2_6^+	17.6	6.0		
3_1^-	1.51	2.0×10^{-3}	2.366 ± 0.005	$(3.4 \pm 0.5) \times 10^{-2}$
1_1^-	3.65	0.30	3.569 ± 0.016	0.315 ± 0.025
2_1^-	4.68	0.42	4.553 ± 0.016	0.260 ± 0.025
4_1^-	5.16	0.12	6.077 ± 0.017	0.375 ± 0.040 [3]
2_2^-	9.31	4.65		
3_2^-	11.0	0.5		
5_1^-	11.4	0.3		

Continuum Level Density of 3 α system

[Ref.] S.Shlomo, NPA 539 (1992) 17.

$$\Delta'(E) = \rho_{3B}(E) - \rho^0_{3B}(E) - \left(\rho_{2B}(E) - \rho^0_{2B}(E)\right)$$
$$= -\frac{1}{\pi}\mathrm{Im}\left\{\mathrm{Tr}\left[\frac{1}{E-H_{3B}} - \frac{1}{E-H^0_{3B}} - \left(\frac{1}{E-H_{2B}} - \frac{1}{E-H^0_{2B}}\right)\right]\right\}$$



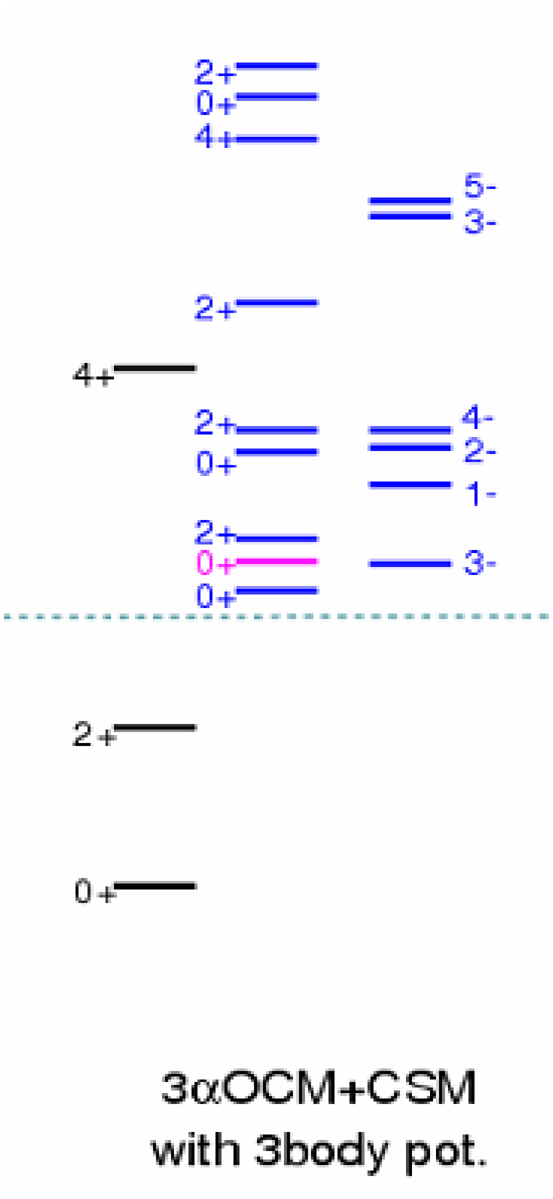
$$H_{2B} = \sum_{i=1}^{3} t_i - T_G + V^{N+Cl+OCM}_{\alpha-\alpha}(\xi_1) + V^{Cl(\mathrm{point})}_{8Be-\alpha}(\eta_1)$$

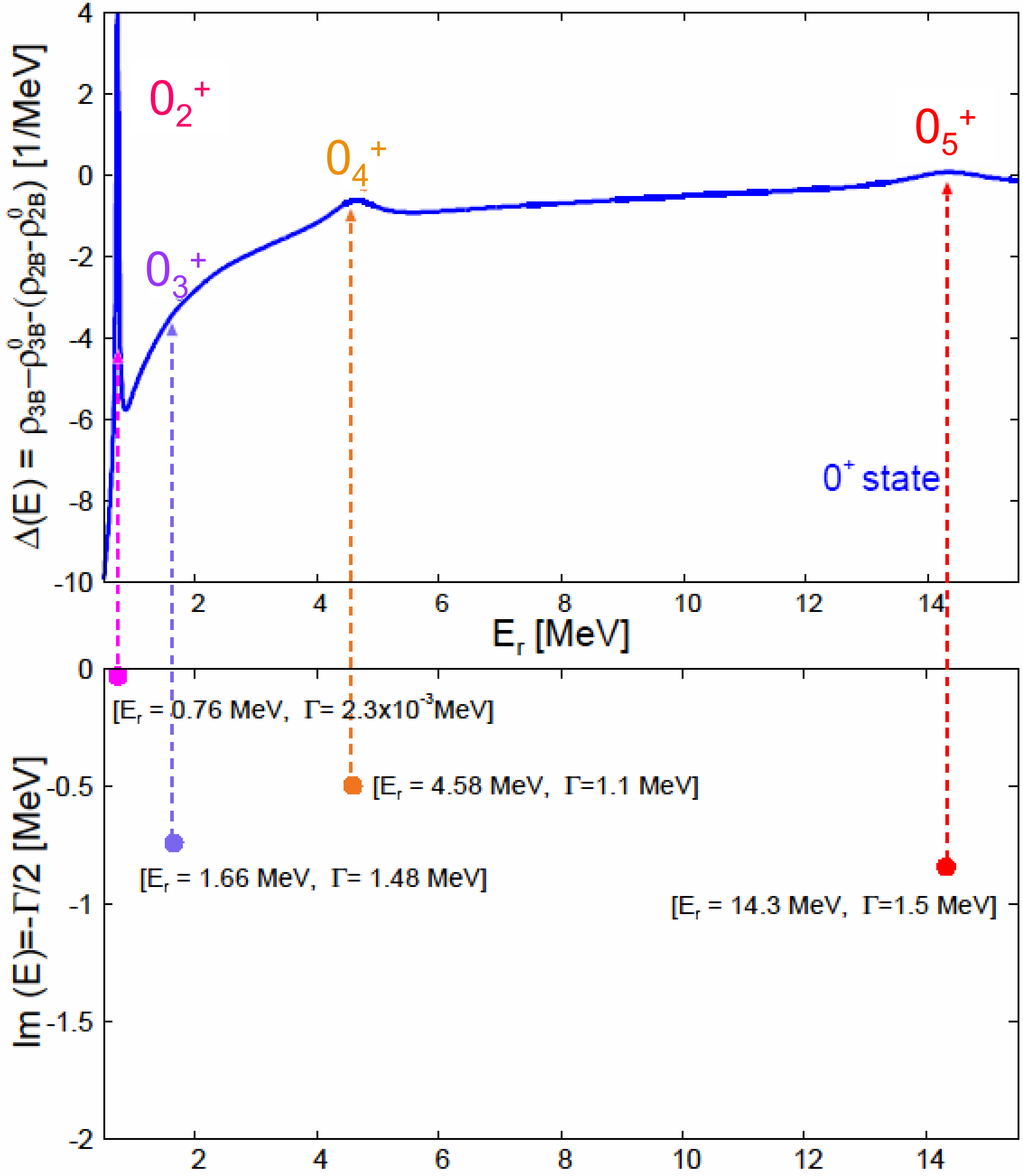
- α_1- α_2: resonance + continuum
- ($\alpha_1 \alpha_2$)- α_3: continuum

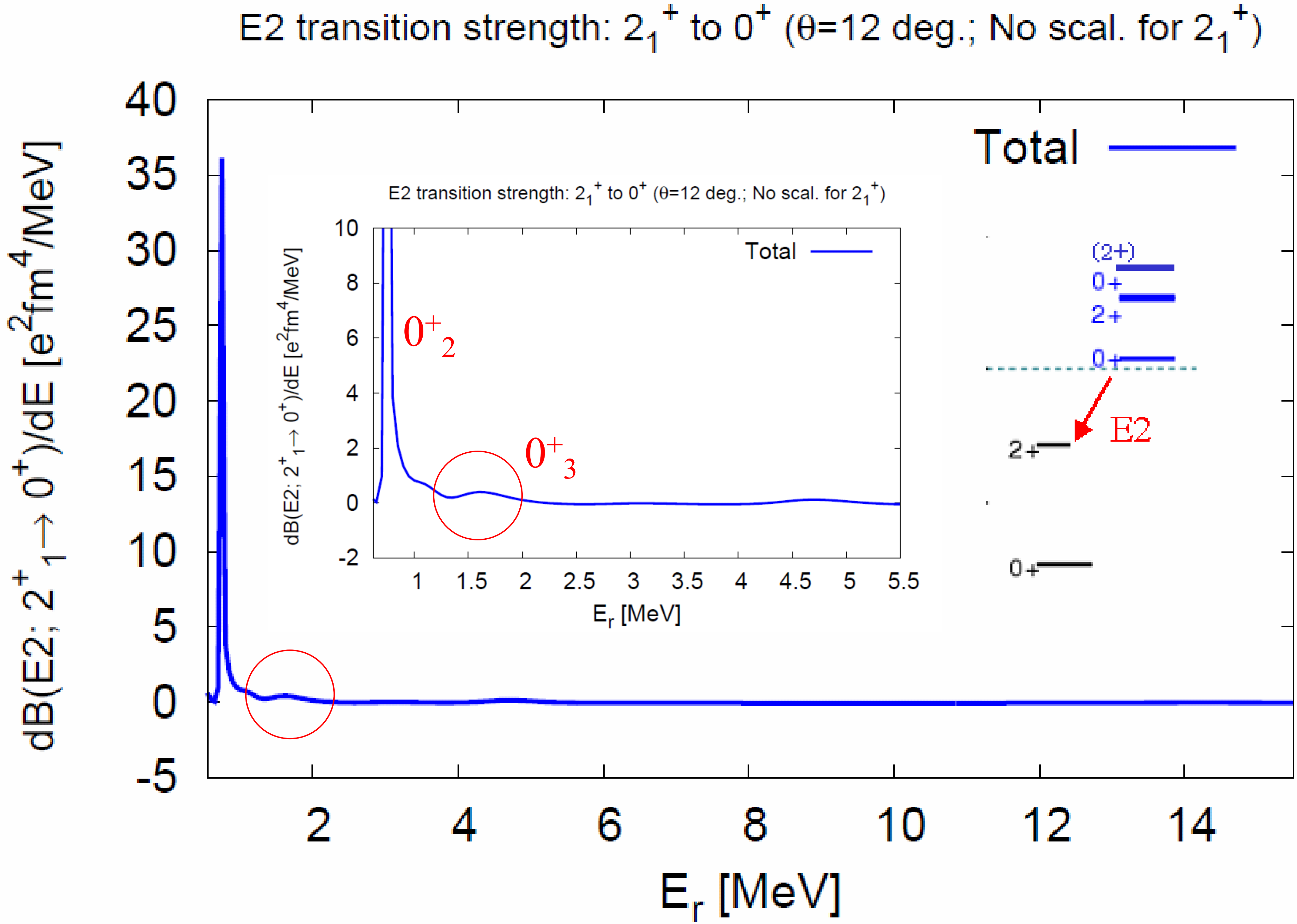
$$H^0_{2B} = \sum_{i=1}^{3} t_i - T_G + V^{Cl(\mathrm{point})}_{\alpha-\alpha}(\xi_1) + V^{Cl(\mathrm{point})}_{8Be-\alpha}(\eta_1)$$

- α_1- α_2: continuum
- ($\alpha_1 \alpha_2$)- α_3: continuum

Continuum Level Density of 0+ states







E2 transition strength: 2_1^+ to 0^+ (θ=12 deg.; No scal. for 2_1^+)
dB(E2; $2^+_1 \rightarrow 0^+$)/dE [e^2fm^4/MeV]
E_r [MeV]
Total
E2 transition strength: 2_1^+ to 0^+ (θ=12 deg.; No scal. for 2_1^+)
dB(E2; $2^+_1 \rightarrow 0^+$)/dE [e^2fm^4/MeV]
E_r [MeV]
Total
0^+_2
0^+_3
(2+)
0+
2+
0+
E2
2+
0+

5. Summary and conclusion

- It is shown that resonant states play an important role in the continuum phenomena.
- The resolution of identity in the complex scaling method is presented to treat the resonant states in the same way as bound states.
- The complex scaling method is shown to describe not only resonant states but also continuum states on the rotated branch cuts.
- We presented several applications of the extended resolution of identity in the complex scaling method; sum rule, strength function and continuum level density.

Collaboration:

S. Aoyama(Niigata Univ.)

H. Masui(Kitami I. T.)

T. Myo（RCNP, Osaka Univ.)

R. Suzuki(Hokkaido Univ.)

C. Kurokawa(Hokkaido Univ.)

K. Ikeda(RIKEN)