Gamow HFB description of loosely bound and resonant medium and heavy nuclei

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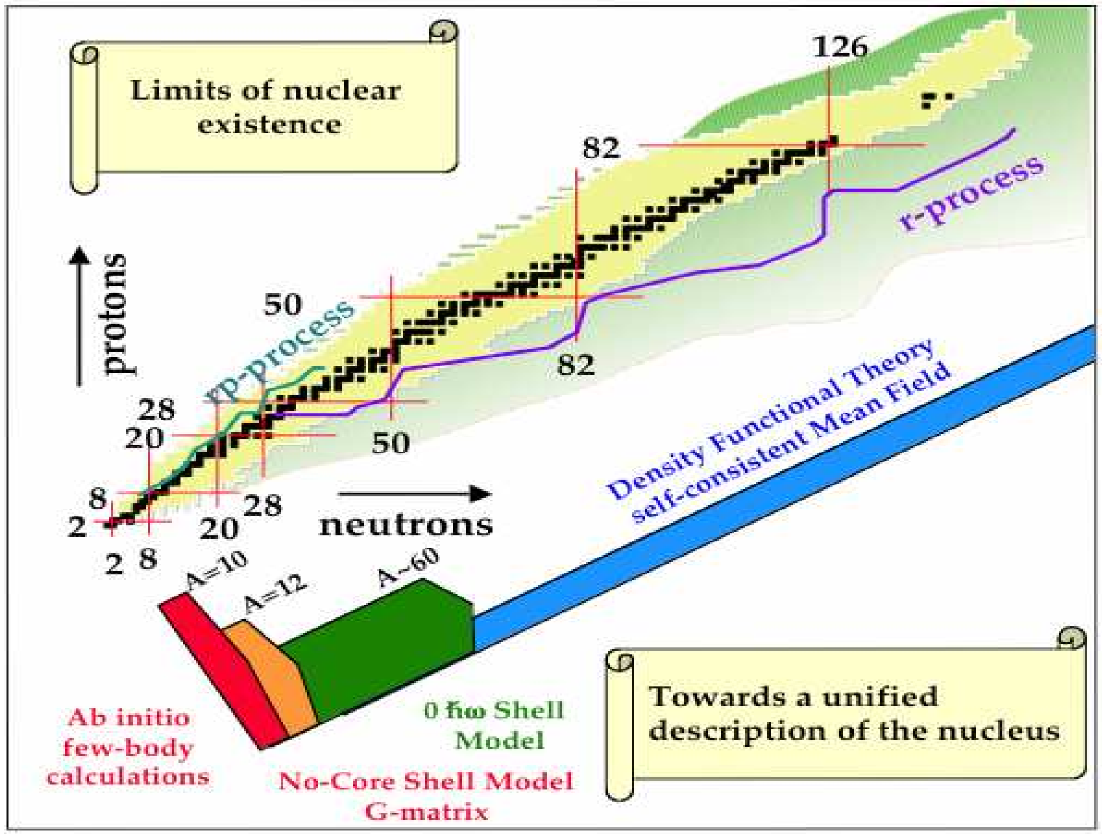
Plan

- Scientific motivation: drip-line nuclei
- Gamow states, Berggren completeness relation
- Gamow quasi-particle states and HFB densities
- Applications: Nickel chain and one proton emitters (spherical)
- Pöschl-Teller-Ginocchio (PTG) basis for loosely bound systems
- Resonant structure with PTG basis
- Applications: Nickel chain (spherical)
  Zirconium and Magnesium (deformed)
- Conclusion and perspectives

Scientific motivation



Gamow states

- Georg Gamow : α decay

  G.A. Gamow, Zs f. Phys. **51** (1928) 204; **52** (1928) 510

- Definition :

$$u''(r) = \left[\frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}V(r) - k^2\right] u(r)$$

$$u(r) \sim C_0 \, r^{l+1} \; , \; r \to 0$$

$$u(r) \sim C_+ \, H^+_{l,\eta}(kr) \; , \; r \to +\infty \text{ (bound,resonant)}$$

$$u(r) \sim C_+ \, H^+_{l,\eta}(kr) + C_- \, H^-_{l,\eta}(kr) \; , \; r \to +\infty \text{ (scattering)}$$

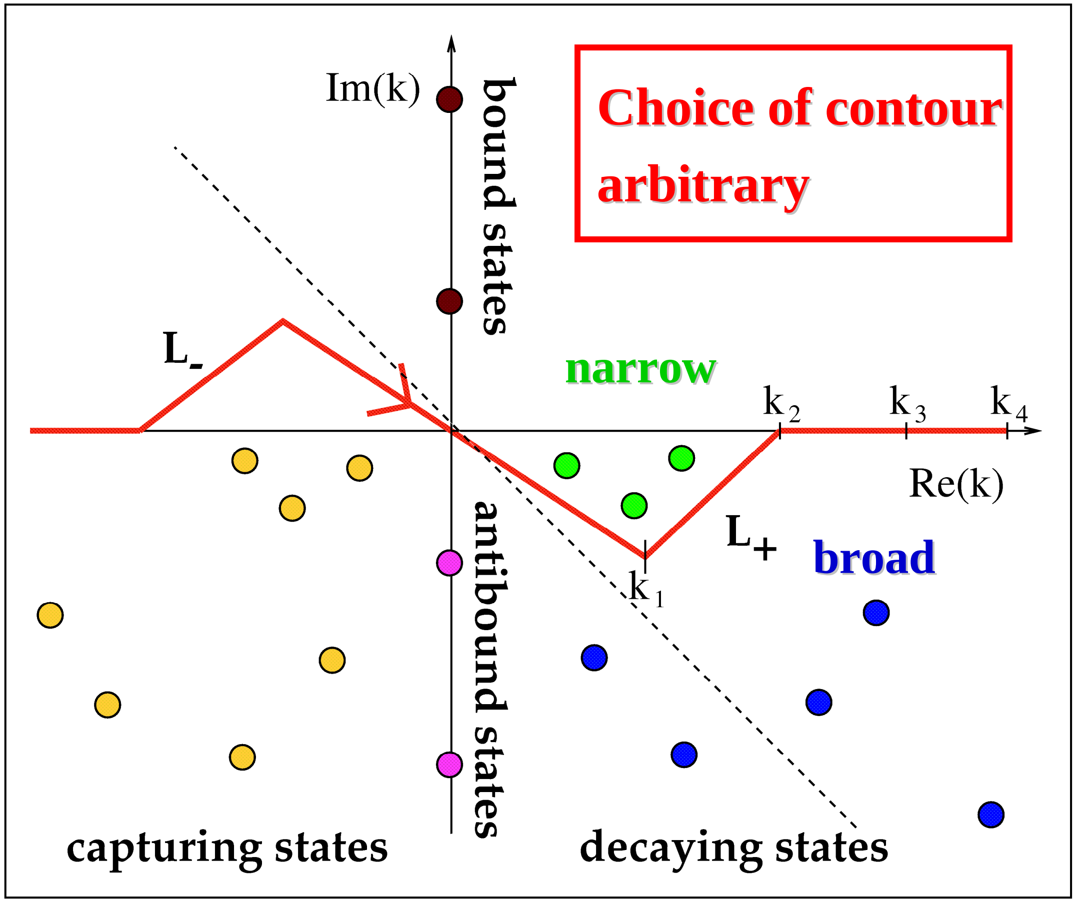
Complex scaling method

- Radial integral calculation : complex scaling

$$\begin{aligned} \langle \widetilde{u_f} | O | u_i \rangle &= \int_0^R u_f(r) O(r) u_i(r) \, dr \\ &+ \sum_{\omega_i, \omega_f} \int_0^{+\infty} u_i^{\omega_i}(z(x)) O(z(x)) u_f^{\omega_f}(z(x)) e^{i\theta_{if}} \, dx \\ z(x) &= R + x e^{i\theta_{if}} \end{aligned}$$

- Analytic continuation : integral independent of R and θ

Gamow states location



Completeness relations with Gamow states

- Berggren completeness relation (l,j) :

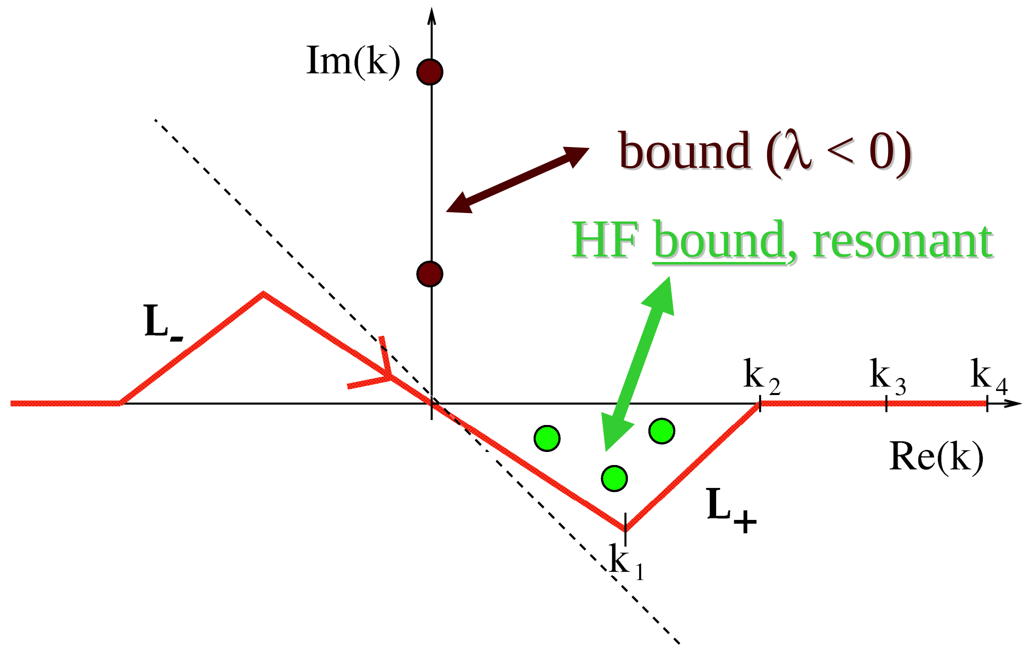
T. Berggren, Nucl. Phys. A **109**, (1967) 205

$$\sum_{n \in (b,d)} |\phi_{nlj}\rangle \langle \widetilde{\phi_{nlj}}| + \int_{L^+} |\phi_{lj}(k)\rangle \langle \widetilde{\phi_{lj}(k)}| \, dk = \mathbb{1}$$

- Continuum discretization : $|\phi_{lj}(k)\rangle \rightarrow \sqrt{\Delta_{k_i}} \cdot |\phi_{lj}(k_i)\rangle$

$$\sum_i |\phi_i\rangle \langle \widetilde{\phi_i}| \sim \mathbb{1}$$

Gamow HFB space



$$E = E_{qp} - |\lambda|$$

Densities with Gamow HFB

- HFB equations: $\begin{pmatrix} h-\lambda & \tilde{h} \\ \tilde{h} & \lambda - h \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = E \begin{pmatrix} u \\ v \end{pmatrix}$

- Complex particle and pairing densities:

$$\rho_{lj}(r) = \sum_{n \in (b,d)} v^2_{nlj}(r) + \int_{L+} v^2_{lj}(k,r)\ dk$$

$$\widetilde{\rho_{lj}}(r) = - \sum_{n \in (b,d)} u_{nlj}(r) v_{nlj}(r) - \int_{L+} u_{lj}(k,r) v_{lj}(k,r)\ dk$$

$$\rho(r) = \sum_{lj} \rho_{lj}(r),\ \tilde{\rho}(r) = \sum_{lj} \widetilde{\rho_{lj}}(r)$$

- HF associated bound and narrow resonant states in discrete sum

Quasi-particle pole states

- Bound, resonant states:

S matrix poles => outgoing wave function behavior

$$u(r) \sim C_u^0 \, r^{l+1}, \; r \to 0$$
$$v(r) \sim C_v^0 \, r^{l+1}, \; r \to 0$$
$$u(r) \sim C_u^+ \; H_{l,\eta_u}^+(k_u r), \; r \to +\infty$$
$$v(r) \sim C_v^+ \; H_{l,\eta_v}^+(k_v r), \; r \to +\infty$$
$$k_u \propto \sqrt{\lambda + E}, \; k_v \propto \sqrt{\lambda - E}$$

Quasi-particle scattering states

- Scattering states:

u(r): incoming and outgoing components
v(r): outgoing wave function behavior

$$u(r) \sim C_u^0 \, r^{l+1}, \; r \to 0$$

$$v(r) \sim C_v^0 \, r^{l+1}, \; r \to 0$$

$$u(r) \sim C_u^+ \, H_{l,\eta_u}^+(k_u r) + C_u^- \, H_{l,\eta_u}^-(k_u r), \; r \to +\infty$$

$$v(r) \sim C_v^+ \, H_{l,\eta_v}^+(k_v r), \; r \to +\infty$$

$$k_u \propto \sqrt{\lambda + E}, \; k_v \propto \sqrt{\lambda - E}$$

Gamow quasi-particle states norm

- Normalization:

S-matrix poles: complex scaling

$$\text{Reg}\left[\int_0^{+\infty} \left(u(r)^2 + v(r)^2\right)\, dr\right] = 1$$

Scattering states: Dirac delta normalization

$$\int (u_k u_{k'} + v_k v_{k'}) = \delta(k - k') \Leftrightarrow C_u^+ \cdot C_u^- \cdot \frac{k_u}{k} = \frac{1}{2\pi}$$

Continuum discretization: $\begin{pmatrix} u(k) \\ v(k) \end{pmatrix} \rightarrow \sqrt{\Delta_{k_i}} \begin{pmatrix} u(k_i) \\ v(k_i) \end{pmatrix}$

Gamow Hartree-Fock diagonalization method

- Two-basis method

Basis generated by ph part of HFB hamiltonian: $h|\varphi\rangle = e|\varphi\rangle$

B. Gall et al., Z. Phys. **A348** 183 (1994)

- HFB matrix structure:

$$\begin{pmatrix} h-\lambda & \tilde{h} \\ \tilde{h} & \lambda-h \end{pmatrix} = \left(\begin{array}{ccc|ccc} e-\lambda & & 0 & & & \\ & \ddots & & & \tilde{h} & \\ 0 & & e-\lambda & & & \\ \hline & & & \lambda-e & & 0 \\ & \tilde{h} & & & \ddots & \\ & & & 0 & & \lambda-e \end{array}\right)$$

- Diagonalization of HFB matrix in Gamow HF basis

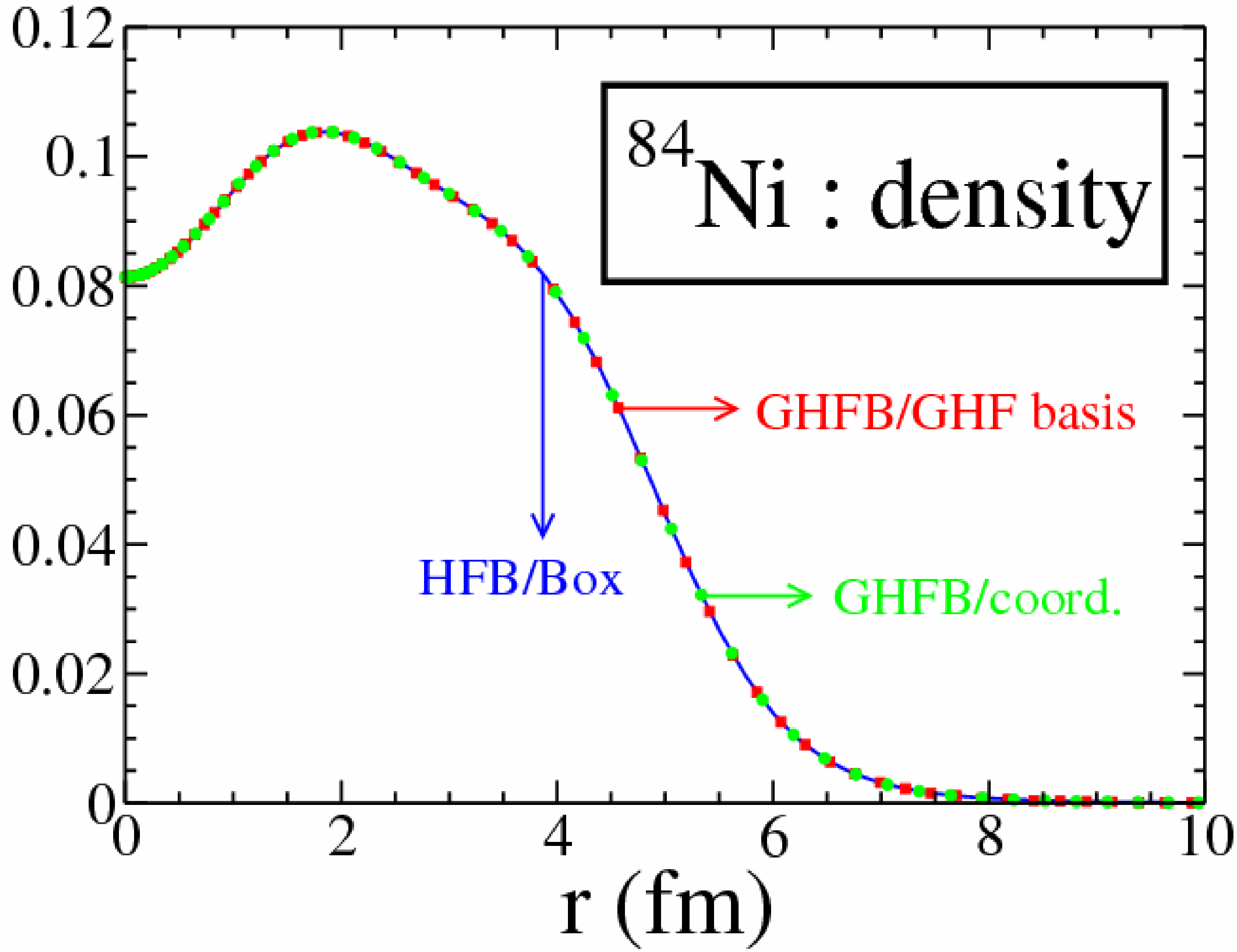
Description of Nickel calculations

- Considered nuclei: ^{84}Ni, ^{86}Ni, ^{88}Ni, ^{90}Ni

- Interaction and space:
  Skyrme interaction: Sly4, E_{cut} = 60 MeV, l: 0 $\rightarrow$ 10

  R_{cut} = 20 fm, k_{max} = 4 fm^{-1}, N_{scat} (l,j) = 100 for GHF basis.

- Interest: Resonant structure directly put in HFB basis



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0.06
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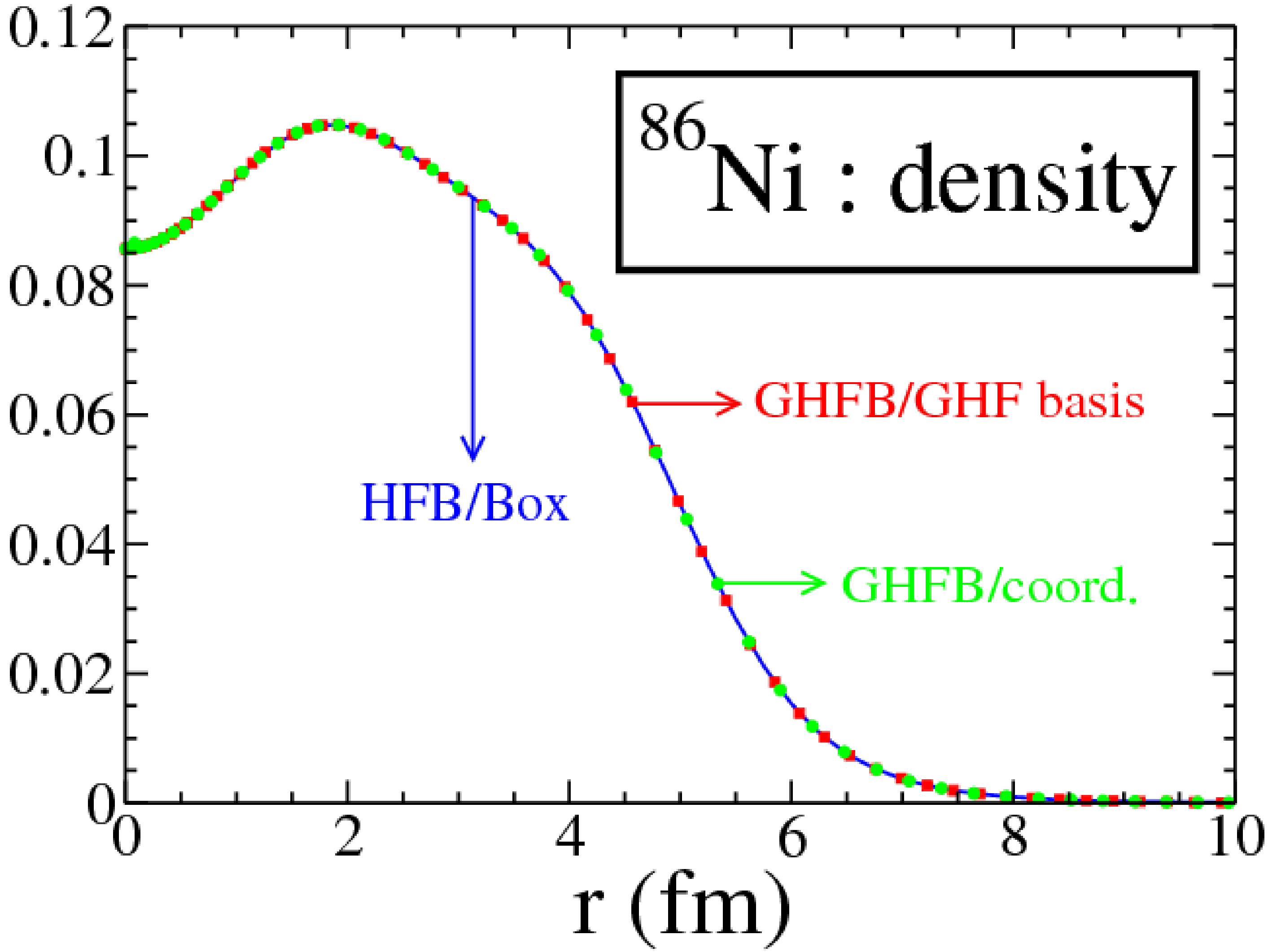
r (fm)
84Ni : density
GHFB/GHF basis
HFB/Box
GHFB/coord.



86Ni : density
GHFB/GHF basis
HFB/Box
GHFB/coord.
r (fm)
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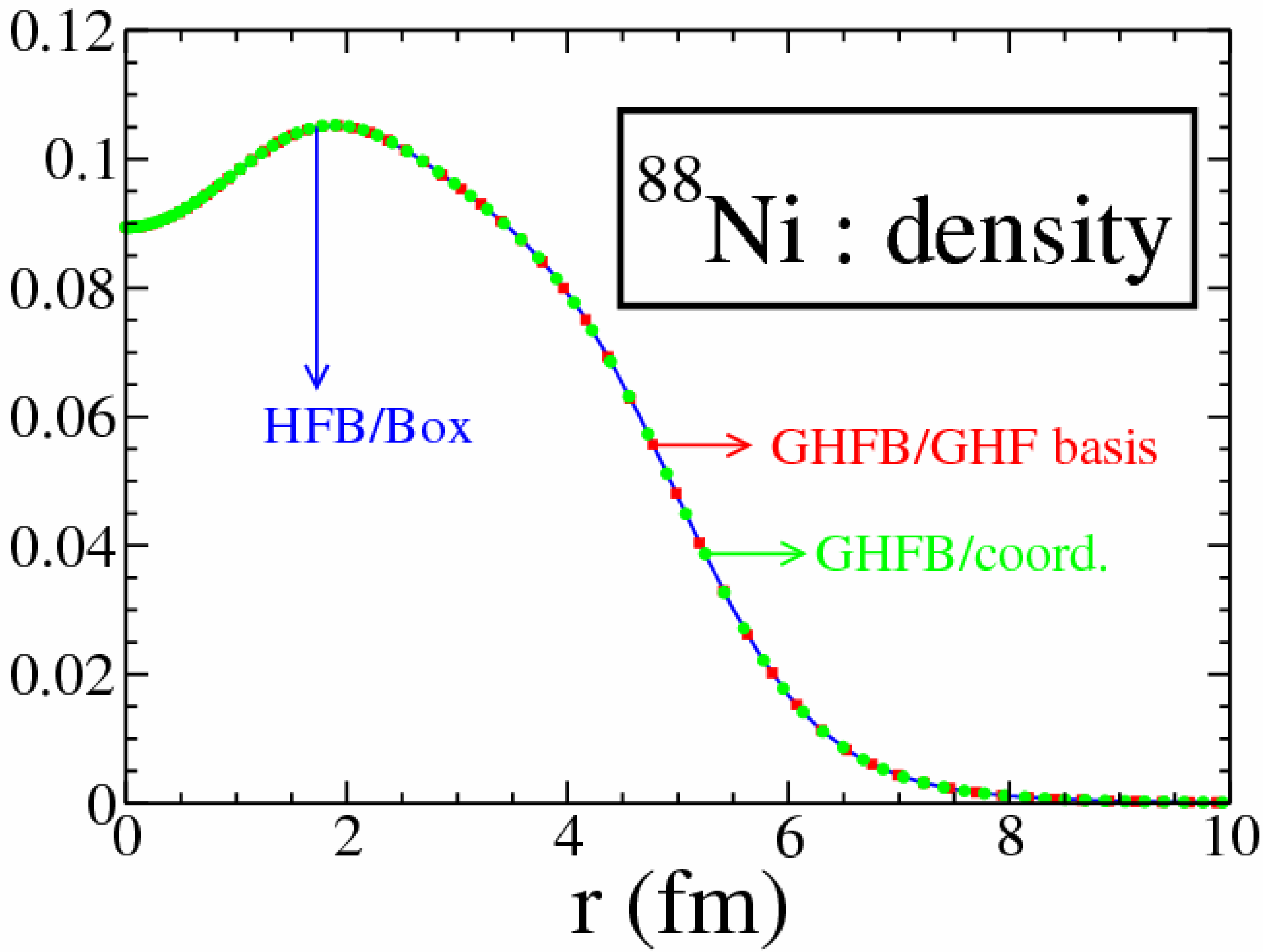
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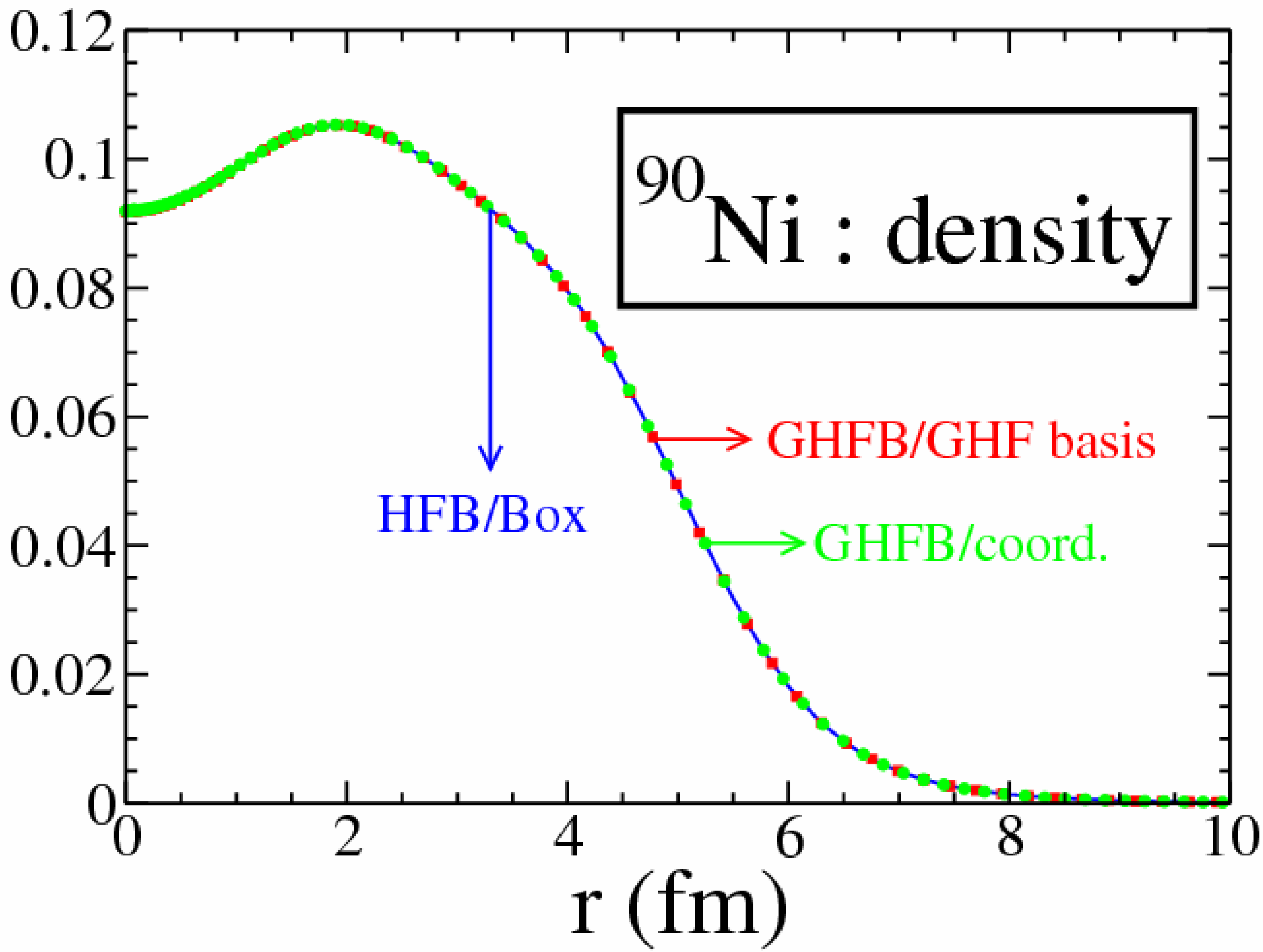
r (fm)
88Ni : density
HFB/Box
GHFB/GHF basis
GHFB/coord.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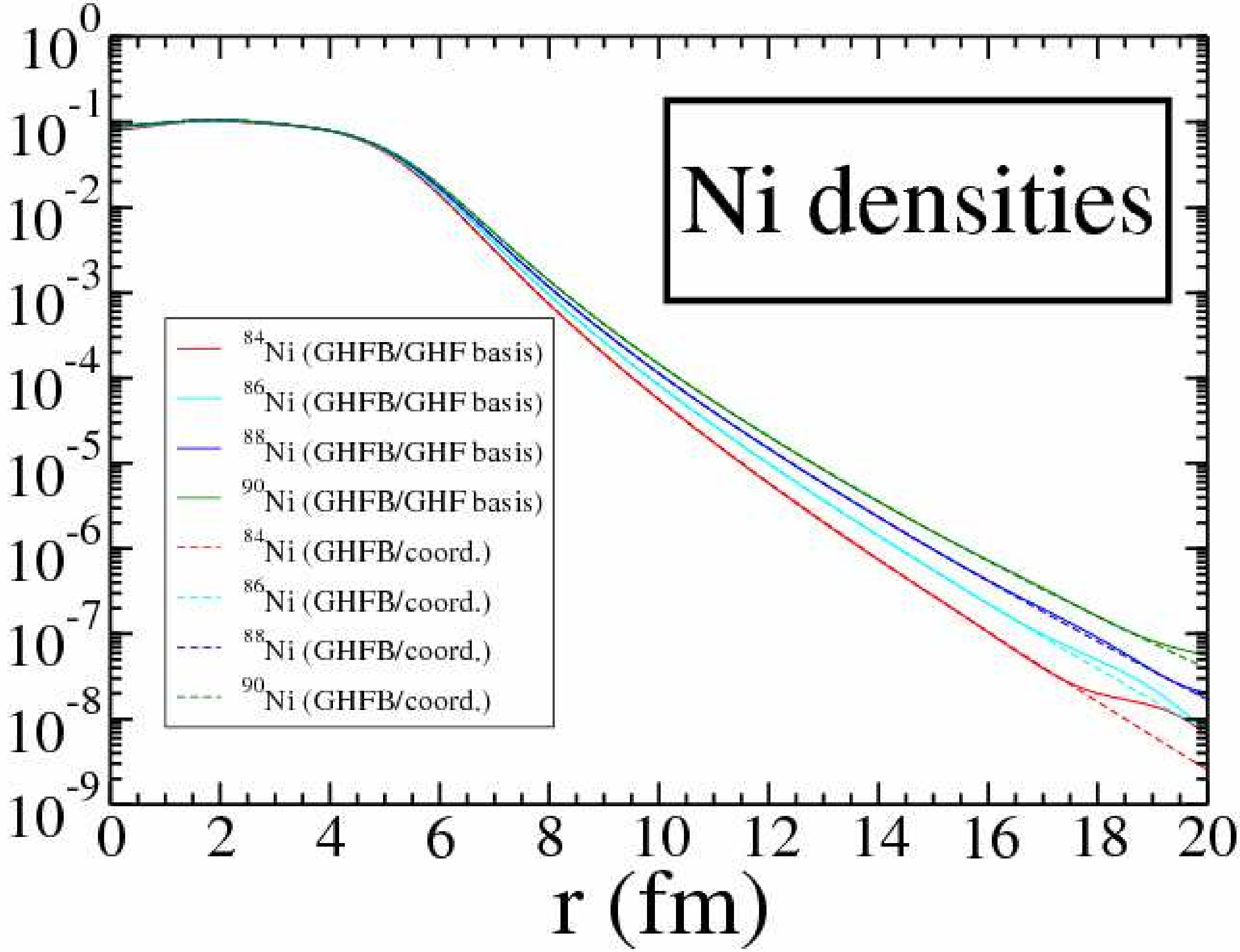
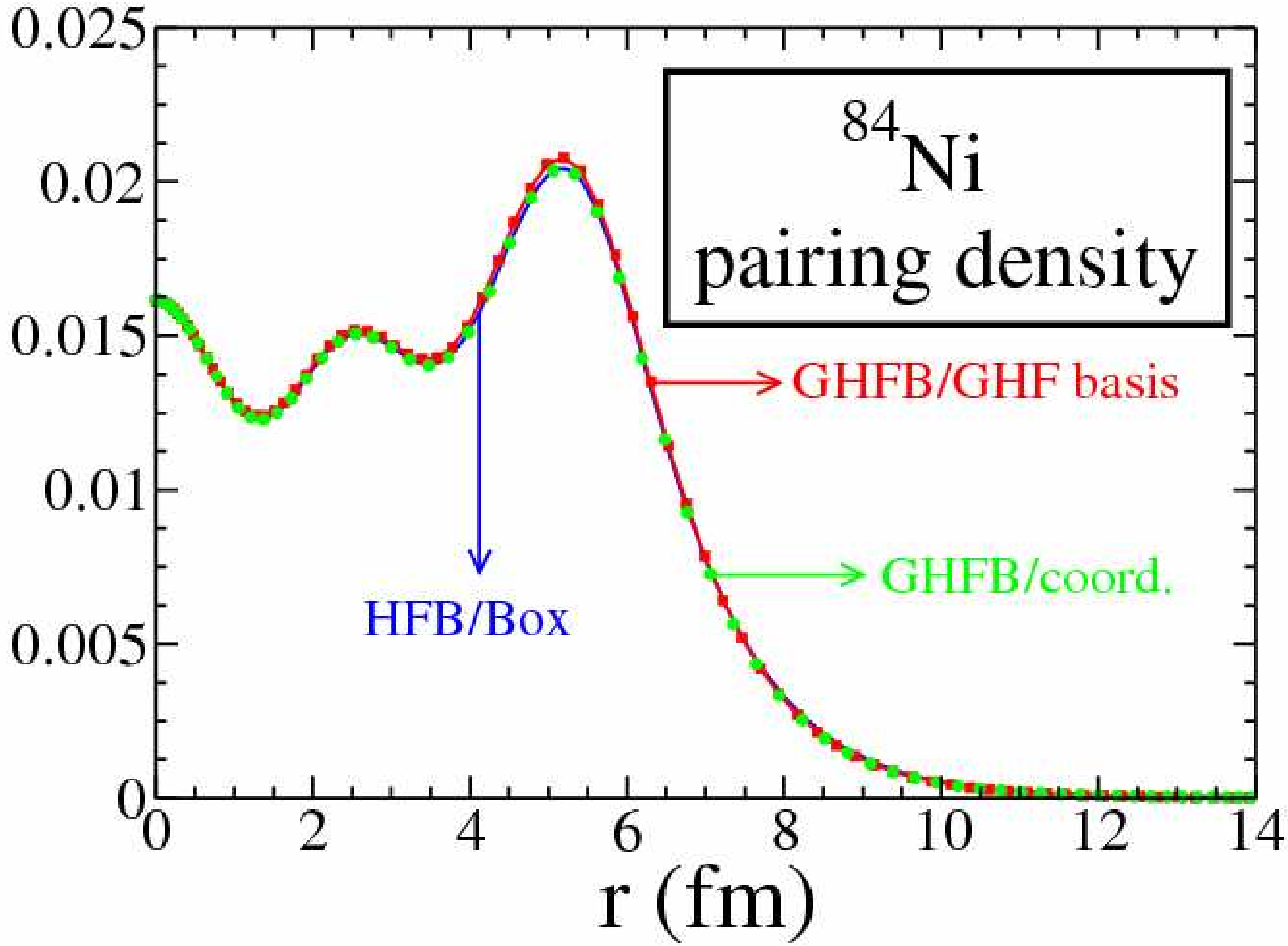
90Ni : density
GHFB/GHF basis
GHFB/coord.
HFB/Box



Ni densities
^{84}Ni (GHFB/GHF basis)
^{86}Ni (GHFB/GHF basis)
^{88}Ni (GHFB/GHF basis)
^{90}Ni (GHFB/GHF basis)
^{84}Ni (GHFB/coord.)
^{86}Ni (GHFB/coord.)
^{88}Ni (GHFB/coord.)
^{90}Ni (GHFB/coord.)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
0 2 4 6 8 10 12 14 16 18 20
r (fm)



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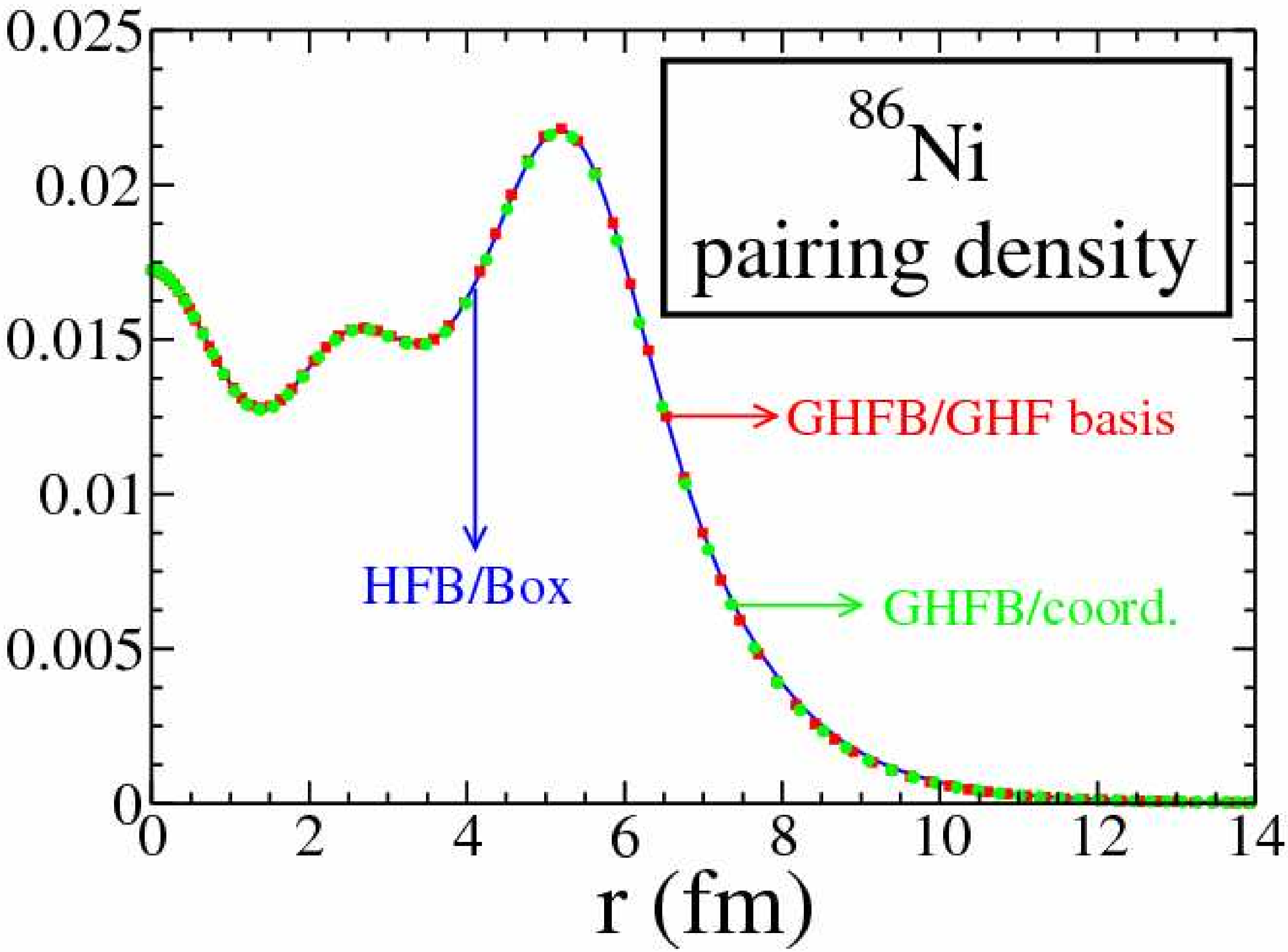
r (fm)
84Ni
pairing density
GHFB/GHF basis
GHFB/coord.
HFB/Box



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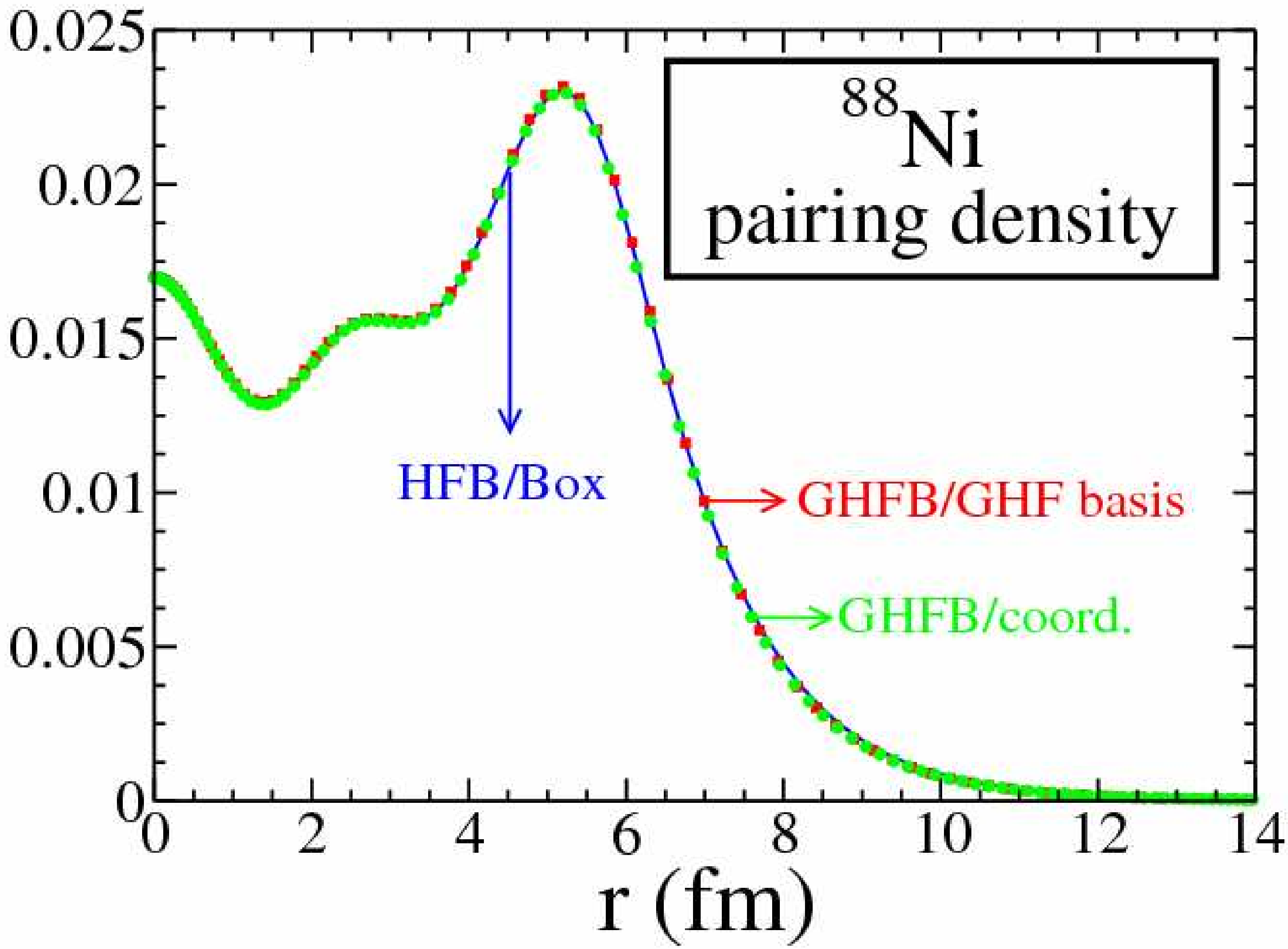
r (fm)
86Ni
pairing density
HFB/Box
GHFB/GHF basis
GHFB/coord.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88Ni
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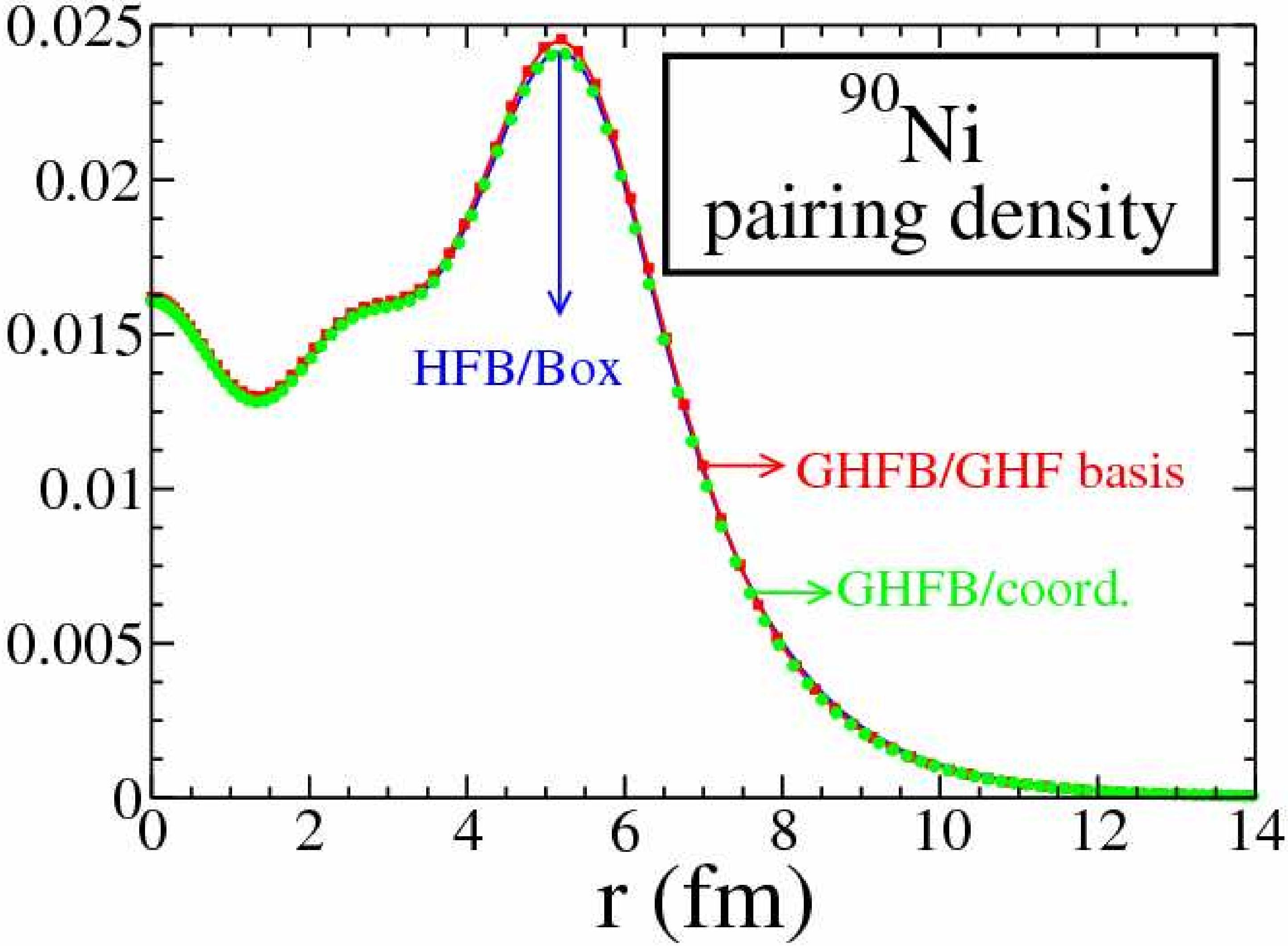

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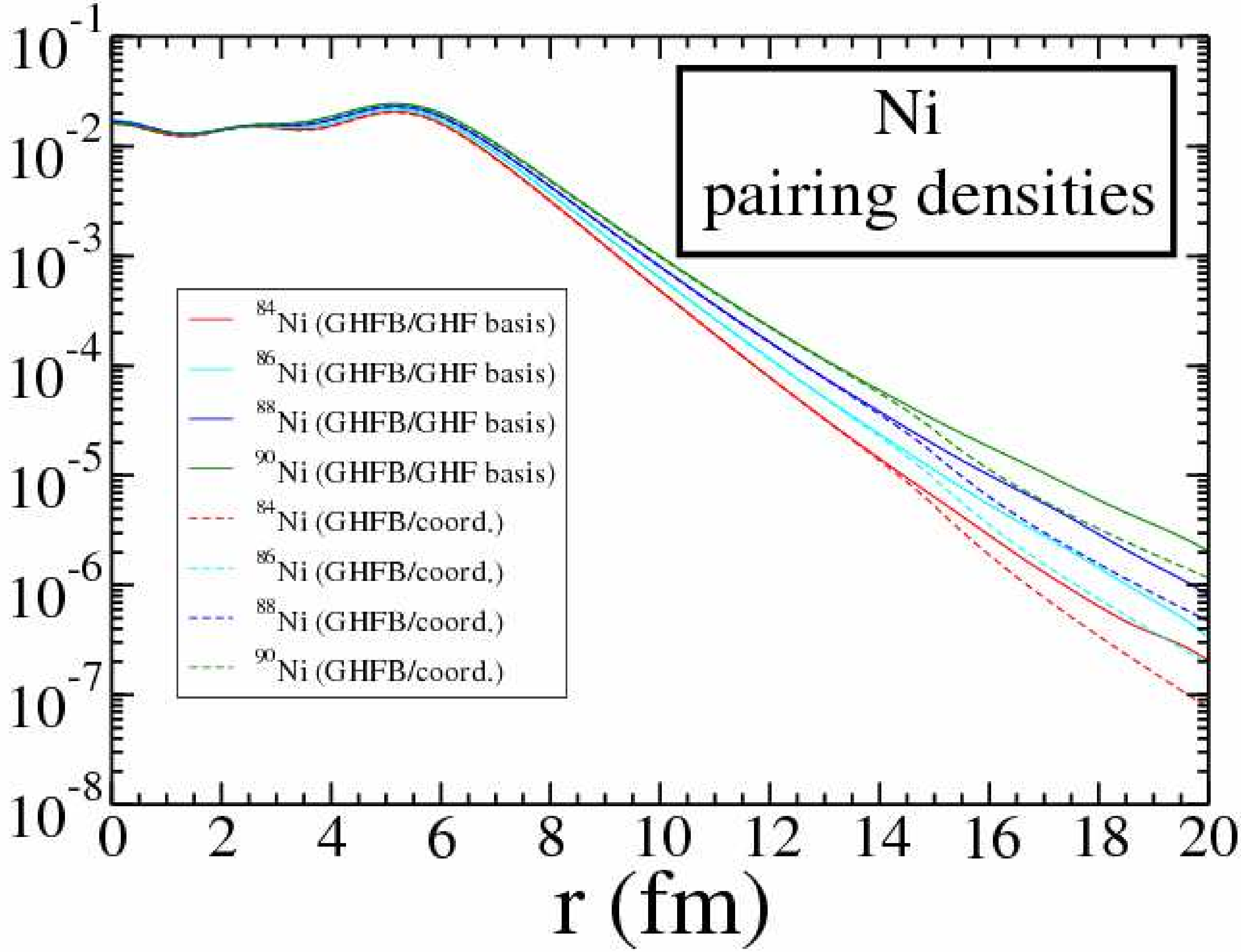
90Ni
pairing density
HFB/Box
GHFB/GHF basis
GHFB/coord.


Ni
pairing densities
^{84}Ni (GHFB/GHF basis)
^{86}Ni (GHFB/GHF basis)
^{88}Ni (GHFB/GHF basis)
^{90}Ni (GHFB/GHF basis)
^{84}Ni (GHFB/coord.)
^{86}Ni (GHFB/coord.)
^{88}Ni (GHFB/coord.)
^{90}Ni (GHFB/coord.)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0 2 4 6 8 10 12 14 16 18 20
r (fm)

Spherical proton emitters with Skyrme density functional

- Simple case: ^{185}Bi. Pairing for neutrons only, protons ~ Gamow HF
  Imaginary part of HF proton potential removed.
  Real bound states, positive width for resonant states
  Sly4 interaction, pairing window of 60 MeV

- ^{185}Bi: HFB/Skyrme: E = 2.720 MeV, $\Gamma = 9.228\ 10^{-11}$ keV ($0h_{9/2}$)
  Experimental: S_p = 1.540 MeV, $\Gamma \sim 10^{-17}$ keV (Phys. Rev. Lett., **76** (1996) 592)

- Proton emitters: Fit of density functionals with respect to resonant s.p. levels

PTG basis for HFB calculations

- Gamow HF(B) basis:

  Advantages: good asymptotics, smoothly varying continuums

  Inconvenients: complex arithmetic, long calculations

- Weakly bound systems: real continuous bases sufficient
- Real Gamow HF basis: problematic due to resonant structure in continuum

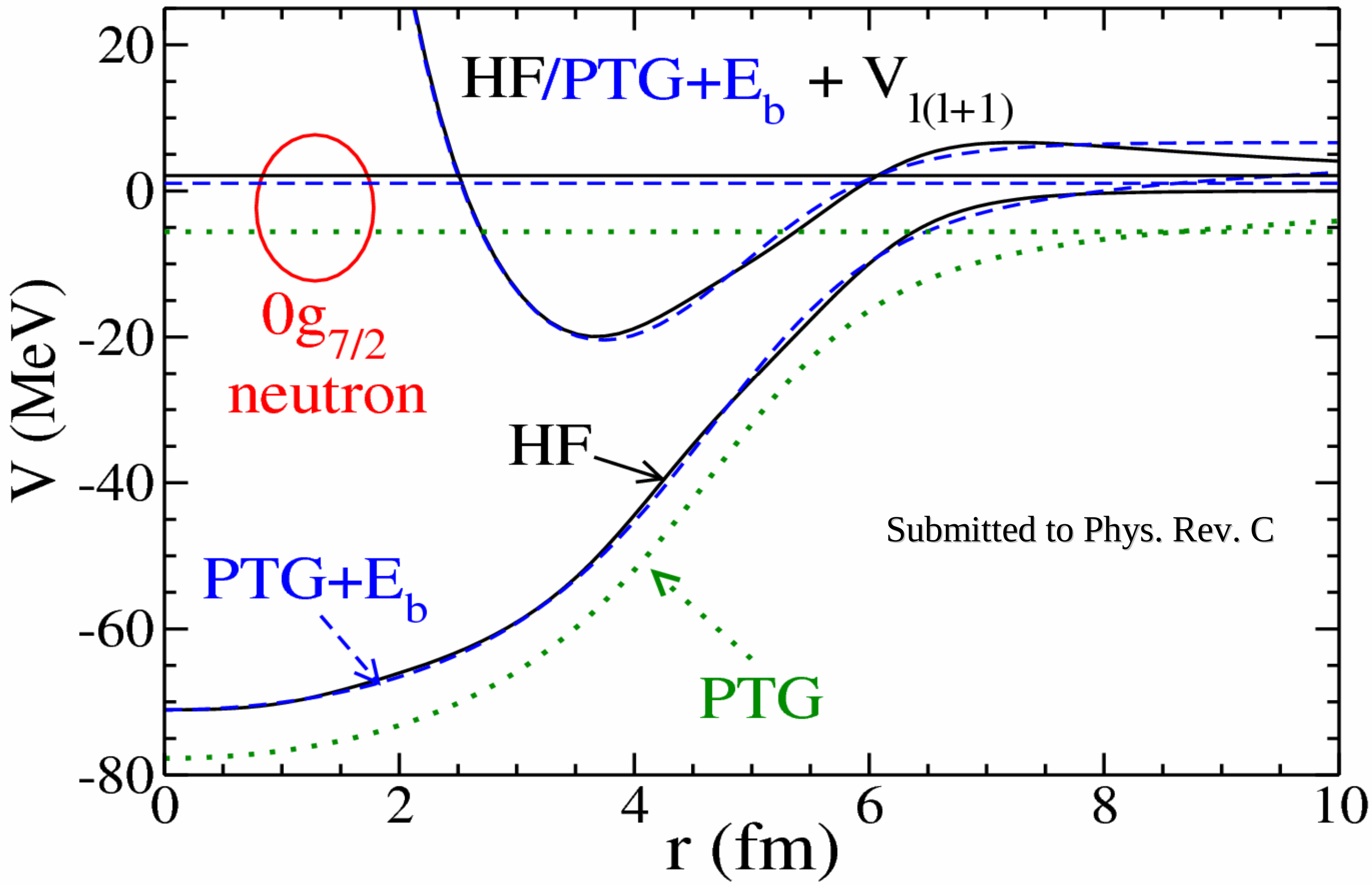
- PTG basis: resonances replaced by bound states

  No resonant state in (l,j) partial wave: Hankel/Coulomb functions

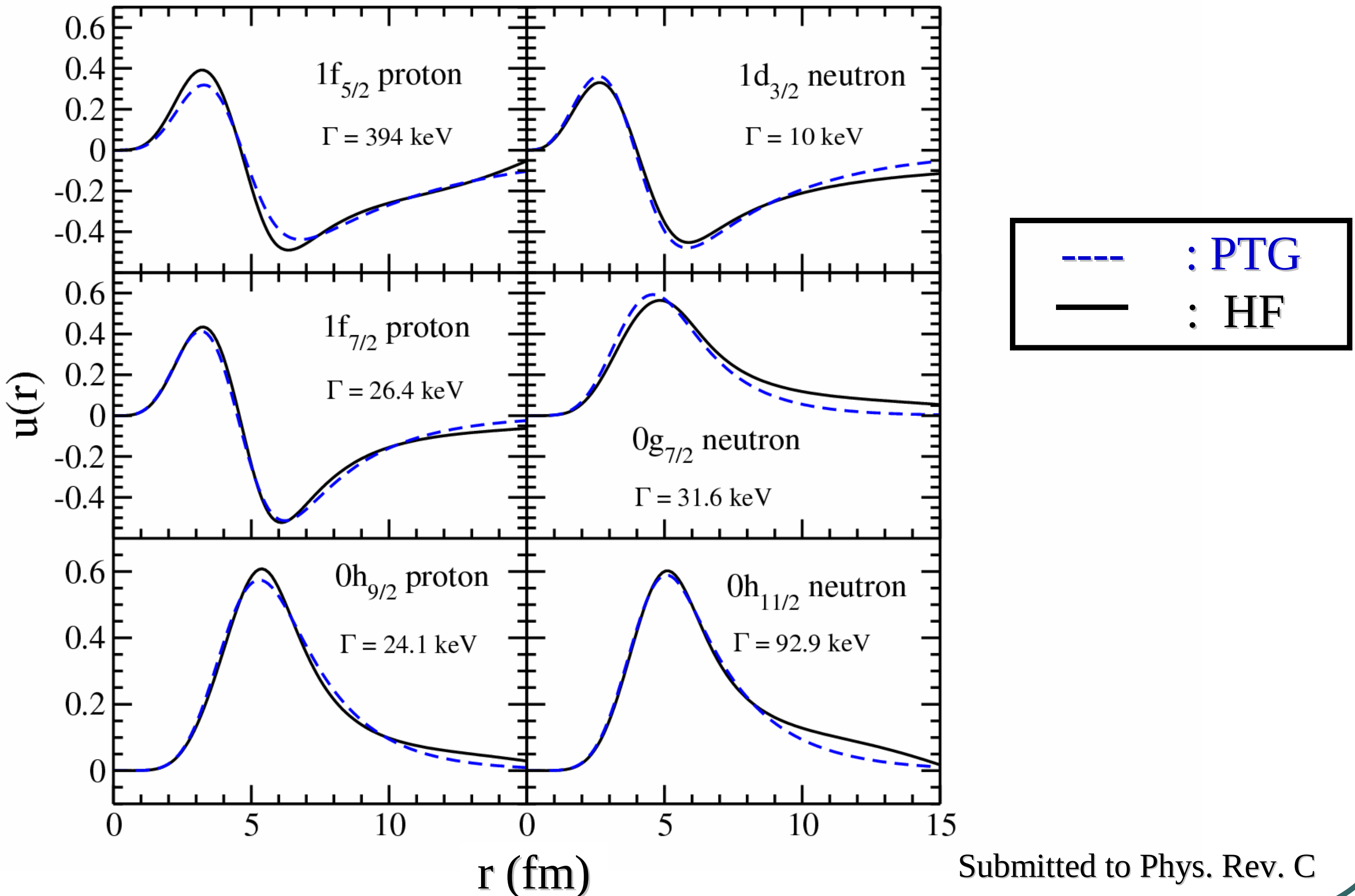
  Smooth continuums for all partial waves

  Weakly bound systems asymptotics well described

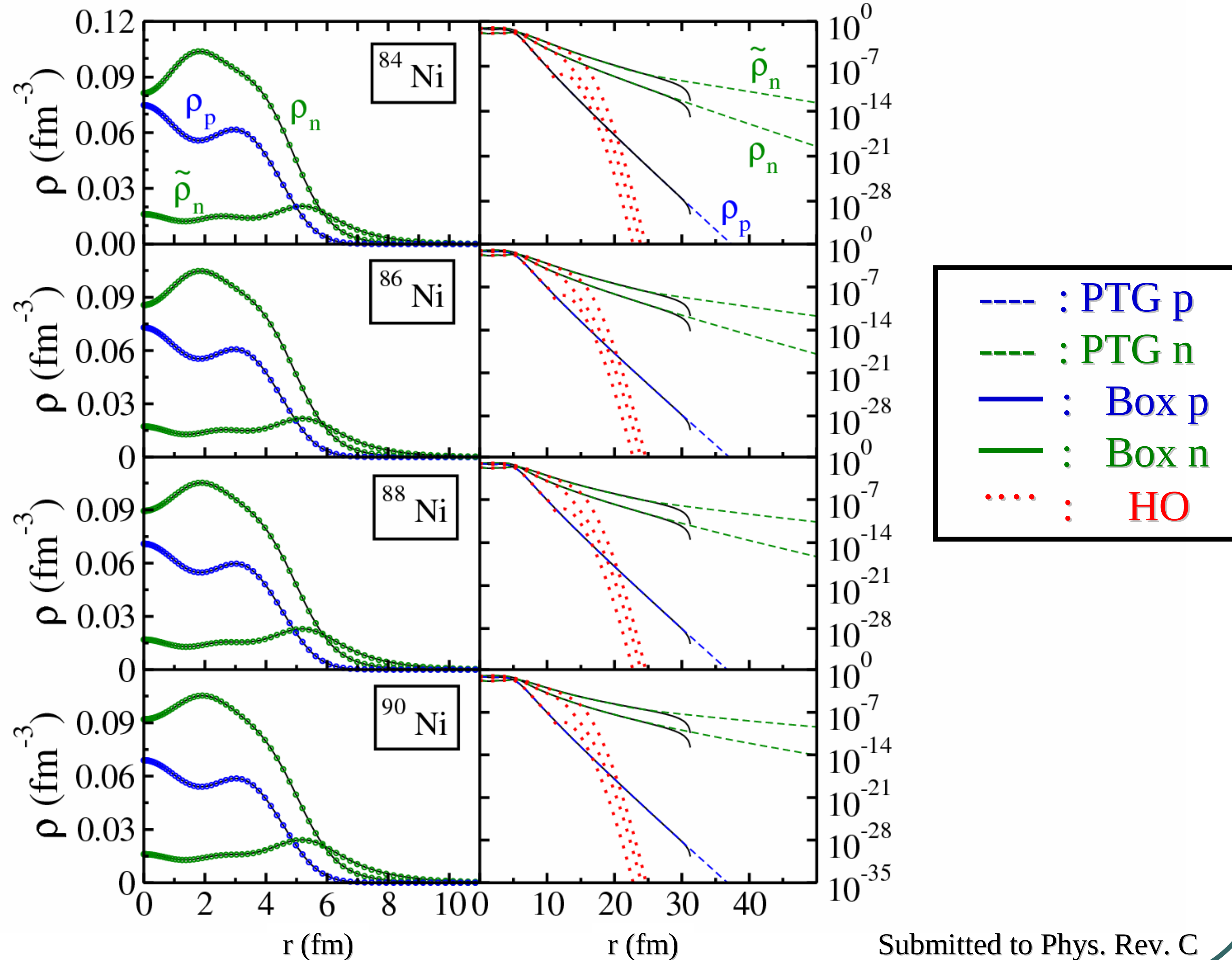
HF/PTG potentials



HF/PTG wave functions



Submitted to Phys. Rev. C



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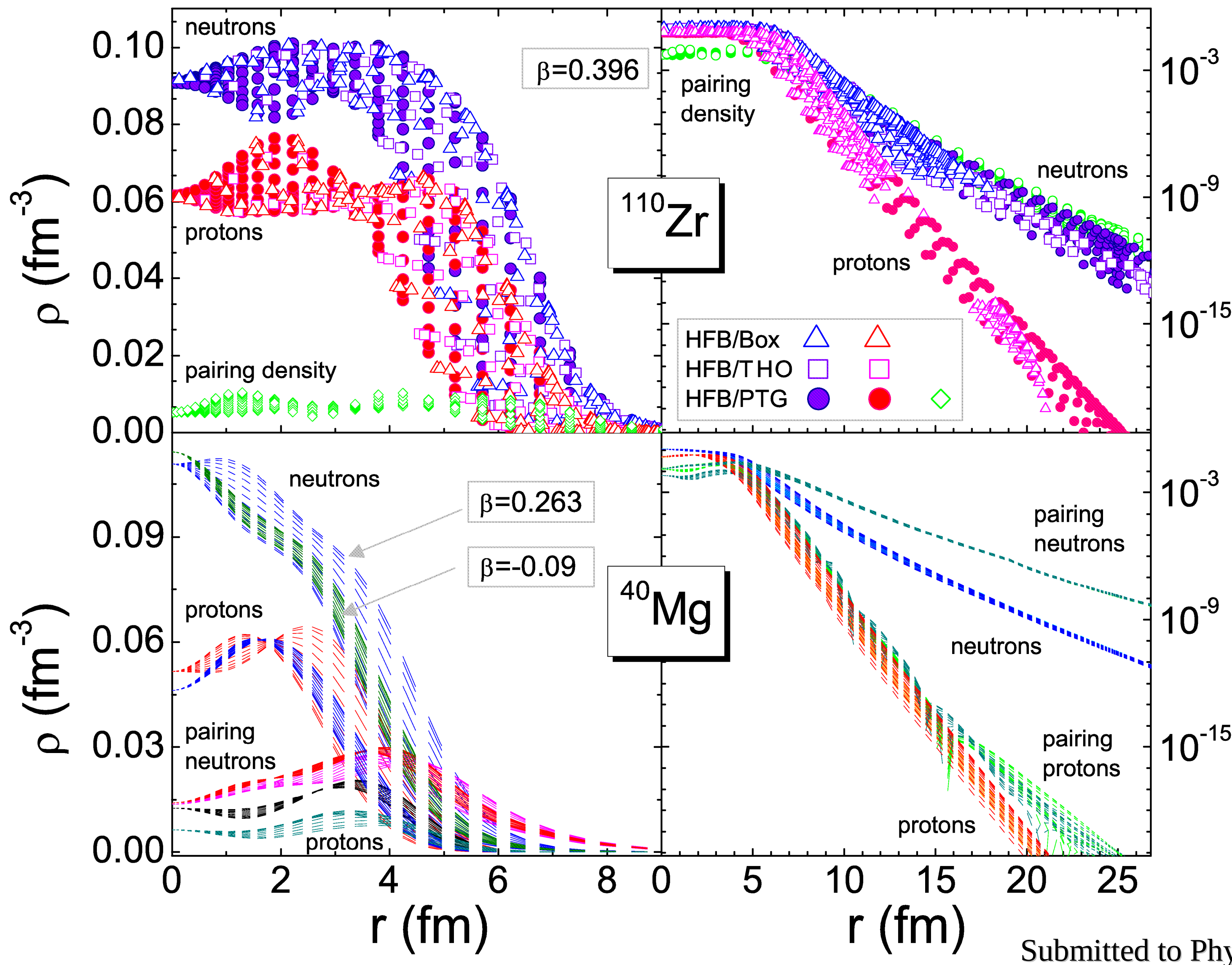

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Conclusion and perspectives

- HFB expansions with Gamow and PTG bases:
  Precise tool to study dripline heavy nuclei
  Continuum fully taken into account
  PTG basis near-optimal for weakly bound systems

- First applications
  Nickel chain close to neutron dripline
  Proton emitter without proton pairing : ^{185}Bi
  Deformed nuclei with Mg and Zn, prolate and oblate deformation

- Conclusions and perspectives
  Weakly bound nuclei : fast and stable method with PTG basis
  Problems remain for unbound nuclei