

# Updating Finite Element Models to Match Ground Vibration Test Data

*Chan-gi Pak, Ph.D.*

Leader, Structural Dynamics Group  
Aerostructures Branch (Code RS)  
NASA Dryden Flight Research Center



# Structural Dynamics Group

---

## ☐ Functionality

- ❖ Aeroelastic & Aeroservoelastic System Analysis, Clearance, Monitoring, & Research

## ☐ Skills

- ❖ Structural Dynamic Finite Element Modeling, Analyses, & Tool Development

- Use ProE, MSC/PATRAN, & MSC/NASTRAN codes for Structural Modeling & Analyses
- In-house Tool Development for Structural Dynamic, Aeroelastic, & Aeroservoelastic Analyses

- ❖ Ground Vibration Test and Finite Element Model Update

- Improve Structural Dynamic FEM if needed

- ❖ Aeroelastic and Aeroservoelastic Analyses

- Flutter, Buzz, Divergence, and Closed-Loop Flutter Analyses
- Subsonic and Supersonic Flight Regimes: Use Linear Lifting Surface Codes (ZAERO or MSC/NASTRAN)
- Transonic Flight Regime: Use 3D CFD Codes (CFL3D version 4 or CAPTSDv etc.)

- ❖ Structural Optimization with Stress/Strain and Flutter Constraints

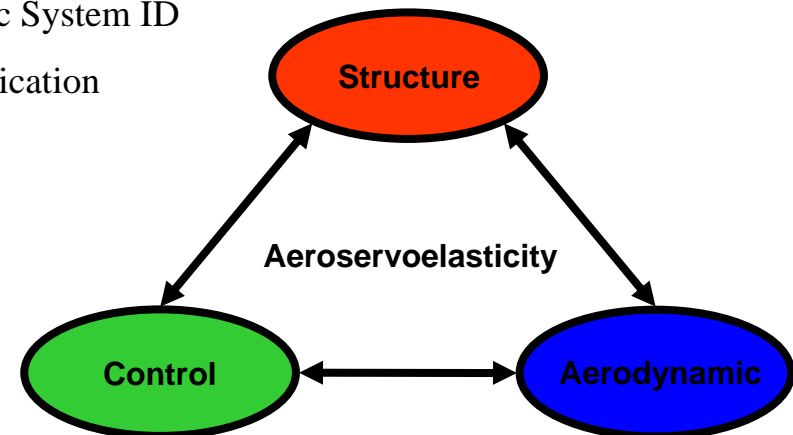
- Based on MSC/NASTRAN code



# Structural Dynamics Group (continued)

## ❑ Skills (continued)

- ❖ Structural Mode Interaction Test and Flight Control Model Update
  - Improve Flight Control Model if needed
- ❖ Maneuver Load Alleviation and Control
  - Based on Minimization of the Maximum Bending Moment and/or Shear Force
- ❖ Active Aeroelastic Control and Vibration Suppression
  - Based on Modern and Adaptive Control Techniques
- ❖ Flight Flutter Testing & On-Line System Identification (Flutterometer)
  - Flutter Boundary Identification based on Flight Test Data
  - Linear and Nonlinear Robust Aeroservoelastic System ID
  - Time-frequency-scale (wavelet, HHT) Identification
- ❖ Structural Health Monitoring
  - Use GVT & Mode Matching Technique
  - Linear/Nonlinear ID Methods





# Introduction

---

- ❑ Everyone believes the test data except for the experimentalist, and no one believes the finite element model except for the analyst.
  - ❖ Some of the discrepancies come from analytical Finite Element modeling uncertainties, noise in the test results, and/or inadequate sensor and actuator locations.
  
- ❑ MIL-STD-1540C Section 6.2.10
  - ❖ Test Requirements for Launch, Upper-Stage, & Space Vehicles
  - ❖ Less than 3% and 10% frequency errors for the primary and secondary modes, respectively
  - ❖ Less than 10% off-diagonal terms in mass matrix
  
- ❑ AFFTC-TIH-90-001 (Structures Flight Test Handbook)
  - ❖ If measured mode shapes are going to be associated with a finite element model of the structure, *it will probably need to be adjusted to match the lumped mass modeling of the analysis.*
  - ❖ Based on the measured mode shape matrix  $[\Phi]$  and the analytical mass matrix  $[M]$ , the following operation is performed.

$$\Phi^T M \Phi$$

The results is near diagonalization of the resulting matrix with values close to 1 on the diagonal and values close to zero in the off-diagonal terms. Experimental reality dictates that the data will not produce exact unity or null values, so *10 percent of these targets are accepted as good orthogonality* and the data can be confidently correlated with the finite element model.



# Orthogonality Requirements for Structural Dynamics

---

$$\Phi^T \mathbf{M} \Phi = [\mathbf{I}]$$

$$\Phi^T \mathbf{K} \Phi = [\omega^2]$$

**M**: Mass matrix

**K**: Stiffness matrix

$\Phi$ : Mode shaped (Eigen matrix)

$\omega$ : Frequencies (Eigen Values)

- Guarantee linear independency between mode shapes
- Superposition principle can be used for the aeroelastic and aeroservoelastic analyses



# FEM Based Flutter Analysis: Approach #1

## □ Update Mass

- ❖ Match Total Weight
- ❖ Match C.G. Location

## □ Update Stiffness

- ❖ Frequency difference
- ❖ Goal=5% (Primary modes) ~ 10% (Secondary modes)

## □ Flutter Analysis

- ❖ Based on analytical mass & modes
- ❖ NOT based on GVT Mode Shapes

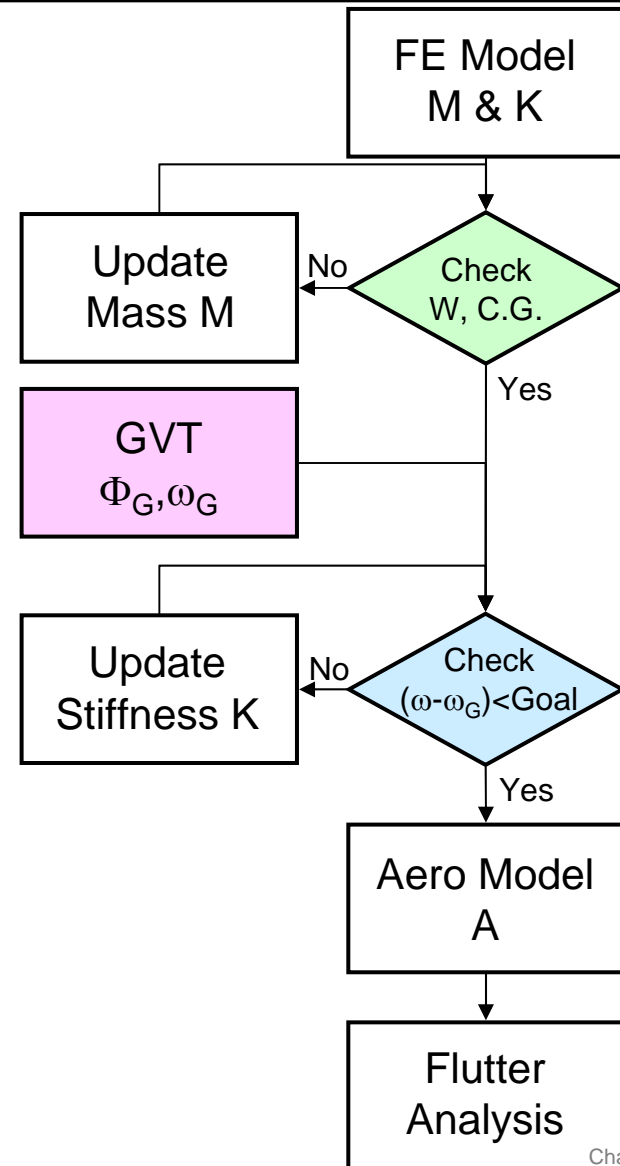
## □ Summarize

- ❖ FEM updated manually
- ❖ Best estimated mass

❖  $\Phi_G^T M \Phi_G \neq [I]$  ←

## □ Applications

- ❖ F-18 SRA, AAW, ATW, & B-52B





# GVT Based Flutter Analysis: Approach #2

## □ Update Mass

- ❖ Mass Model has to be created
- ❖ Match Total Weight
- ❖ Match C.G. Location

## □ Flutter Analysis

- ❖ Based on GVT modes & Analytical Mass Matrix

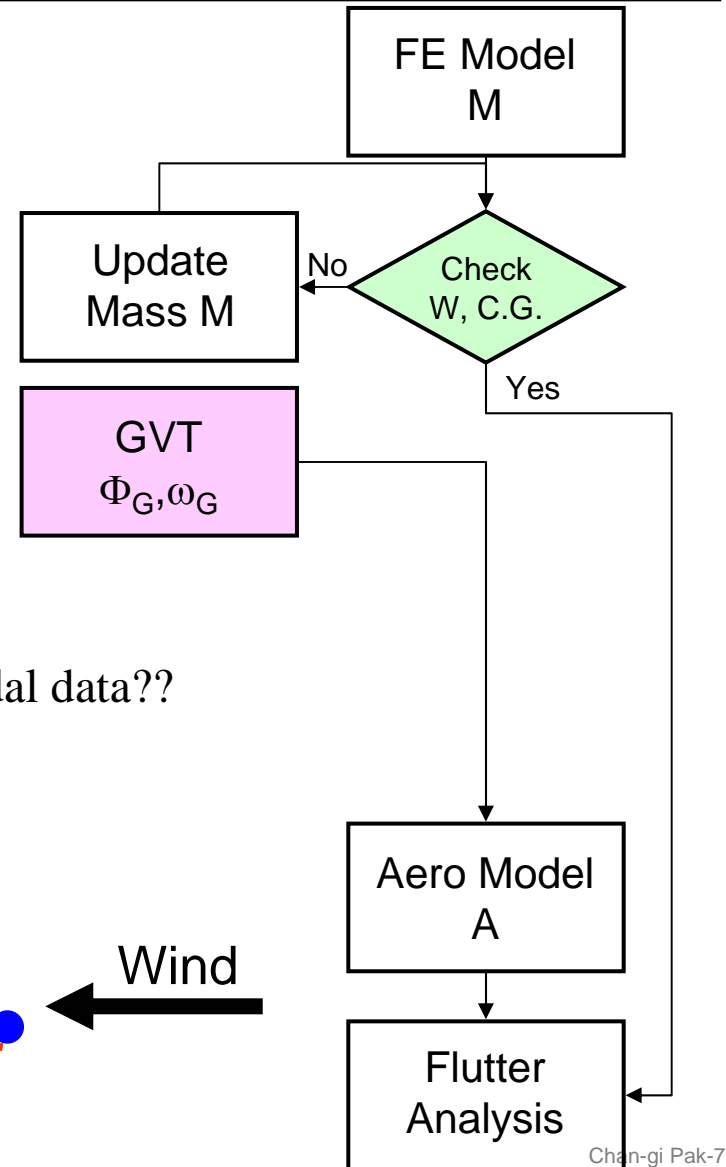
## □ Summarize

- ❖ Accuracy and completeness of the measured modal data??
- ❖ Best estimated mass

❖  $\Phi_G^T M \Phi_G \neq [I]$

## □ Applications

- ❖ All F-15B experiments





# Updated FEM Based Flutter Analysis: New Approach

- ❑ Update Mass
  - ❖ Minimize errors in total weight, C.G. location, and mass moment of inertia
  - ❖ Minimize off diagonal terms in orthogonal mass matrix

- ❑ Update Stiffness
  - ❖ Minimize errors in frequencies
  - ❖ Minimize errors in mode shapes and/or minimize off diagonal terms in orthogonal stiffness matrix

- ❑ Flutter Analysis

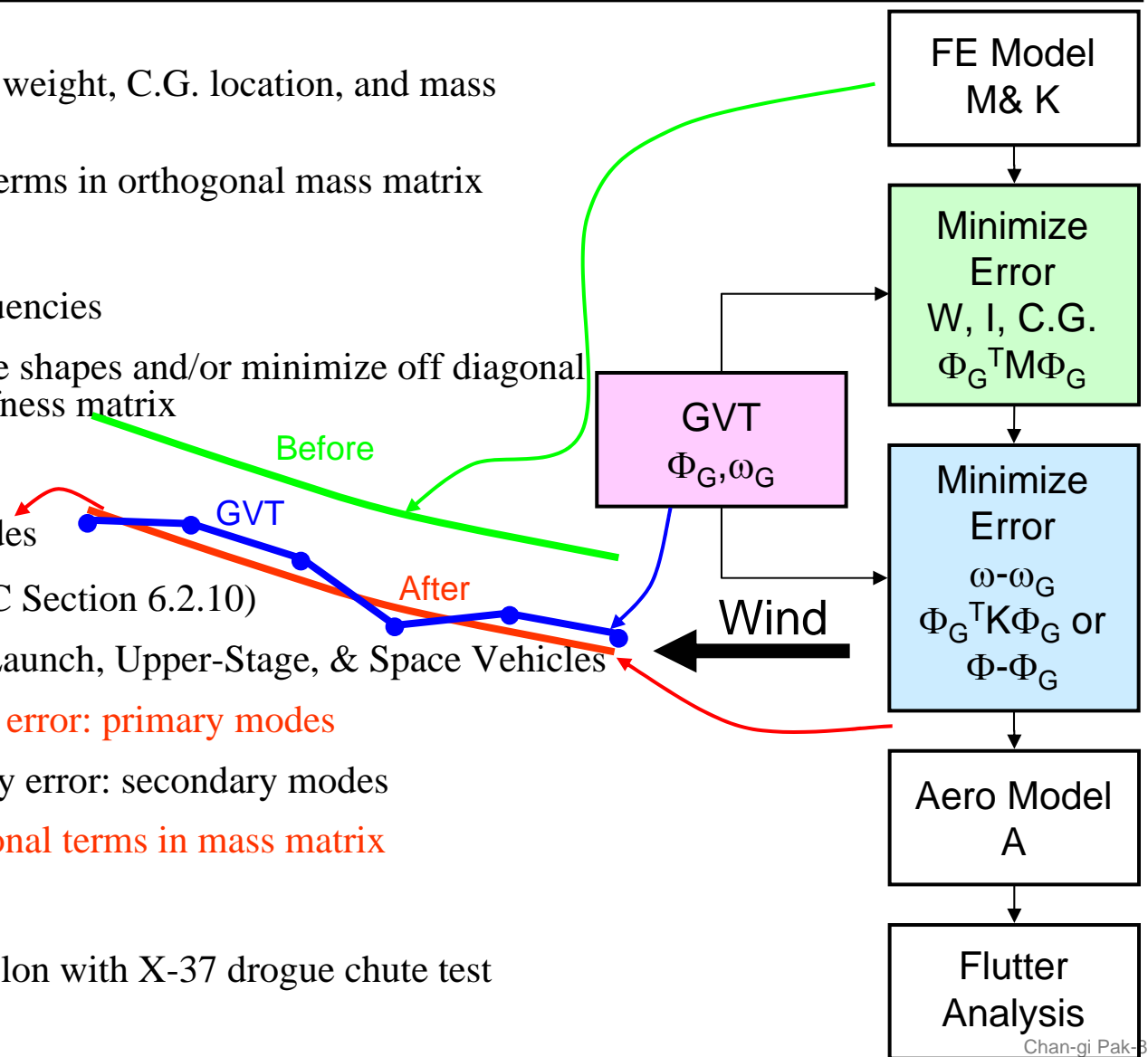
- ❖ Based on analytical modes

- ❑ Discussion (MIL-STD-1540C Section 6.2.10)

- ❖ Test Requirements for Launch, Upper-Stage, & Space Vehicles
  - ❖ **Less than 3% frequency error: primary modes**
  - ❖ Less than 10% frequency error: secondary modes
  - ❖ **Less than 10% off-diagonal terms in mass matrix**

- ❑ Applications

- ❖ X-43A Stack, B-52H Pylon with X-37 drogue chute test fixture, & X-37 ALTV



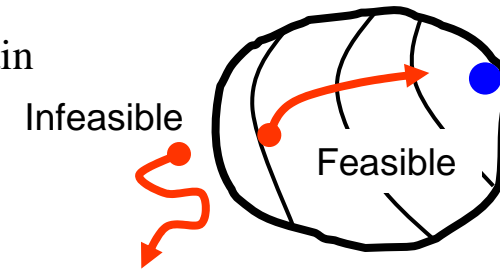




# Model Update Technique

## □ Step 1: Mass Properties

- ❖ To start optimization procedure inside the feasible domain
- ❖ Match Total Mass
- ❖ Match CG Locations
- ❖ Match Mass Moment of Inertias



Statement Number	Objective Function	Constraints
1	$J_1 = W - W_G$	Unconstraint
2	$J_2 = X - X_G$	$ J_1  < \epsilon$
3	$J_3 = Y - Y_G$	$ J_i  < \epsilon \quad i=1,2$
4	$J_4 = Z - Z_G$	$ J_i  < \epsilon \quad i=1, \dots 3$
5	$J_5 = I_{XX} - (I_{XX})_G$	$ J_i  < \epsilon \quad i=1, \dots 4$
6	$J_6 = I_{YY} - (I_{YY})_G$	$ J_i  < \epsilon \quad i=1, \dots 5$
7	$J_7 = I_{ZZ} - (I_{ZZ})_G$	$ J_i  < \epsilon \quad i=1, \dots 6$
8	$J_8 = I_{XY} - (I_{XY})_G$	$ J_i  < \epsilon \quad i=1, \dots 7$
9	$J_9 = I_{YZ} - (I_{YZ})_G$	$ J_i  < \epsilon \quad i=1, \dots 8$
10	$J_{10} = I_{ZX} - (I_{ZX})_G$	$ J_i  < \epsilon \quad i=1, \dots 9$



# Model Update Technique (Continued)

---

- Step 2: Improve Mass Matrix
  - ❖ Orthonormalized Mass Matrix:  $\underline{\mathbf{M}} = \Phi^T \mathbf{M} \Phi$
  - ❖ Minimize J

$$J \equiv \sum_{i=1, j=1, i \neq j}^n \underline{\mathbf{M}}_{ij}$$

Such that,

- $|W - W_G| < \varepsilon$  : Total Mass
- $|X - X_G| < \varepsilon, |Y - Y_G| < \varepsilon, \& |Z - Z_G| < \varepsilon$ : CG Locations
- $|I_{XX} - (I_{XX})_G| < \varepsilon, |I_{YY} - (I_{YY})_G| < \varepsilon, |I_{ZZ} - (I_{ZZ})_G| < \varepsilon, |I_{XY} - (I_{XY})_G| < \varepsilon, |I_{YZ} - (I_{YZ})_G| < \varepsilon, |I_{ZX} - (I_{ZX})_G| < \varepsilon$ : Mass Moment of Inertia at CG
- Positive Definiteness of Lumped Masses



# Model Update Technique (continued)

## □ Step 3: Frequencies and Mode Shapes

### ❖ Option 1: Minimize Errors in Frequencies and off-diagonal terms in $\underline{\mathbf{K}}$

■ Orthonormalized Stiffness Matrix:  $\underline{\mathbf{K}} = \Phi^T \mathbf{K} \Phi$

■ Minimize J

$$J \equiv \sum_{i=1}^n \left| \frac{\Omega_i^2 - \underline{\mathbf{K}}_{ii} / \underline{\mathbf{M}}_{ii}}{\Omega_i^2} \right| + \beta \sum_{i=1, j=1, i \neq j}^n \underline{\mathbf{K}}_{ij}$$

### ❖ Option 2: Minimize Errors in Frequencies and Mode Shapes

■ Eigen-Solver is based on

- Subspace Iteration Method
- Simplified Approach

■ Minimize J

$$J \equiv \sum_{i=1}^n \left| \frac{\Omega_i^2 - \underline{\mathbf{K}}_{ii} / \underline{\mathbf{M}}_{ii}}{\Omega_i^2} \right| + \beta \sum_{j=1}^m (\Phi_j - \Phi_{jG})$$

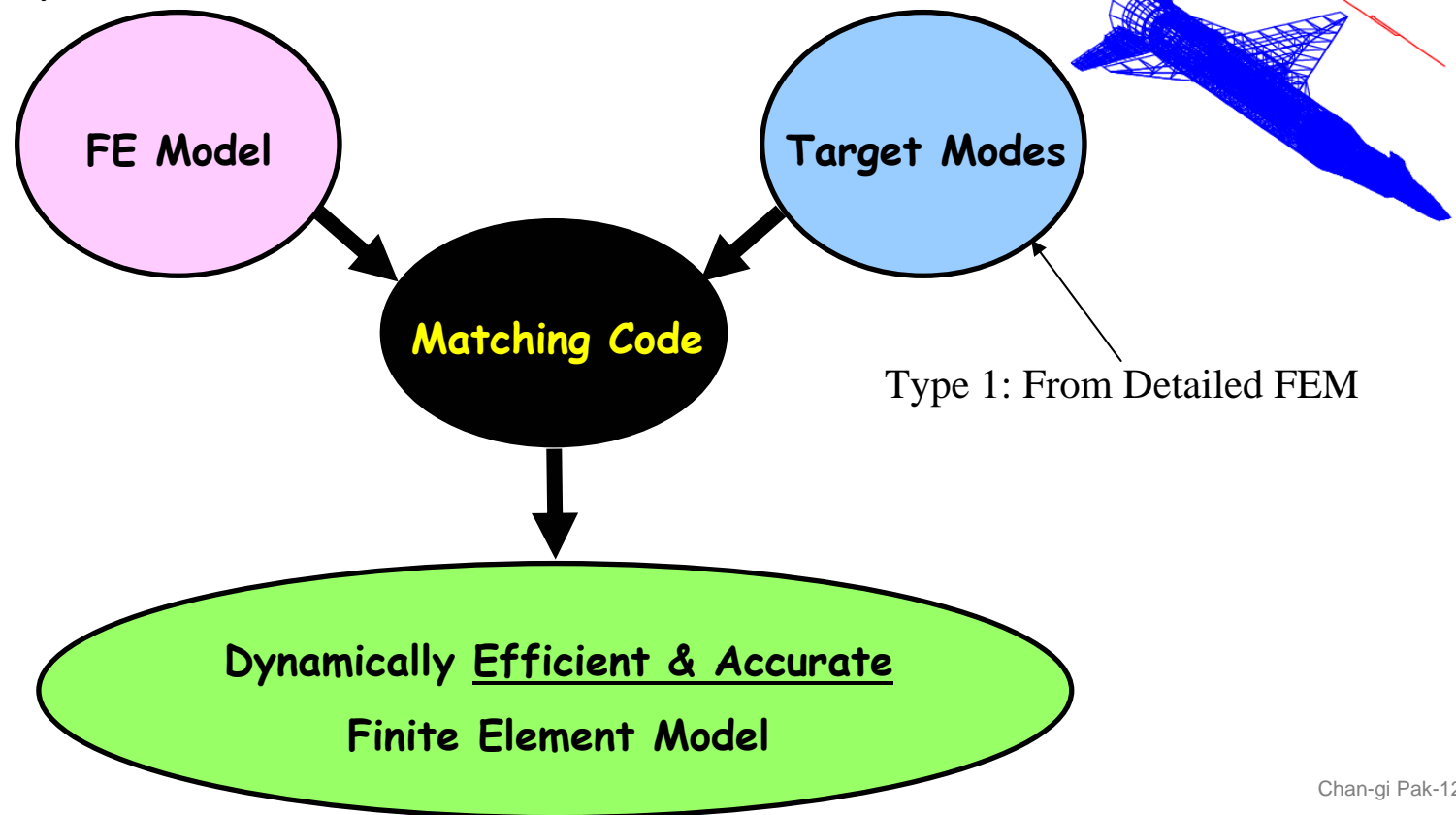
where,  $i=1, \dots, n$   $j=1, \dots, m$   $n$ : number of modes  $m$ : number of sensors



# Application of Mode Matching Technique: Type 1

## □ Generation of a Reduced Order Finite Element Model

- ❖ More Accurate than a Simple Beam Model
- ❖ More Efficient than Detailed FEM
- ❖ Maintain Accuracy of Detailed FEM
- ❖ Match Analytical Modes Obtained from Detailed and Reduced Order FEMs

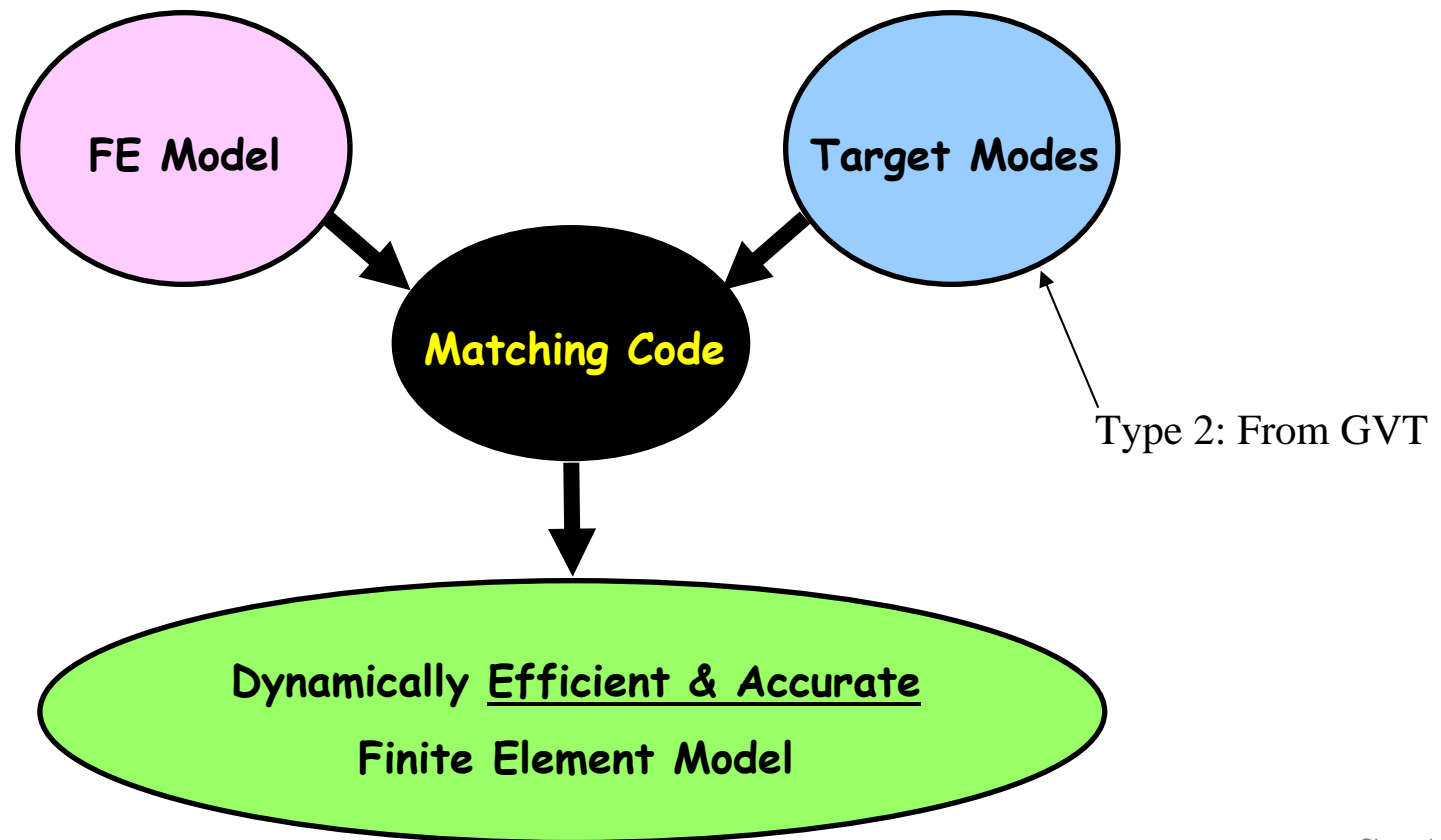




## Application of Mode Matching Technique: Type 2

### □ Finite Element Model Update using GVT Data

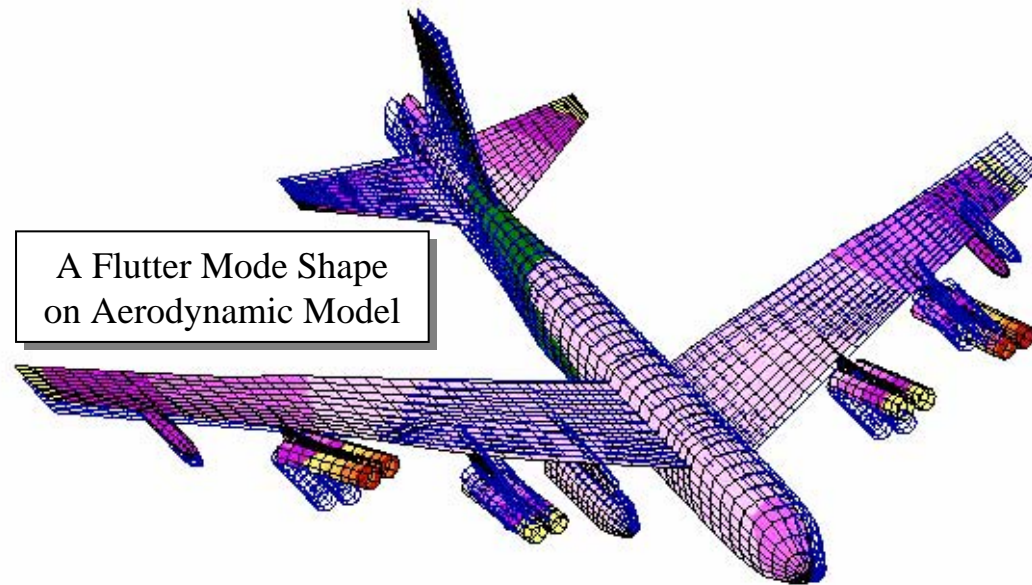
- ❖ Minimize the structural modeling error in aeroelastic and/or aeroservoelastic stability analyses.





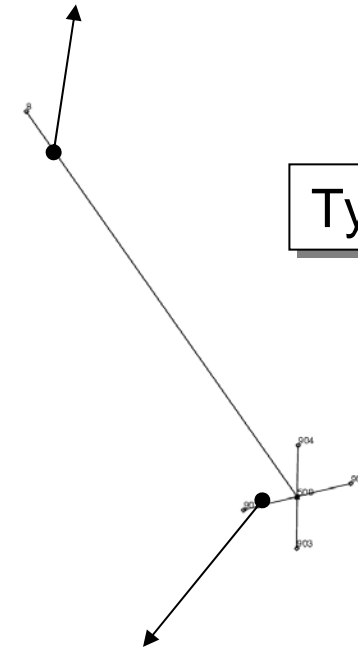
# Case 1: B-52H Engine Modeling using GVT Data

- ❑ Half aircraft model from Boeing Wichita
- ❑ Make tip-to-tip model
- ❑ Use B-52B Engine Properties as an initial B-52H Engine Properties
  - ❖ GVT data for B-52H engines
- ❑  $I_1$ ,  $I_2$ , and  $J$ : Design Variables



A Flutter Mode Shape on Aerodynamic Model

## Single Beam Engine Model



## Rigid Bars For Mode Visualization



# Case 1: Results

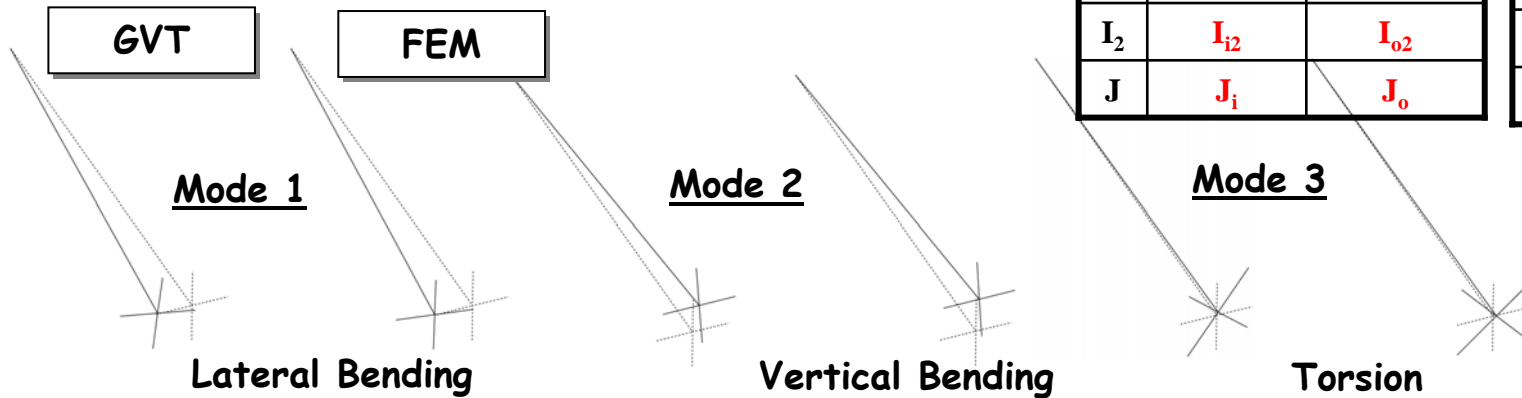
## Initial Beam Properties

## Updated Beam Properties

Mode	Inboard Engine (Hz)		Outboard Engine (Hz)	
	Initial FEM	Final FEM	Initial FEM	Final FEM
1	-8.3%	0%	-7.6%	0%
2	-14%	-0.02%	-6.2%	-0.03%
3	-3.7%	0%	-4.7%	-0.02%

	Inboard Engine	Outboard Engine
E	$E_i$	$E_o$
v	$v_i$	$v_o$
A	$A_i$	$A_o$
$I_1$	$I_{i1}$	$I_{o1}$
$I_2$	$I_{i2}$	$I_{o2}$
J	$J_i$	$J_o$

	Inboard Engine	Outboard Engine
E	$E_i$	$E_o$
v	$v_i$	$v_o$
A	$A_i$	$A_o$
$I_1$	<b>1.34 <math>I_{i1}</math></b>	<b>1.14 <math>I_{o1}</math></b>
$I_2$	<b>1.19 <math>I_{i2}</math></b>	<b>1.17 <math>I_{o2}</math></b>
J	<b>1.16 <math>J_i</math></b>	<b>1.18 <math>J_o</math></b>



Mode	Inboard Engine (MAC*)			Outboard Engine (MAC*)		
	Initial FEM	Final FEM	GVT	Initial FEM	Final FEM	GVT
1	98.95	98.98	<b>100</b>	97.79	97.92	<b>100</b>
2	96.37	98.30	<b>100</b>	97.99	99.33	<b>100</b>
3	92.22	89.16	<b>100</b>	88.77	82.71	<b>100</b>

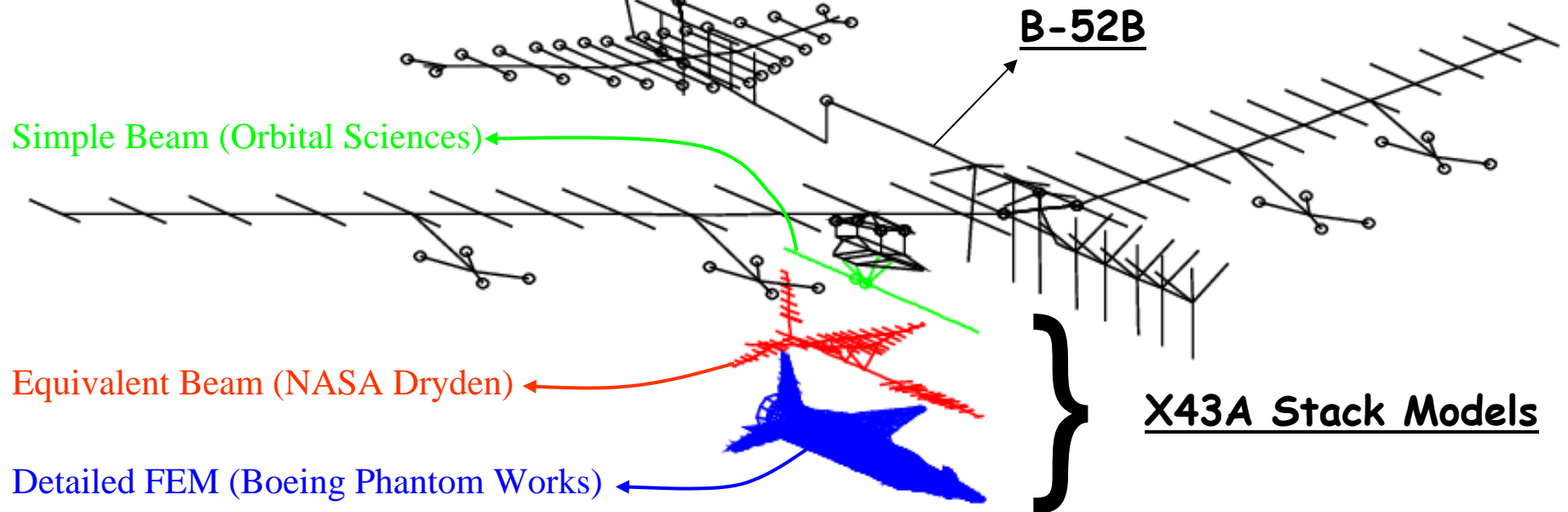


# Case 2: X-43A Stack Equivalent Beam Model



Type 1

		Number of Nodes
B-52B		375
X-43A Stack	Simple Beam	69
	Equivalent Beam	107
	Detailed FEM	31338



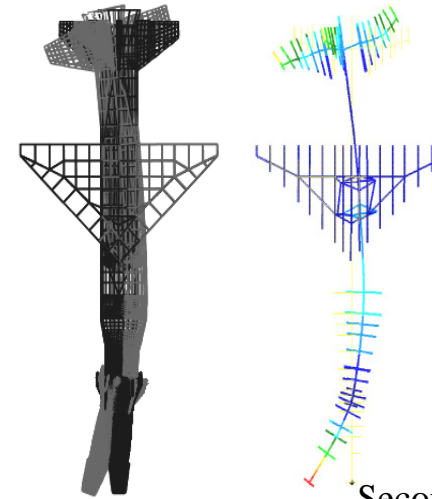




# Case 2: Results

## Frequencies (Hz)

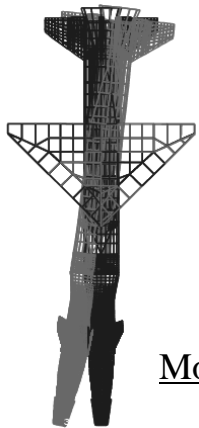
Mode	Simple Beam	Equivalent Beam	Detailed FEM
1	31%	-.09% (99.3)	$f_1$
2	83%	-.02% (89.6)	$f_2$
3	168%	2% (94.8)	$f_3$
4	200%	1% (91.1)	$f_4$
5	159%	-1.1% (82.1)	$f_5$



Mode 5  
Second Lateral Bending

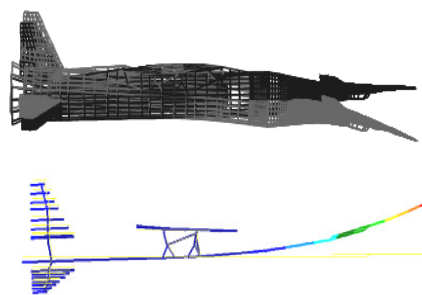
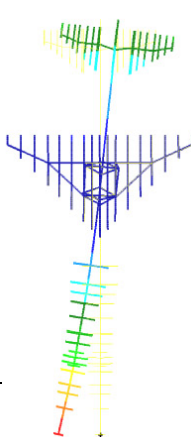
Detail      Equivalent

(\*): MAC Value



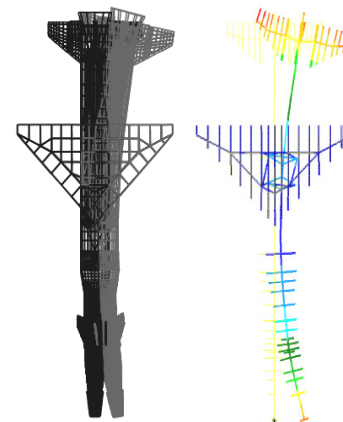
Mode 1

Yawing



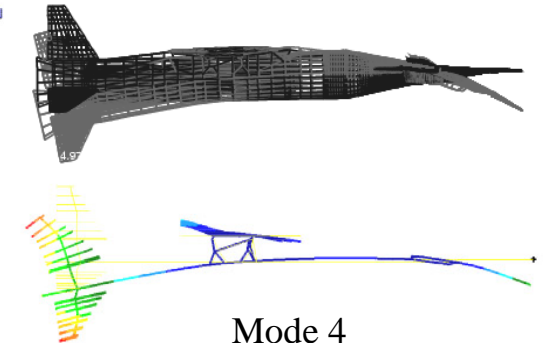
Mode 2

Pitching



Mode 3

Lateral Bending



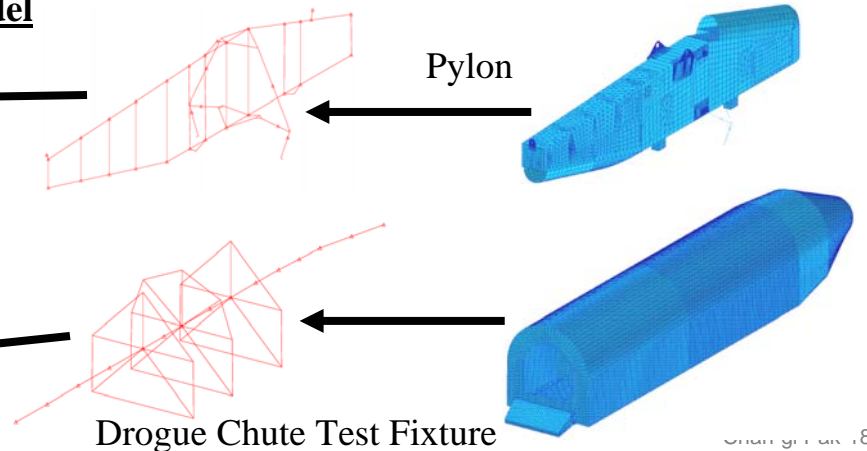
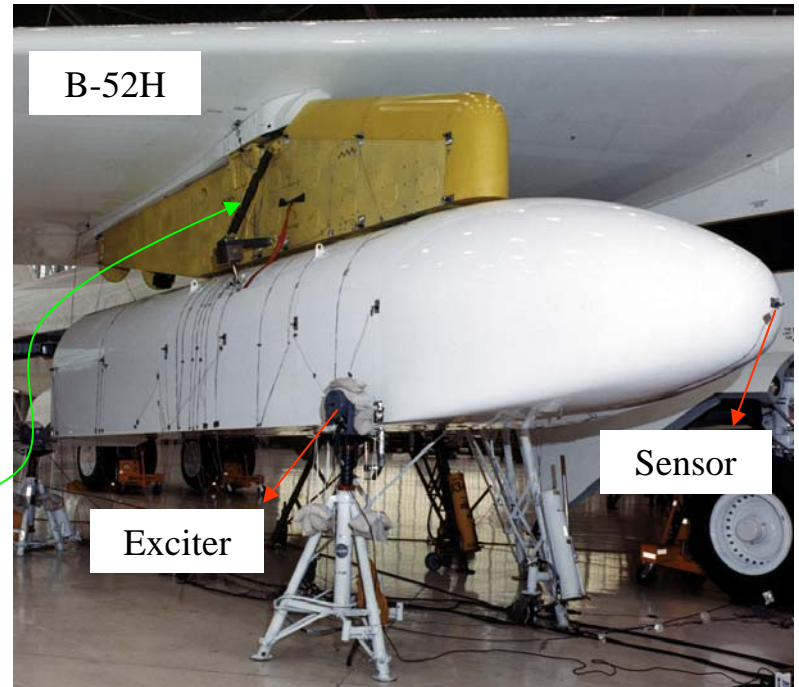
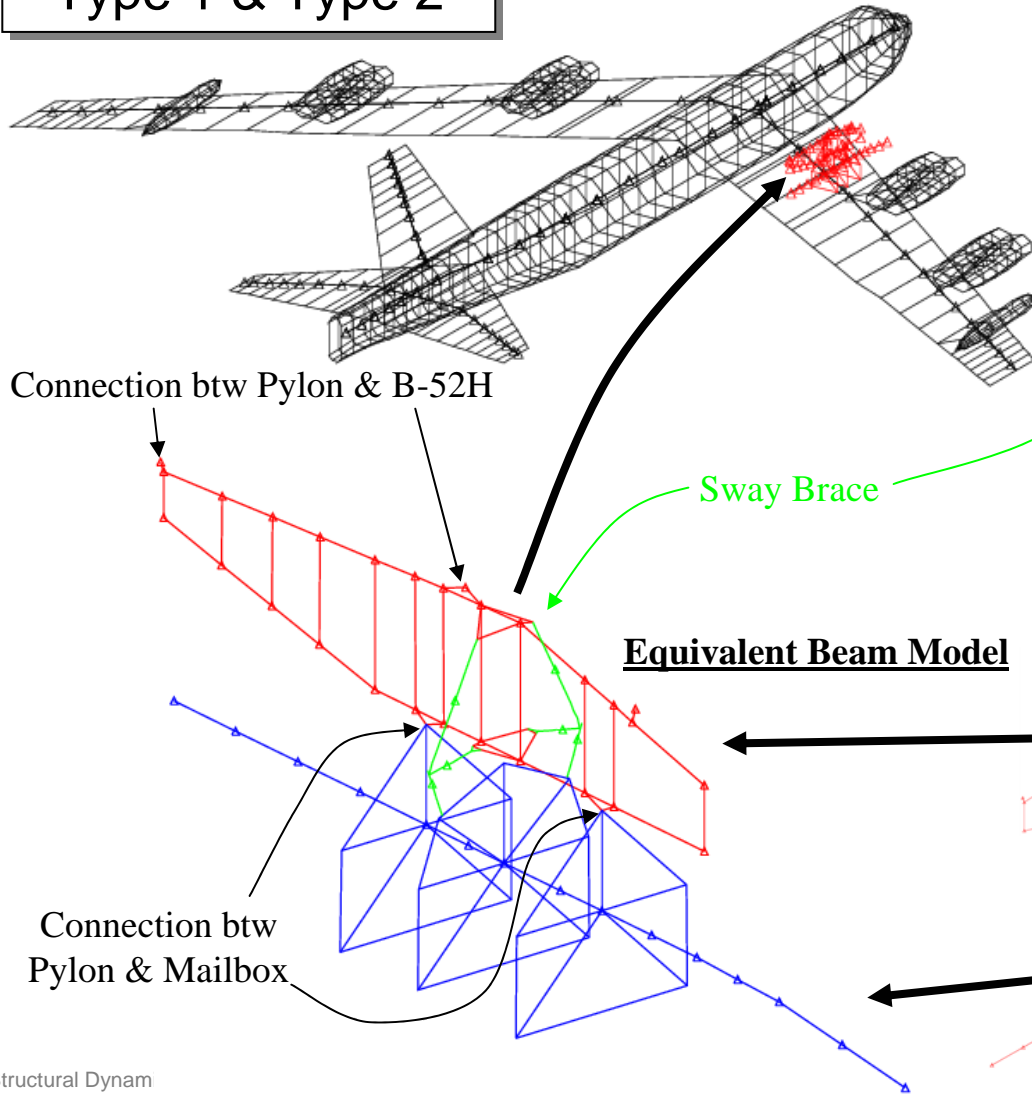
Mode 4

Vertical Bending



# Case 3: B-52H Pylon + X-37 DCTF Model Update using GVT Data

Type 1 & Type 2





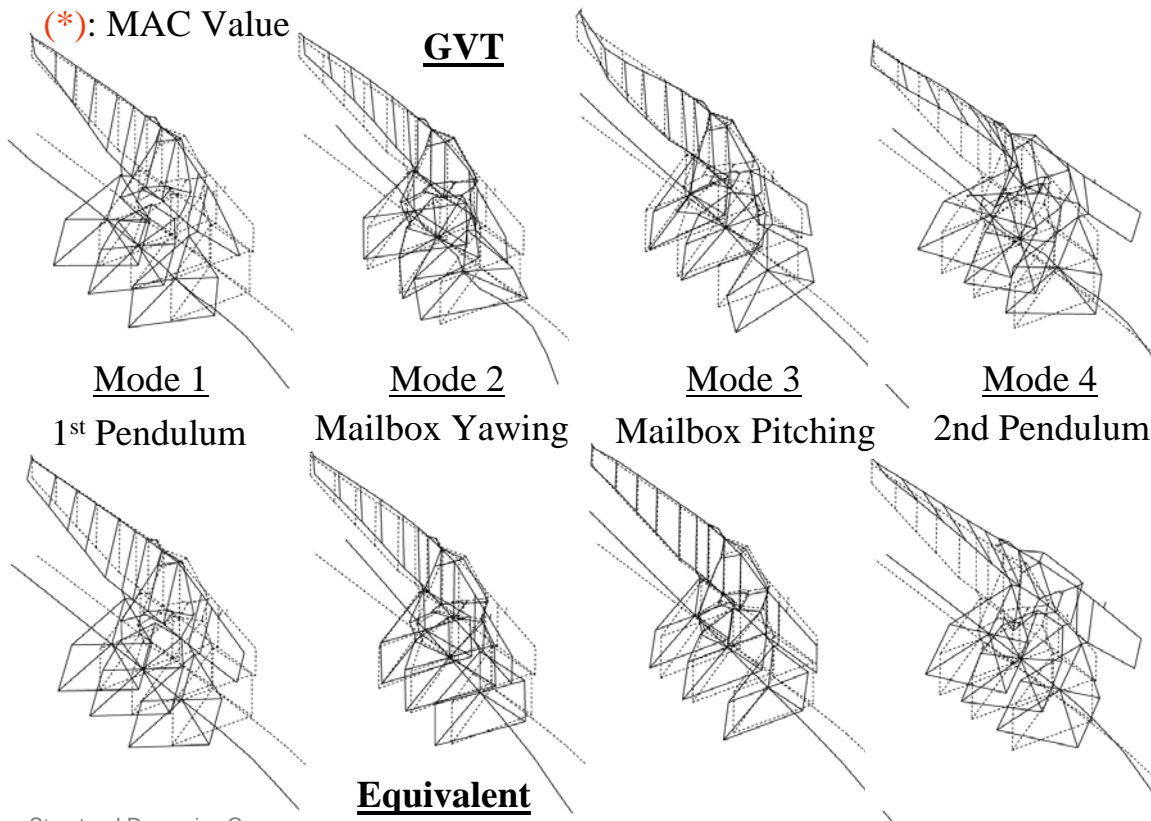
# Case 3: Results

Mode	Equivalent Beam (% error)		MAC	GVT
	Guyan Reduction	Full Order		
1	-0.04%	-0.08%	94.4	$f_1$
2	-0.01%	-0.03%	84.7	$f_2$
3	-0.01%	-0.03%	50.6	$f_3$
4	-0.05%	-4.1%	82.3	$f_4$

Generalized Mass			
1	1%	-2%	-8%
.011	1	-6%	5%
-0.016	-0.063	1	2%
-0.076	.046	.020	1

Generalized Stiffness			
1	2%	-2%	1%
.022	1	-8%	-1%
-0.017	-0.075	1	0%
-0.005	-0.010	-0.001	1

(\*): MAC Value



	Detailed FEM	Equiv. Beam	Error
Weight	-	-	-0.10%
$X_{CG}$	Measured	-	.07%
$Y_{CG}$		-	-0.01%
$Z_{CG}$	-	-	.18%
$I_{XX}$	-	-	.21%
$I_{YX}$	-	-	-0.20%
$I_{YY}$	Computed	-	-0.19%
$I_{ZX}$		-	-0.11%
$I_{ZY}$	-	-	.15%
$I_{ZZ}$	-	-	-0.16%



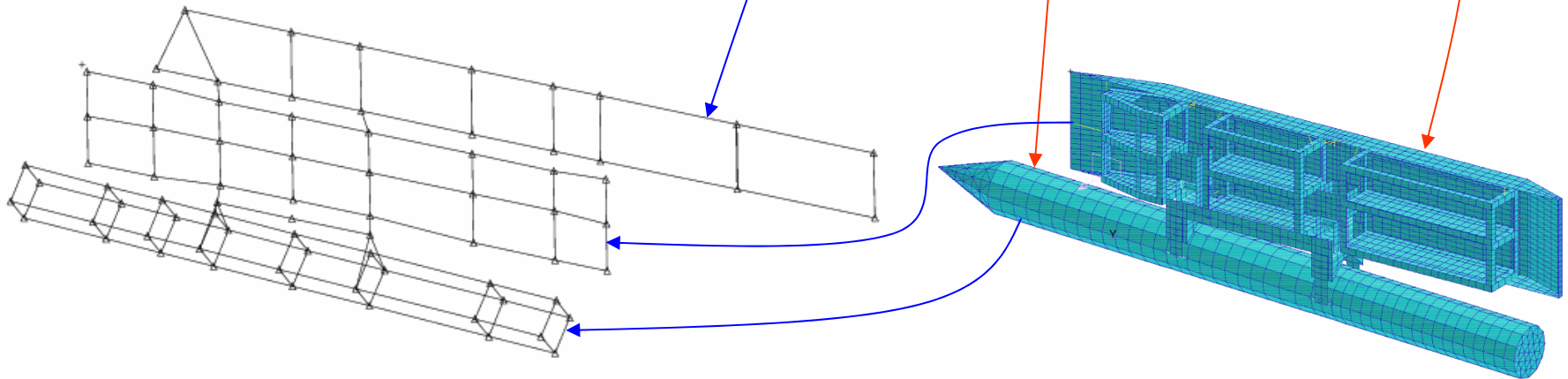
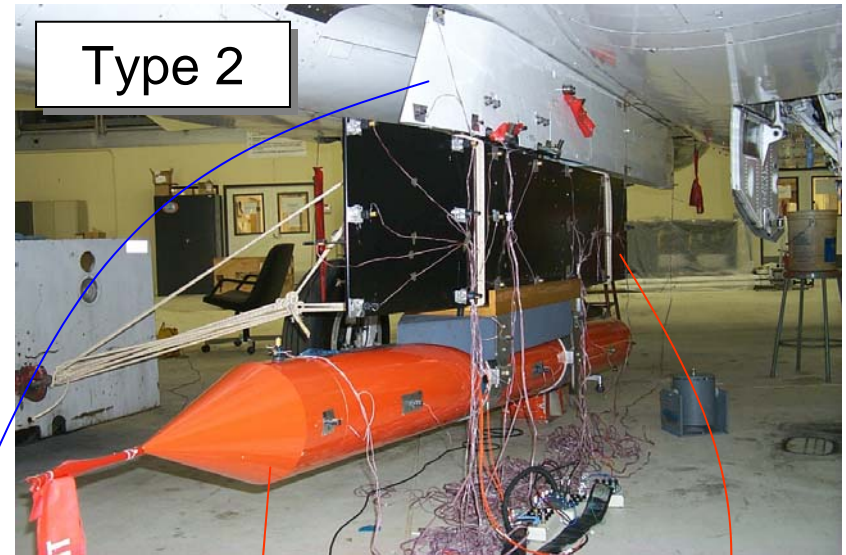
# Case 4: F-15B Cone Drag Experiment

## Task Statements

- ❑ Compare Flutter Boundaries from Previous and New Methodologies for the Flutter Analysis

## Approaches

- ❑ Previous Flutter Analysis: Approach #2
  - ❖ Frequencies & Mode Shapes: From GVT
  - ❖ Mass Matrix: Best Guess Mass Distribution
- ❑ New Flutter Analysis
  - ❖ Frequencies & Mode Shapes: From Equivalent Beam
    - Equivalent Beam is obtained from GVT Mode Matching Technique
  - ❖ Mass Matrix: Orthogonal to GVT mode shapes





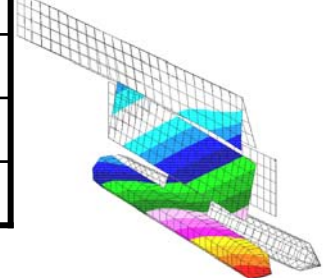


# Case 4: Results

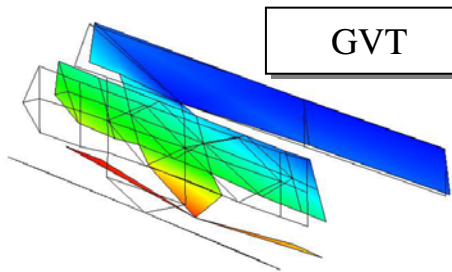
Mode	Equivalent Beam	GVT	Mach #	Approach #2	New Approach	% Diff.
1	-4.65%(98.7)	$f_1$	0.9	-	-	-17.7
2	1.30%(93.5)	$f_2$	1.2	-	-	-18.0
3	2.86%(85.9)	$f_3$	1.6	-	-	-17.3
4	-1.00%(92.9)	$f_4$	2.0	-	-	-16.0

Divergence Speed

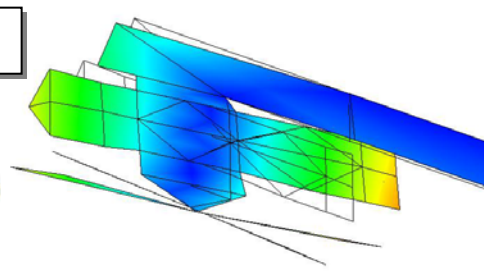
Divergence Mode Shape



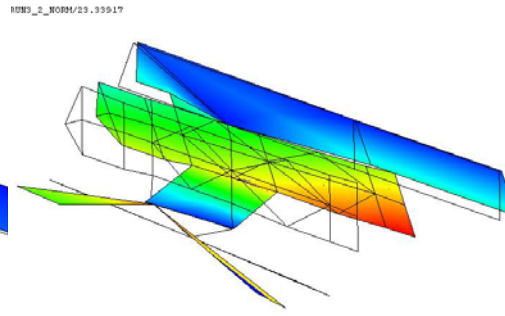
( ): MAC Value



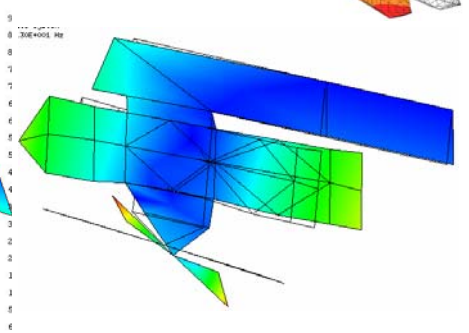
Mode 1  
1<sup>st</sup> Bending



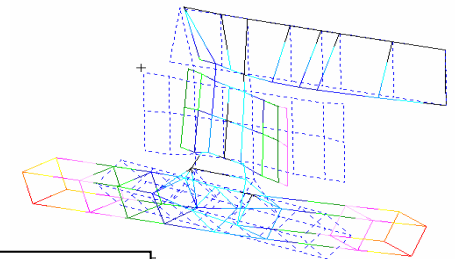
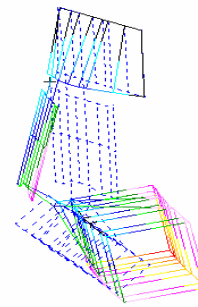
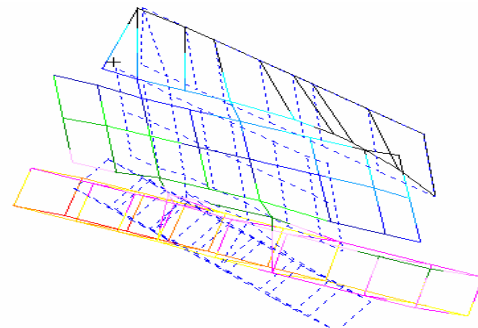
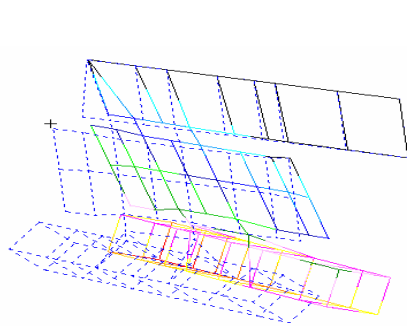
Mode 2  
1<sup>st</sup> Torsion



Mode 3  
2<sup>nd</sup> Bending



Mode 4  
2<sup>nd</sup> Torsion



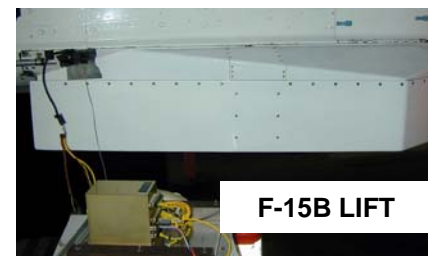
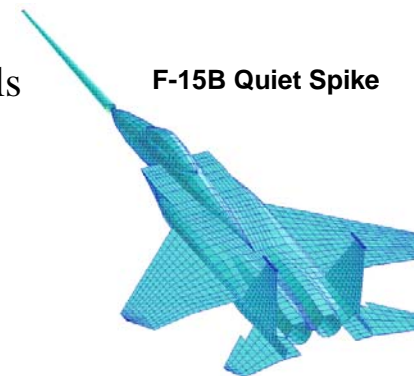
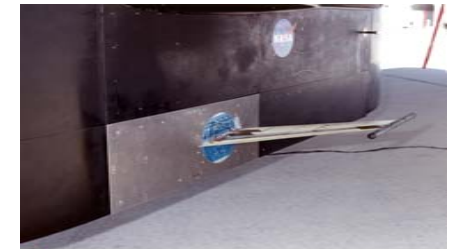
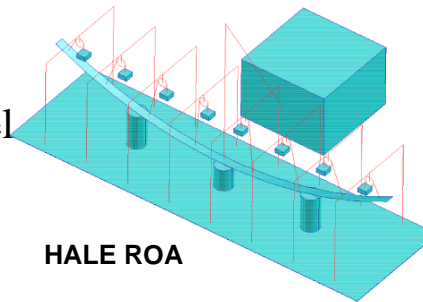
Equivalent

# Structural Dynamic Research Activities at NASA Dryden Flight Research Center



# Project Supports & Researches (FY05 - Present)

- ❑ High Altitude Long Endurance Remotely Operated Aircraft
  - ❖ Create and Update Beam Equivalent Model for a High Aspect Ratio Wing
  - ❖ Develop New GVT Methodology
  - ❖ Preparing Structural Dynamics R&D Proposals for Modeling/Simulation/Control
- ❑ F-15B Quiet Spike Boom
  - ❖ Update F-15B & Quiet Spike Boom Models for the Open-Loop Flutter Clearance
- ❑ F-15B LIFT
  - ❖ For Space Shuttle Return to Flight
  - ❖ Flutter Clearance
- ❑ ATW2
  - ❖ Flutter Clearance and Sensor Research
- ❑ AAW
  - ❖ ASE Flight Research
- ❑ F-15 IFCS
  - ❖ ASE Clearance with Adaptive Controller





# Project Supports & Researches (FY02 - FY04)

- ❑ Helios Mishap Investigation
  - ❖ Structural Dynamic & Flutter Analyses
- ❑ X-43A Ship1
  - ❖ Independent Mishap Investigation
  - ❖ Closed-Loop Flutter Analysis
- ❑ X-43A Ship2 & Ship3
  - ❖ B-52B Captive Carry Flutter Clearance
- ❑ X-37 ALTV, Pylon, and DCF
  - ❖ B-52H Captive Carry Flutter Clearance
- ❑ ALTAIR (UAV)
  - ❖ Structural Dynamic & Flutter Analyses
- ❑ F-15B CDE
  - ❖ Flutter Clearance
- ❑ ATW1
  - ❖ Flutterometer Research
- ❑ X-45A (UCAV)
  - ❖ GVT



Helios



ALTAIR



X-43A Ship 1



F-15B CDE



X-43A Ship 2 & 3



ATW1



X-37 ALTV, Pylon, & DCF



X-45A