An Analytic Technique to Separate Cochannel FM Signals

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Abstract— **A new technique is presented to separate two cochannel FM signals. Two candidate solutions for the phases are analytically derived, and a sequence of phase solutions is chosen to match the expected power spectral density of each constituent signal. This is accomplished with a one-step linear predictor and a simple two-state Viterbi algorithm. Simulations on recorded RF data indicate improved separation capability over other techniques such as the joint Viterbi algorithm and cross-coupled phase-locked loop.**

Index Terms— **Cochannel signal separation, cochannel interference, blind adaptation, MMSE, FM**

I. Introduction

When two or more signals use the same frequency band, they create cochannel interference. Such interference arises from in airborne reception where multiple ground transmitters are visible at the receiver, and in frequency reuse in cellular systems, where it is one of the major factors limiting system capacity [1].

The complex baseband representation of a sampled cochannel FM signal is

$$r[n] = A[n]e^{j\theta[n]} + B[n]e^{j\phi[n]}. \qquad (1)$$

When no confusion can result, the subscript n will be dropped. Initially, assume that there is no noise and that A and B are known positive quantities and vary slowly compared to θ and ϕ. These assumptions will be dropped later. Given A, B and r, accurate estimates of θ and ϕ are desired.

II. Analysis

One might expect that with one equation and two unknowns there are two possible solutions for the phases. However, previous work [2–8] has not attempted to explicitly determine these solutions, relying instead on tracking loops or other estimators. In this section, the phase solutions are derived analytically. From Eq. (1),

$$\|r\|^2 = \|Ae^{j\theta} + Be^{j\phi}\|^2 = A^2 + B^2 + 2AB\cos(\phi - \theta),$$

and thus

$$\cos(\phi - \theta) = \frac{\|r\|^2 - A^2 - B^2}{2AB}. \qquad (2)$$

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From $r = e^{j\theta}(A + Be^{j(\phi-\theta)})$, it follows that

$$\begin{aligned} e^{j\theta} &= \frac{r}{A + Be^{j(\phi-\theta)}} = \frac{r(A + Be^{-j(\phi-\theta)})}{A^2 + B^2 + 2AB\cos(\phi - \theta)} \\ &= \frac{r(A + Be^{-j(\phi-\theta)})}{\|r\|^2}. \end{aligned}$$

Hence, $\theta = \arg[r(A + Be^{-j(\phi-\theta)})] = \arg[r(A + B\cos(\phi - \theta) - jB\sin(\phi - \theta))]$. By the symmetry of Eq. (1), it also immediately follows that $\phi = \arg[r(B + A\cos(\phi - \theta) + jA\sin(\phi - \theta))]$. By letting $D \triangleq (\|r\|^2 - A^2 - B^2)/(2AB) = \cos(\phi - \theta)$, the solutions are given by

$$\begin{aligned} \theta &= \arg\left[r(A + BD + jsB\sqrt{1 - D^2})\right] \\ \phi &= \arg\left[r(B + AD - jsA\sqrt{1 - D^2})\right], \end{aligned} \qquad (3)$$

where $s \in \{-1, +1\}$. The phases have now been determined exactly as a function of A, B, and r, to within one of two possibilities.

III. The Tracking Algorithm

For a single sample $r[n]$, there is no reason to prefer one solution for $\theta[n]$ and $\phi[n]$ over the other possible solution. However, a sequence of solutions $\ldots, \theta[n-2], \theta[n-1], \theta[n], \ldots$, and $\ldots, \phi[n-2], \phi[n-1], \phi[n], \ldots$, can be chosen that has the bandwidth (or spectral density, if known) expected of the underlying modulated phase.

A two-state trellis is set up. The first state corresponds to $s = 1$ in Eq. (3), and the second state corresponds to $s = -1$. The sequence of solution choices will be traced through the trellis using a Viterbi algorithm. Stored at each state at time n are hypothesized phase solutions $(\hat{\theta}[n], \hat{\phi}[n])$ determined from Eq. (3), as well as the instantaneous frequencies $(\hat{\theta}'[n], \hat{\phi}'[n])$, calculated by finite differences with phases traced back to time $n-1$. In addition, each state at time n stores a prediction $(\hat{\theta}'_p[n+1], \hat{\phi}'_p[n+1])$ of the instantaneous frequencies at time $n+1$.

The branch metric from state i at time $n-1$ to state j at time n is the squared difference between the predicted instantaneous frequency $\theta'_p[n]$ from state i and the hypothesized solution $\theta'[n]$ from state j, added to the similar squared difference for $\phi'[n]$. The Viterbi algorithm operates by computing the four branch metrics at each time step, storing an accumulated metric, and tracing backwards in the trellis to find the correct solution.

An mth order Levinson-Durbin linear predictor is used to compute the one-step instantaneous frequency prediction.

It has the form

$$\hat{\theta}'_p[n+1] = \sum_{i=1}^{m} a_i \hat{\theta}'[n+1-i].$$

A similar linear predictor is used for $\hat{\phi}'[n+1]$. The Levinson-Durbin algorithm is the linear minimum-mean squared error (LMMSE) estimator for the instantaneous frequencies. This is a standard technique, and is closely related to a Kalman filter and the EM method for parameter estimation for Markov models [9]. The coefficients a_i are determined as follows. Let $r_i = E(\theta'[n]\theta'[n-i])$, let $v = (r_1, \ldots, r_m)^T$, and let

$$R_m = \begin{bmatrix} r_0 & r_1 & \cdots & r_{m-1} \\ r_1 & r_0 & \cdots & r_{m-2} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m-1} & r_{m-2} & \cdots & r_0 \end{bmatrix}.$$

Then the coefficients are given by $a = R_{k-1}^{-1} v$. There is an efficient iterative technique to determine the mth order coefficients from the $(m-1)$th order coefficients, so that computing the inverse of the large autocorrelation matrix is not necessary [10]. An estimate of r_i may be obtained by assuming a flat power spectrum density for θ' with a sharp cutoff at B Hz. Taking the inverse Fourier transform gives $r_i = \mathrm{sinc}(2iBT_s)$, where T_s is the sampling rate. For example, if the bandwidth is 4 kHz. and $T_s = 1/132300$ sec., then for a fourth order linear predictor, $v = (0.993996, 0.976115, 0.946741, 0.906506)^T$ and $a = (3.96459, -5.92481, 3.95553, -0.995421)^T$. If more is known about the spectral characteristics of θ', then the coefficients may be determined from more accurately determined r_i values.

IV. Amplitude tracking

The analytic technique above relies on known values of A and B, but in a real system the amplitudes are not known, and furthermore, will slowly vary. A heuristic algorithm is used to track these variations. For the purposes of amplitude tracking one can assume that θ and ϕ are uniformly distributed in $[0, 2\pi)$. Consequently, $(\theta - \phi) \mod 2\pi$ is also uniform in $[0, 2\pi)$.

One simple heuristic for amplitude tracking is to compute the maximum and minimum values of $\|r[i]\|$ over $i = 1, \ldots, n$, where n is chosen such that $A[i]$ and $B[i]$ do not vary appreciably. Assuming without loss of generality that $A \geq B$, the minimum $\|r[i]\|$ will be close to $A - B$ and the maximum will be close to $A + B$, from which estimates of A and B can be determined. Unfortunately, it turns out that this heuristic does not perform well in the presence of noise.

A better performing heuristic is obtained by using the median M of $\|r[1]\|^2, \ldots, \|r[n]\|^2$. Define

$$X \triangleq \frac{1}{n}\sum_{i=1}^{n} \|r[i]\|^2 \tag{4}$$

$$Y \triangleq \frac{2}{n}\sum_{i:\|r[i]\|^2 > M} \|r[i]\|^2 \tag{5}$$

$$Z \triangleq \frac{2}{n}\sum_{i:\|r[i]\|^2 \leq M} \|r[i]\|^2. \tag{6}$$

Since the expected value of $\|r\|^2$ is given by

$$E[\|r\|^2] = E[A^2 + 2AB\cos(\theta - \phi) + B^2] = A^2 + B^2,$$

it follows that $X \approx A^2 + B^2$ for reasonably large n. Conditioning on the event $\cos(\theta - \phi) > 0$, it follows that $(\theta - \phi) \mod 2\phi$ is uniform in $[0, \pi/2) \cup [3\pi/2, 2\pi)$ Therefore,

$$\begin{aligned} &E[\|r\|^2 \mid \cos(\theta - \phi) > 0] \\ &\quad = E[A^2 + 2AB\cos(\theta - \phi) + B^2 \mid \cos(\theta - \phi) > 0] \\ &\quad = A^2 + B^2 + 2AB\, E\left[\int_0^{\pi/2} (1/\pi)\cos\alpha d\alpha \right. \\ &\qquad \left. + \int_{3\pi/2}^{2\pi} (1/\pi)\cos\alpha d\alpha\right] \\ &\quad = A^2 + B^2 + 4AB/\pi \end{aligned}$$

Similarly,

$$E[\|r\|^2 \mid \cos(\theta - \phi) < 0] = A^2 + B^2 - 4AB/\pi.$$

Thus, $Y \approx A^2 + B^2 + 4AB/\pi$, and similarly $Z \approx A^2 + B^2 - 4AB/\pi$. If

$$\hat{A} \triangleq \frac{1}{2}\left(\sqrt{X + \frac{\pi}{2}(Y - X)} + \sqrt{X + \frac{\pi}{2}(Z - X)}\right) \tag{7}$$

$$\hat{B} \triangleq \frac{1}{2}\left(\sqrt{X + \frac{\pi}{2}(Y - X)} - \sqrt{X + \frac{\pi}{2}(Z - X)}\right), \tag{8}$$

then $\hat{A} \approx A$ and $\hat{B} \approx B$. Using only the norms of r over a set of samples, A and B can be estimated fairly accurately.

V. Noise

When noise $N[n]$ is added in the right-hand side of Eq. (1), the solution in Eq.s (3), (7), and (8) is no longer accurate. In particular, since the noise $N[n]$ is unknown, $\cos(\phi - \theta) = (\|r - N\|^2 - A^2 - B^2)/(2AB)$ is also unknown. For small levels of noise, D may be defined as in Section II as an approximation for $\cos(\theta - \phi)$. Or, the following correction can be applied to obtain a slightly improved, unbiased estimate:

$$D \triangleq (\|r\|^2 - \sigma^2 - A^2 - B^2)/(2AB) \approx \cos(\phi - \theta),$$

where $\sigma^2 = E[\|N\|^2]$. Similarly, Eq.s (7) and (8) may be bias-corrected with

$$\hat{A} \triangleq \frac{1}{2}\left(\sqrt{X + \frac{\pi}{2}(Y - X) - \sigma^2} + \sqrt{X + \frac{\pi}{2}(Z - X) - \sigma^2}\right)$$

$$\hat{B} \triangleq \frac{1}{2}\left(\sqrt{X + \frac{\pi}{2}(Y - X) - \sigma^2} - \sqrt{X + \frac{\pi}{2}(Z - X) - \sigma^2}\right).$$

TABLE I
COMPARISON OF COCHANNEL SEPARATION METHODS ON SIMULATED DATA.

Freq. Dev. (kHz)	SIR (dB)	Cross-coupled PLL (MSE)	Joint Viterbi (MSE)	Analytical Technique
12	6	0.06/0.48	0.09/0.45	0.00/0.00
12	1	1.28/0.75	0.59/0.66	0.00/0.00
8	6	0.09/0.87	0.16/0.96	0.00/0.00
8	1	1.32/0.79	0.71/0.97	0.00/0.00
12	30	–/–	–/–	0.00/0.00

VI. PERFORMANCE

Two types of data were used in the simulations. The first type consisted of software generated signals, in which voice signals were FM modulated, adjusted in power, and added to form a simulated cochannel FM signal. This represents a completely noiseless scenario. The second type of simulation was conducted on signals recorded directly from UHF narrowband FM radios.

Both types of signals were processed on a Pentium 166 running the separation algorithm offline. Parameters used in the program included the sampling rate, decoding delay, order of the Levinson-Durbin linear predictor, and the modulating signal bandwidths of the cochannel signals. The program read in the parameters, computed the linear predictor coefficients from the sampling rate and bandwidths, and then began the Viterbi algorithm as described in the previous section.

There were five test cases for the simulated data set. In each case the the first signal is a male voice and the second signal is a female voice. In all cases, the sampling rate is 132300 Hz., (to match with previous work for the joint Viterbi [7] and cross-coupled phase-locked loop [8]), the SNR is infinity, a fifth order linear predictor is used under the assumption of flat 4 kHz. bandwidth modulating signals, and the decoding delay is 1. The SIR and frequency deviations were varied. Table I gives the normalized mean-squared error (MSE) between the true and estimated instantaneous frequencies, i.e., $\sum_n (\hat{\theta}'[n] - \theta'[n])^2 / \sum_n \theta'[n]^2$ and $\sum_n (\hat{\phi}'[n] - \phi'[n])^2 / \sum_n \phi'[n]^2$, and compares it to the cross-coupled phase-locked loop and the joint Viterbi algorithm. In all cases, both the dominant and subdominant signals were separated perfectly, to within the floating point precision of the computer (normalized MSE of 10^{-20} or less), i.e., the correct branch of the trellis was chosen at every step. In addition, the one-step linear predictor itself is very accurate; in every case, the average difference between the linearly predicted phase and the phase given by the chosen state is 1.5 degrees or less. Furthermore, the amplitude estimation is nearly perfect as well, since the signals are noiseless.

Each test case in the recorded RF data set consists of a female dominant voice and a male subdominant voice. Here, the sample rate is reduced to 40kHz. Added impairments included noise (SNR = 10-50 dB), Doppler offset (0Hz or 1000Hz), and SIR (6dB, 10dB, or 20dB). The SNR is defined as the ratio of the *dominant* signal energy to the noise energy.

Fig. 1 shows the normalized mean-squared error of the instantaneous frequency tracking. As a rule of thumb, an MSE less than 1.0 corresponds to an intelligible output. As can be seen, the dominant signal was extracted with excellent quality throughout the SNR range from 10-50dB, and the subdominant signal was intelligible whenever the SNR was higher than the SIR by at least 20dB. The algorithm demonstrates robustness against varying SIR's, and also against varying amplitudes— the recorded signals varied in amplitude by about 2 dB over the length of the sample. When the amplitudes are assumed to be known, Fig. 1 indicates that the performance does not flatten out as quickly in the high SNR region. A final test in which a Doppler offset of 1kHz was added to the subdominant signal did not appreciably alter the phase-tracking performance shown in Fig. 1.

Fig. 2 shows the normalized mean-squared error of the amplitude tracking using the analytic algorithm when a window length of 5000 samples is used, i.e., $\sum_{i=n}^{i=n-4999} (\hat{A}[i] - A[i])^2 / \sum_{i=n}^{i=n-4999} A[i]^2$ and $\sum_{i=n}^{i=n-4999} (\hat{B}[i] - B[i])^2 / \sum_{i=n}^{i=n-4999} B[i]^2$. As SIR increases, the dominant signal amplitude is easier to track (because of less interference), and the subdominant signal itself is harder to track (because of its lower power).

These results are in stark contrast to the joint Viterbi algorithm and cross-coupled phase-locked loop, which were not able to separate the recorded field data reliably in any SNR or SIR region in Fig. 1. For example, when the joint Viterbi algorithm was used with SIR=6dB and SNR=∞, the resulting phase MSE was 0.34 for the dominant and 1.66 for the subdominant. The cross-coupled phase-locked loop had similar performance.

VII. CONCLUSIONS

This new analytic technique is capable of excellent separation in a low noise environment. This has been verified by tests of both simulated and field data. An extension to $M \geq 2$ signals is straightforward, with the use of a trellis with 2^{M-1} states. It appears that in order to successfully demodulate a subdominant signal, about a 20dB power separation must exist between the subdominant signal and the noise. Further refinements of the algorithm may make it more resistant to noise.

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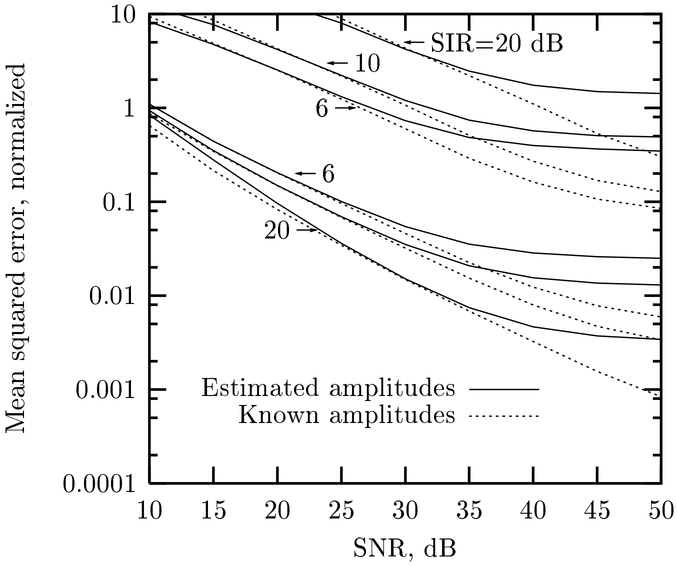

Fig. 1. Instantaneous frequency tracking performance of the analytic cochannel separation technique. The lower set of curves are for the dominant signal; the upper set, for the subdominant signal.

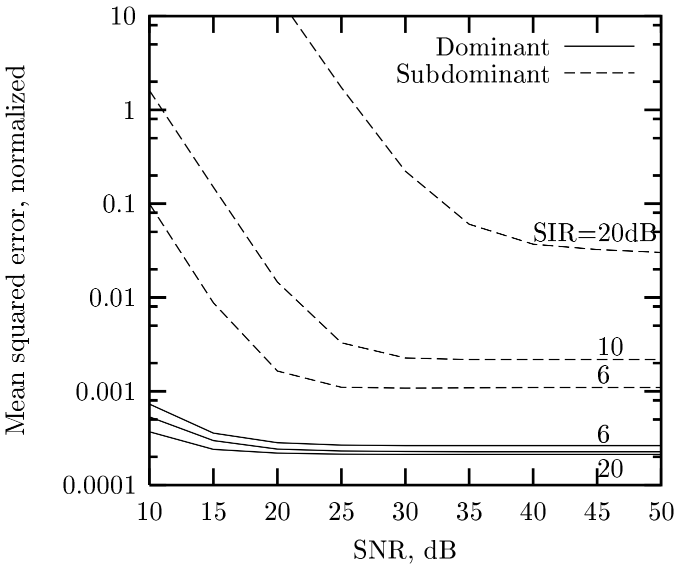

Fig. 2. Amplitude tracking performance of the analytic cochannel separation technique.