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# How VLBI Contributes to Ionospheric Research

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**Abstract**

Geodetic VLBI observations are carried out at two distinct frequencies in order to determine ionospheric delay corrections. Each ionospheric delay corresponds to the total electron content (TEC) along the ray path through the ionosphere. Because VLBI is a differential technique the observed ionospheric delays represent the differences of the behaviour of the propagation media above each two stations. Additionally, there is a constant instrumental delay offset per baseline that contributes to the observed ionospheric delay. This instrumental offset is independent of azimuth and elevation in which the antennas point which allows us to separate it from the variable ionospheric parameters for each station which can be represented by different functional approaches. If horizontal gradients in the ionosphere above the stations are neglected we are able to separate the instrumental offsets from the ionospheric parameters by a least-squares fit. A weakness of this approach is the assumption that the TEC values are assigned to the station coordinates but not to the geographical coordinates of the intersection point of the ray path and the infinitely thin ionospheric layer. Nevertheless, the results agree well with results of other techniques like GPS on short and long time scales.

## 1. Estimation of Ionospheric Parameters

Each dual-frequency VLBI observation provides the baseline dependent ionospheric delay ($\tau_{measured}$) in the form of equation (1).

$$\tau_{measured}(t) = \tau_{ion,1}(t) - \tau_{ion,2}(t) + \tau_{offset,1} - \tau_{offset,2} \tag{1}$$

The ionospheric delay at X-band measured at station $i$ can be described by equation (2) with an appropriate mapping function $M_f(\varepsilon_i)$ depending on the elevation angle $\varepsilon_i$ (e.g., Schaer, 1999, [1], Hobiger and Schuh, 2004, [2]).

$$\tau_{ion,i}(t) = \frac{1.34 \cdot 10^{-7}}{f_x^2} \cdot M_f(\varepsilon_i) \cdot VTEC_i(t) \tag{2}$$

If the Vertical Total Electron Content (VTEC) above a single station shall be estimated the problem occurs that each measurement contains 4 unknowns. Thus, some assumptions and simplifactions have to be made and an estimation method has to be used that takes the physical behaviour of the ionosphere into account (TEC values cannot be negative).

1. As indicated in equation (1), instrumental offsets caused by the receiving system (e.g., Ray, 1991, [3]) are supposed to be constant within a 24h VLBI session. In GPS there is a similar problem called differential code biases, DCBs [1], when solving for absolute ionospheric parameters. In VLBI the same assumption as in GPS is applied by postulating that the sum of all station-dependent offsets equals to zero.

$$\sum_{i=1}^{N_{stat}} \tau_{offset,i} = 0 \tag{3}$$

2. However, measurements are made at different elevation angles and azimuths and the observed VTEC value has to be assigned to the intersection point of the ray path with the ionospheric shell which is not vertically located above the station. By negelecting horizontal gradients within an area around each station it can be concluded that the values at the intersection points correspond to the values in zenith direction above the antenna.

3. Under the previous assumption the VTEC values for each station can be modeled by means of least-squares. Different elevation angles for each station will enable the separation of the station-dependent parameters.

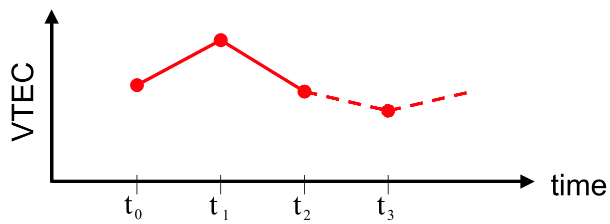


Figure 1. Piece-wise linear function as a representation of vertical total electron content.

4. The ionospheric parameters are modeled in the form of piece-wise linear functions (figure 1, equation 4) with only positive VTEC values allowed (solution within positive half-space).

$$VTEC_{PLF}(t) = offset + rate_1(t_1 - t_0) + rate_2(t_2 - t_1) + \ldots + rate_n(t_n - t) \qquad (4)$$

$$\begin{array}{lcl} (offset_0 + \Delta offset) & \geq & 0 \\ (offset_0 + \Delta offset) + (rate_{1,0} + \Delta rate_1)(t_1 - t_0) & \geq & 0 \\ (offset_0 + \Delta offset) + (rate_{1,0} + \Delta rate_1)(t_1 - t_0) + (rate_{2,0} + \Delta rate_2)(t_2 - t_1) & \geq & 0 \\ \vdots & \vdots & \vdots \end{array} \qquad (5)$$

Applying the non-negative constraint to all points at the interval boundaries yields equation (5). Parameters indicated with $_0$ stand for the initial guess and a $\Delta$ symbolizes the improvement from any adjustment process. If all conditions are summed up using matrix notation equation 6 is obtained.

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ 1 & t_1 - t_0 & 0 & 0 & \cdots \\ 1 & t_1 - t_0 & t_2 - t_1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}}_{\mathbf{B}} \mathbf{X_0} + \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ 1 & t_1 - t_0 & 0 & 0 & \cdots \\ 1 & t_1 - t_0 & t_2 - t_1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}}_{\mathbf{B}} \mathbf{\Delta X} \geq 0 \qquad (6)$$

Re-aranging equation (6) yields

$$\mathbf{B} \cdot \mathbf{\Delta X} \geq -\mathbf{B} \cdot \mathbf{X_0} \quad \Rightarrow \quad \mathbf{B} \cdot \mathbf{\Delta X} \geq \mathbf{C} \qquad (7)$$

Incorporation of equation (7) into the adjustment process (least-squares method, Gauss-Markov model) leads to

$$\Delta \mathbf{X} = \left(\mathbf{A^T W A}\right)^{-1} \mathbf{A^T W} \Delta \mathbf{Y} \quad \downarrow \rightarrow \text{"}reflective\ Newton\ method\text{"} \leftarrow \downarrow \quad \mathbf{B} \cdot \Delta \mathbf{X} \geq \mathbf{C} \tag{8}$$

As pointed out by equation (8) the usage of a reflective Newton method allows us to combine the classical least-squares approach with non-negative conditions.

5. Normally piece-wise linear functions are modelled with constant interval lengths (figure 2). For periods with no data additional constraints are needed to stabilize the solution. To

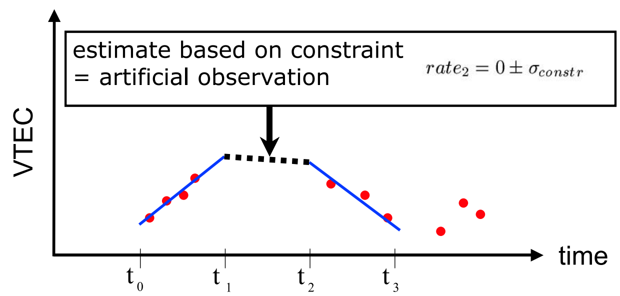


Figure 2. Piece-wise linear function - constant interval lengths.

avoid the introduction of constraints an adaptive piece-wise linear function was devoloped that shifts the interval boundaries that every interval contains the same (given) number of observations (figure 3).

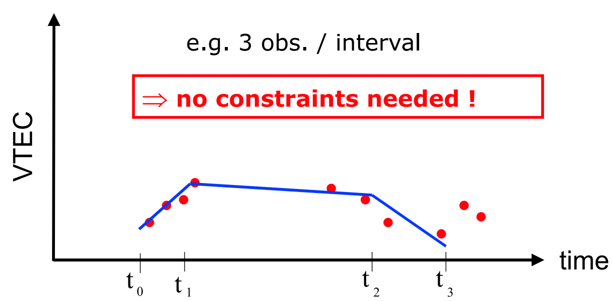


Figure 3. Piece-wise linear function - adaptive interval lengths with 3 observations per interval.

## 2. Results

### 2.1. VTEC on Short Time Scales

Results on daily or weekly time scales can be found in this volume (Hobiger and Schuh, 2004, [2] or Hobiger et al., 2004, [4]).

## 2.2. VTEC on Long Time Scales

A two year period of NEOS-A sessions covering the years 2000 and 2001 was used to demonstrate the capability of VLBI to monitor long-term trends or signals in the ionosphere. As an example for station Fortaleza, Brazil, VTEC values were determined for sunrise (local time 6 am), noon (local time 12 am) and sunset (local time 6 pm) for each session during the relevant period. The results are shown in figure 4 (red, thick line - VLBI; blue, thin dashed line - CODE/GPS, [5]). The agreement between both techniques is very high and especially local noon values differ very little. As for many VLBI stations almost two complete solar cyles ($\sim$ 22 years) are covered by dual-frequency observations it will be possible to provide input for long-term ionospheric studies.

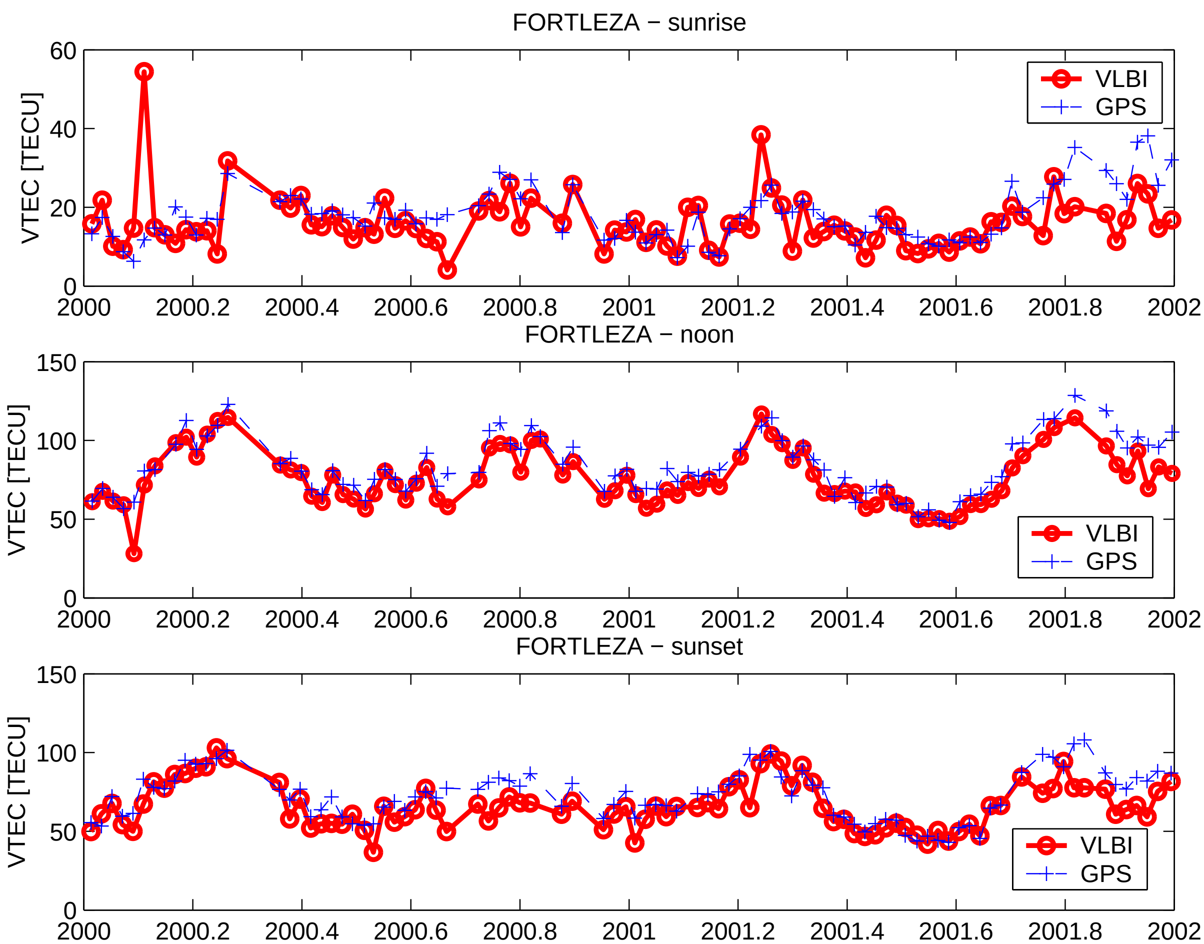


Figure 4. VTEC for station Fortaleza, NEOS-A sessions of the years 2000 and 2001, results from VLBI (red, thick line) and GPS (blue, thin dashed line) for sunrise, noon, sunset.

## 3. Conclusions and Outlook

VLBI is able to provide absolute ionospheric parameters above globally distributed stations. Our first results agree well with values derived by GPS and it is possible to study the behaviour of the ionosphere on long- and short-term time scales. Therefore, VLBI can be used as a source for global ionospheric models.

## 4. Acknowledgements

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## References

[1] Schaer, S., Mapping and Predicting the Earths Ionosphere Using the Global Positioning System, Inaugural Dissertation der Philosophisch-naturwissenschaftlichen Fakultaet der Universitaet Bern, Bern, 1999

[2] Hobiger, T., H. Schuh, Modeling Vertical Total Electron Content from VLBI Observations, this volume, 2004

[3] Ray, J.R, B.E. Corey, Current Precision of VLBI Multi-Band Delay Observables, Proceedings AGU Chapman Conference on Geodetic VLBI: Monitoring Global Change, Washington D.C., 22-26 April, 1991

[4] Hobiger, T., S. Todorova, H. Schuh, Ionospheric Parameters Obtained by Different Space Geodetic Techniques During CONT02, this volume, 2004

[5] `http://igscb.jpl.nasa.gov/components/dcnav/cddis_products_ionex.html`