Horizontally Differentiated Market Makers*

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I analyze a model of competition between two market makers who are located at the opposite ends of the Hotelling line, both of which are also occupied by a search market. Buyers and sellers are uniformly distributed between the two market places, yet can only trade at one of them. In equilibrium, market makers net a positive profit and trade with buyers and sellers close to them, while buyers and sellers further away participate in search markets. Interestingly, a duopolistic market maker nets a larger profit than a monopoly because search markets make the quantities market makers trade strategic complements.

1 Introduction

Firms compete for buyers and suppliers in many different ways and on many different levels. Firms that set ask and bid prices at which they are willing to sell and buy perform the task of market making, the role traditionally attributed to the Walrasian auctioneer. Within a given market place, such market making firms usually not only compete amongst themselves, but face additional competition from alternative exchanges. For example, used car dealers, employment agencies and real estate intermediaries all compete with search markets based on newspaper and internet ads. Similarly, market makers in stock markets and ticket scouts at sports events and concerts face competition from the search markets where buyers and sellers of stocks and tickets, respectively, can meet and trade bilaterally. On top of that, market makers compete across markets. For example, market makers at NYSE compete with Nasdaq dealers, the dealers at the Fish market in NYC compete with those in New Jersey and more generally, retailers and shops in any given city compete with those in neighboring cities.[1]

In this paper, I analyze competition between market makers when these are active in different market places and when each of them faces competition from an alternative exchange in its market place. The basic set-up is the following. Two market places are located at the West and East end of the Hotelling line, labelled W and E. Each market place hosts a market maker and a search market. A continuum of buyers and sellers who face transportation costs are uniformly distributed between the two places, yet can only trade at one of them. Market makers simultaneously and independently choose ask and bid prices at which they stand ready to sell and buy, respectively. This basic model is then extended to what can be called an appropriately defined game of matchmaker or platform competition. The feature that distinguishes matchmakers from market makers is that the former set participation fees but do not participate in the transactions themselves, whereas the latter set ask and bid prices at which they stand ready to trade.[2]

The following results obtain. First, as in Spulber (1999, Ch.3), competing market makers net a positive profit in equilibrium. This is so because differentiation mitigates price competition in exactly the same way as it does in product market competition. However, these equilibrium profits are smaller by orders of magnitude compared to the model without search markets. The reason is that the demand and supply functions an intermediary faces are constrained by the outside option of participating in the search market. The more attractive a search market for buyers and sellers, the worse the demand and supply functions from the point of view of the intermediary.

Second, buyers and sellers close to the market places trade with intermediaries while the other ones participate in the search markets because those closer to intermediaries have relatively more to gain from intermediated trade than those further away.

Third, an intermediary's profit increases in the quantity traded by the other intermediary, i.e., intermediaries' quantities are *strategic complements*. The intuition is the following. As in standard models of horizontal product differentiation like Hotelling (1929) and Salop (1979), equilibrium is determined by indifferent buyers (and sellers). In the present model, these are indifferent between joining the search market in W and E. Consider now a quantity decrease by the intermediary in E. From the point of view of more distant buyers and sellers, the search market in E is now more attractive since sellers (buyers) closer to E, who have relatively low costs (high valuations), are now active in this search market. Consequently, the location of the indifferent buyers and sellers moves towards W, which in turn renders the search market in W more attractive from the point of view of buyers (sellers) close to W because the least efficient sellers (the lowest valuation buyers) in W have now lower costs (higher valuations). Thus, the demand and supply functions the intermediary in W faces worsen, and hence its profit decreases as the quantity traded by the one in E decreases.

Fourth, every equilibrium outcome in the market maker game is also an equilibrium outcome in an appropriately defined game of matchmaker or platform competition. Thus, the paper also contains a simple model of platform competition, where platforms not only compete with each other but on top of that with alternative matching mechanisms such as search markets. These outside options are of obvious importance for many applications, such as matchmakers for men and women and for business-to-business platforms.

Related Literature To the best of my knowledge, this is the first paper to analyze competition between market makers in horizontally differentiated market places who compete with alternative exchanges within their market place as well. Spulber (1999, Ch.3) analyzes Bertrand competition between market making intermediaries who are horizontally differentiated. He shows that in equilibrium these market makers net a positive profit. Competition of a monopolistic market maker with an alternative exchange and Bertrand competition between market makers within a market place has been analyzed by Stahl (1988), Gehrig (1993), Fingleton (1997), Spulber (1996, 1999) and Rust and Hall (2003). Ju, Linn and Zhu (2006) and Loertscher (2007) analyze price competition between market makers when goods are homogeneous and market makers set capacity constraints prior to competing in prices. Neeman and Vulkan (2003) and Kugler, Neeman and Vulkan (2006) study the conditions under which centralized markets, whose microstructure is not modelled, drive out trade based on bilateral negotiations. Gehrig (1998) studies competition between and optimal taxation of two horizontally differentiated stock exchanges, where each stock exchange is the only exchange at its location. Ellison and Fudenberg (2003) and Ellison, Fudenberg and Möbius (2004) study competition between two a priori homogenous auction markets when the numbers of buyers and sellers are finite and when the auction markets are the only exchanges. Nocke, Peitz and Stahl (2004) and Smith and Hay (2005) study competition between market places with a focus on the ownership structure of the market places (or platforms). Armstrong (2006), Caillaud and Jullien (2001, 2003) and Rochet and Tirole (2006) study platform competition when buyers and sellers do not have an outside option to trading on a platform.

The remainder of the paper is structured as follows. Section 2 develops the basic model. Section 3 first derives the equilibrium for the model when there are no search markets. Then the search markets assumptions are described in detail, and I derive, in turn, the search market equilibrium when there are no market makers and when there are market makers. Section 4 discusses the main results and performs some comparative static analysis. It shows that, and why, the presence of an intermediary exerts a positive externality on the other intermediary in the presence of search markets, introduces competition between intermediaries within a market place and compares equilibrium price dispersions when the number of intermediaries differs across markets. Section 5 introduces the matchmaker game and shows that every equilibrium outcome in the market maker game is also an equilibrium outcome in the matchmaker game. Section 6 concludes. Proofs are in the Appendix.

2 The Model

The following is a natural adaptation of the models of Hotelling (1929) and Salop (1979) to market making intermediaries. Assume that there is a continuum of buyers and a continuum of sellers, each with measure $\frac{1}{2}$. Buyers and sellers are uniformly distributed along the North and South semi-circles of a circle with circumference 1, respectively. Each buyer has a gross valuation for the good v, and he either buys one unit of the good or none. Each seller has production costs of zero, and he either produces one unit or none. Both sellers and buyers bear a constant cost $t > 0$ per unit of distance they have to travel with the good. I assume that v is so large in comparison to t that in any equilibrium all buyers consume, i.e., I assume $\frac{v}{t} > \frac{3}{2}$.

This condition guarantees that competition between intermediaries will be so tough that equilibrium profits of the intermediaries are independent of v. It is the same as the condition that guarantees full market coverage in equilibrium in the standard Hotelling model.[3] Because of this assumption, no buyers and sellers will be inactive in equilibrium, which contrasts with the models of Gehrig (1993) and Rust and Hall (2003).

There are two market places, one located at the Westernmost point of the circle and the other one at the Easternmost, as illustrated in Figure 1. The former is labelled W and the latter E. As an accounting convenience, I let the locations of buyers and sellers increase from 0 to $\frac{1}{2}$ from West to East. That is, the buyer and seller with location 0 are situated in W, and the buyer and seller with location $\frac{1}{4}$ are at the North and South pole of the circle. Accordingly, the location of the buyer and seller in E is $\frac{1}{2}$. Each market place can host a market making intermediary or a search market or both. An intermediary in k with $k = E, W$ sets a pair of ask and bid prices (a^k, b^k) at which it is willing to sell and buy. An intermediary is obliged to buy any quantity supplied at the bid price it sets.[4] The quantity it sells is the minimum of the quantity buyers demand at its ask price and the quantity supplied.

Motivation and Discussion Under the assumption that search markets and market makers occupy the same locations, the buyers and sellers who have the most to gain from search market participation, i.e., the high valuation buyers and the low cost sellers, also have the most to gain from trading with intermediaries.[5] Modelling the economy in this way amounts to locating all buyers and sellers on separate Hotelling lines, and connecting these at two points.[6]

The present specification is most appropriate when the horizontal differentiation is geographical. For example, suppose that W and E are two cities, each of which has its own regional newspaper, and consider intermediaries in labor markets. A firm that seeks an employee can either place an ad in one of the newspapers, whereby it participates in the search market in this city. Alternatively, it can contact an intermediary in this city. Consequently, search markets and intermediated markets have the same locations.

The industrial organization literature offers two interpretations for horizontal differentiation, both of which are relevant for the present paper. According to the first, which has been implicitly invoked above, firms (or products) are geographically differentiated.

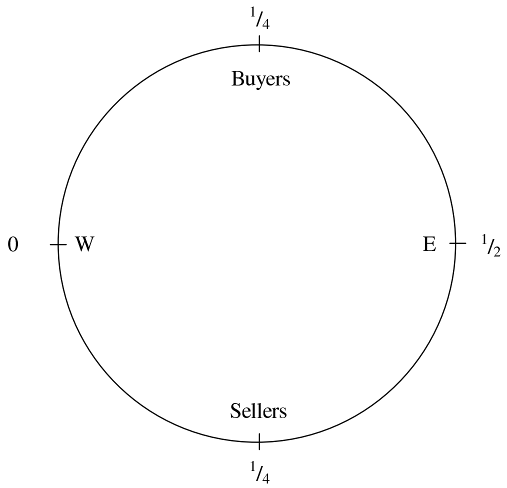

Figure 1: Buyers and sellers uniformly distributed along a circle.

According to the second, the differentiation is in a characteristics space. Consumers and producers can, e.g., choose between trading with a retailer specialized in organic food and another one specialized in genetically manipulated food. In all likelihood, some consumers and producers will prefer trading with the former, while others will prefer buying from or selling to the latter. However, because buyers and sellers can also meet in search markets, but can do so only either at W or E, it is necessary for this interpretation to go through to assume that there is an economy wide standardization, such that a farmer can produce only either organic or genetically manipulated food, but nothing in between.

3 Equilibrium Analysis

As a point of reference, I first derive the equilibrium for the model when there are no search markets.[7]

3.1 Equilibrium Without Search Markets

Let $\tilde{x}$ be the location of the buyer who at ask prices a^W and a^E is indifferent between buying from the intermediary in W and the intermediary in E.[8] As is well known, this buyer's location is a $\tilde{x}$ such that $v - t\tilde{x} - a^W = v - t(\frac{1}{2} - \tilde{x}) - a^E$, implying $\tilde{x} = \frac{1}{4} + \frac{a^E - a^W}{2t}$. Analogously, the location $\tilde{y}$ of the seller who at bid prices b^W and b^E is indifferent between W and E is a $\tilde{y}$ that satisfies $b^W - t\tilde{y} = b^E - t(\frac{1}{2} - \tilde{y})$, which implies $\tilde{y} = \frac{1}{4} + \frac{b^W - b^E}{2t}$. Thus, the profit maximization problem of the market maker in W is

$$\max_{(a^W, b^W)} \Pi^W = a^W \tilde{x} - b^W \tilde{y} \tag{1}$$

subject to the constraint $\tilde{x} \leq \tilde{y}$. Clearly, there is no use for an intermediary to sell less than it buys. Thus, $\tilde{x} = \tilde{y}$ holds in equilibrium. Therefore, the reaction functions associated with the problem (1) are $a^W(a^E) = \frac{t}{4} + \frac{\mu}{2} + \frac{a^E}{2}$ and $b^W(b^E) = -\frac{t}{4} + \frac{\mu}{2} + \frac{b^E}{2}$, where $\mu > 0$ is the Lagrange multiplier associated with the constraint $\tilde{x} = \tilde{y}$. In a symmetric equilibrium, $a^W = a^E \equiv \hat{a}$ and $b^W = b^E \equiv \hat{b}$ hold, yielding

$$\hat{a} = \hat{b} + t. \tag{2}$$

Hence, the equilibrium spread is $\hat{z} \equiv \hat{a} - \hat{b} = t$ and the equilibrium quantity traded by each intermediary is $\frac{1}{4}$. Consequently, the equilibrium profit of each intermediary is $\Pi^* = \frac{t}{4}$.

Note that because $\hat{a}$ depends on $\hat{b}$, there is a continuum of equilibria. These are payoff equivalent for market makers but affect buyers' and sellers' welfare in opposite ways. The only constraints imposed on the set of equilibria are the participation constraints for the indifferent buyers and sellers, who travel the largest distances. For the seller and buyer at $\frac{1}{4}$ to derive nonnegative utility, $\hat{b} \geq \frac{1}{4}t$ and $\hat{a} \leq v - \frac{1}{4}t$ has to hold. By equality (2), $\hat{b} \geq \frac{1}{4}t$ and $\hat{a} \leq v - \frac{1}{4}t$ imply, respectively, $\hat{a} \geq \frac{5}{4}t$ and $\hat{b} \leq v - \frac{5}{4}t$. Consequently, all ask and bid prices satisfying $\hat{a} \in \left[\frac{5}{4}t, v - \frac{1}{4}t\right]$, $\hat{b} \in \left[\frac{1}{4}t, v - \frac{5}{4}t\right]$ and $\hat{a} = \hat{b} + t$ are consistent with equilibrium. This set of equilibrium prices is illustrated in Figure 2.

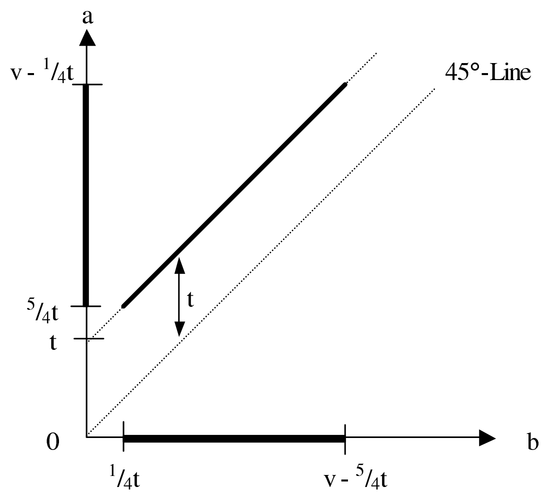

Figure 2: The Set of Equilibrium Ask and Bid Prices.

3.2 Search Markets

Assume now that there is a search market in W and one in E. As before, buyers and sellers have a cost per unit of transportation $t > 0$. In a given search market, buyers and sellers are uniformly randomly matched. If there are, say, more buyers than sellers in a search market, sellers are matched with probability one, while buyers are matched with a probability proportional to the ratio of the number of sellers over the number of buyers. Once matched, a buyer and a seller share the gains from trade evenly.[9] Again, transportation costs are incurred only when trade occurs.

The assumptions underlying the organization of search markets are similar to those of Gehrig (1993). A difference is that he assumes take-it-or-leave-it offers, whereas here sellers and buyers share gains from trade evenly. Because here there are two search markets whereas in his model there is but one, the equilibrium outcome will be somewhat different in my model. Another difference to Gehrig's model that has already been mentioned is that there are no inactive buyers and sellers because all possible buyer-seller matches generate positive surplus. It seems also possible to model the search market as a dynamic matching market à la Spulber (1996) and Rust and Hall (2003), where buyers and sellers search for an opportunity to trade with middlemen. As in the homogenous goods model I expect both models to yield very similar inverse demand and supply functions facing intermediaries.

3.3 Equilibrium Without Market Makers

For this subsection, assume that there are no market makers. Let $\underline{v}_c$ and $\overline{v}_c$ be, respectively, the lowest and highest net valuations of consumers present in a given search market. Similarly, denote by $\underline{v}_p$ and $\overline{v}_p$, respectively, the lowest and highest net cost of producers present in a given search market. Notice that the maximal distance a producer travels is no larger than $\frac{1}{2}$. Consequently, $\overline{v}_p \leq \frac{1}{2}t$ will hold. Similarly, no buyer will travel more than $\frac{1}{2}$. Thus, $\underline{v}_c \geq v - \frac{1}{2}t > t$, where the strict inequality follows from the assumption $v > \frac{3}{2}t$. Therefore, $\underline{v}_c > t > \overline{v}_p$ will hold in any search market.[10]

Let $V_c^W(x)$ and $V_c^E(x)$ denote the expected utility of a buyer at location x when participating in the search market in W and E, respectively. Similarly, let $V_p^W(y)$ and $V_p^E(y)$ denote the expected utility of a seller at location y when participating in the search market in W and E.

Lemma 1 *In any equilibrium with search market participation in both cities, the following holds: If $V_c^W(x') \gtreqless V_c^E(x')$ for some x', then $V_c^W(x) \gtrless V_c^E(x)$ for all $x \lesseqgtr x'$. Analogously, if $V_p^W(y') \gtreqless V_p^E(y')$ for some y', then $V_p^W(y) \gtrless V_p^E(y)$ for all $y \lesseqgtr y'$.*

The single-crossing result reported in Lemma 1 has first been stated by Gehrig (1993). The result implies that the sets of buyers and sellers joining a search market will be convex sets and will include the buyer with the highest net valuation and the lowest cost seller. That is, buyer and seller with location 0 will join the search market in W and the buyer and seller at $\frac{1}{2}$ will join the search market in E. Moreover, because all buyers and all sellers in these sets will join the same search market, the distribution of buyers and sellers active in a search market will be uniform.

Let v_c be the net valuation of a buyer joining a search market, where the highest and lowest cost sellers have costs $\overline{v}_p$ and $\underline{v}_p$ and where the probability of a match for a buyer is γ_c. Then, the expected utility of the buyer v_c is[11]

$$V_c(v_c) = \frac{\gamma_c}{2} \int_{\underline{v}_p}^{\overline{v}_p} (v_c - v_p) \frac{1}{\overline{v}_p - \underline{v}_p} dv_p = \frac{\gamma_c}{2} \left[v_c - \frac{1}{2}(\overline{v}_p + \underline{v}_p) \right], \tag{3}$$

where $\frac{1}{\overline{v}_p - \underline{v}_p}$ is the density for the uniform distribution from which sellers' costs are drawn. The formula after the second equality has a neat interpretation. Consider the second term inside the bracket, $\frac{1}{2}(\overline{v}_p + \underline{v}_p)$. This is the expected cost of the producer to whom the buyer will be matched. Consequently, the difference between v_c and the expected cost is the aggregate surplus buyer v_c expects to generate. If matched, the buyer just gets one half of this surplus because of the even sharing assumption, which explains the fraction $\frac{1}{2}$ that pre-multiplies the bracket.

Similarly, consider a seller with cost v_p who joins a search market where the highest and lowest valuations of buyers are $\overline{v}_c$ and $\underline{v}_c$ and where the probability of being matched is γ_p. Then,

$$V_p(v_p) = \frac{\gamma_p}{2} \int_{\underline{v}_c}^{\overline{v}_c} (v_c - v_p) \frac{1}{\overline{v}_c - \underline{v}_c} dv_c = \frac{\gamma_p}{2} \left[\frac{1}{2}(\overline{v}_c + \underline{v}_c) - v_p \right]. \tag{4}$$

Perfect Equilibria Clearly, not joining a given search market if no one else joins it is a best response for every buyer and seller. Therefore, there is always a trivial equilibrium where one or both search markets are inactive. However, these equilibria are not perfect. If a small number of agents whose mass is positive deviates and joins a hitherto inactive search market, then it is a best response for any other agent to join this search market, too. On the other hand, equilibria where both search markets are open are perfect. Even if a small number of agents deviates and becomes inactive, it will still be optimal for the other agents to join this search market.

Proposition 1 *In the unique perfect equilibrium, all buyers and sellers to the left (right) of* $\frac{1}{4}$ *join the search market in* $W(E)$.

3.4 Equilibrium with Market Makers and Search Markets

To see the potential for market making, observe from equation (3) that the expected utility of search market participation for a buyer with net valuation v_c is less than $\frac{1}{2}v_c$ if $\overline{v}_p > 0$. Thus, there is an ask price $a > 0$ such that the buyer with v_c is indifferent between buying at this price and joining the search market, i.e.,

$$v_c - a = V_c(v_c). \tag{5}$$

Denote $a(v_c) \equiv v_c - V_c(v_c)$ and replace the expression for $V_c(v_c)$ in equation (5) by the one given in (3) to get $v_c - a = \frac{1}{2}\left[v_c - \frac{1}{2}(\overline{v}_p + \underline{v}_p)\right]$, so that $a(v_c) = \frac{1}{2}v_c + \frac{1}{4}(\overline{v}_p + \underline{v}_p)$. Notice that $a(v_c)$ decreases as $\overline{v}_p$ and $\underline{v}_p$ decrease: The more efficient producers on average, the higher buyers' expected utility from search and thus the lower their willingness to pay for intermediated trade. Of particular interest is the reservation price of the buyer with the highest net valuation active in the search market, i.e.,

$$a(\overline{v}_c) \equiv \frac{1}{2}\overline{v}_c + \frac{1}{4}(\overline{v}_p + \underline{v}_p), \tag{6}$$

since this is the reservation price that is relevant for the market maker. If it sets $a = a(\overline{v}_c)$, all buyers with greater net valuations will prefer buying from the market maker to search market participation, and all buyers with smaller net valuations will prefer participating in the search market.[12]

Analogously, one can derive a reservation price $b(v_p)$ for a seller with net cost v_p such that seller v_p is indifferent between participating in the search market and selling to the intermediary, i.e., $b(v_p) - v_p = V_p(v_p)$ implying $b(v_p) = v_p + V_p(v_p)$. Inserting the expression for $V_p(v_p)$ given in (4) yields $b(v_p) = \frac{1}{2}v_p + \frac{1}{4}(\overline{v}_c + \underline{v}_c)$. Observe that $b(v_p)$ increase in $\overline{v}_c$ and $\underline{v}_c$: The higher consumers' valuations on average, the larger the expected utility derived from search for a producer. Consequently, intermediaries need to increase the bids they offer in order to attract a given producer. For reasons analogous to those for buyers, the bid price relevant for an intermediary will be the one that makes the most efficient seller in the search market, $\underline{v}_p$, indifferent between participating in the search market and selling to the intermediary. This bid price is

$$b(\underline{v}_p) = \underline{v}_p + V_p(\underline{v}_p) = \frac{1}{2}\underline{v}_p + \frac{1}{4}(\overline{v}_c + \underline{v}_c). \tag{7}$$

I now derive the inverse demand and supply function facing the intermediaries and then solve for the equilibrium prices. In order to do so, however, one first needs to determine the location of the buyer and seller who are indifferent between the search market in W and E. The reason is that the expected utility of search market participation of the buyer and seller at q^W (who are indifferent between trading with the intermediary in W and joining the search market in W) depend on the net cost and the net valuation of the seller and buyer who are indifferent between the search markets.

Indifference Between Search Markets As a function of q^W and q^E, there is a buyer $\tilde{x}(q^W, q^E)$ who is indifferent between the two search markets. Similarly, denote the location of the seller who is indifferent between the two markets by $\tilde{y}(q^W, q^E)$. The derivation of these indifferent agents is very similar to the model of market making without search markets. The only difference is that they are now not indifferent between trading with the two intermediaries, but rather between joining the two search markets. Nonetheless, it is via these agents that the decisions of the market maker in W and E have an impact on each other's payoff.

So, the sellers at q^W and $\frac{1}{2} - q^E$ are the sellers with the lowest costs in the search markets in W and E, respectively, while the sellers at $\tilde{y}$ are the sellers with the highest cost in either search market. Consequently, the expected utility of the buyer at $\tilde{x}$, who is indifferent between the two search markets, is $V_c^W(\tilde{x}) = \frac{1}{2}(v - t\tilde{x}) - \frac{1}{4}(tq^W + t\tilde{y})$ and $V_c^E(\tilde{x}) = \frac{1}{2}\left[v - t\left(\frac{1}{2} - \tilde{x}\right)\right] - \frac{1}{4}\left[tq^E + t\left(\frac{1}{2} - \tilde{y}\right)\right]$ and satisfies $V_c^W(\tilde{x}) = V_c^E(\tilde{x})$. Analogously, the expected utilities from search in W and E for seller $\tilde{y}$ satisfy $V_p^W(\tilde{y}) = \frac{1}{2}(v - t\tilde{y}) - \frac{1}{4}(tq^W + t\tilde{x})$ and $V_p^E(\tilde{y}) = \frac{1}{2}\left[v - t\left(\frac{1}{2} - \tilde{y}\right)\right] - \frac{1}{4}\left[tq^E + t\left(\frac{1}{2} - \tilde{x}\right)\right]$. Solving $V_c^W(\tilde{x}) = V_c^E(\tilde{x})$ and $V_p^W(\tilde{y}) = V_p^E(\tilde{y})$ for $\tilde{x}$ and $\tilde{y}$ yields

$$\tilde{x}(q^W, q^E) = \tilde{y}(q^W, q^E) = \frac{1}{4} + \frac{1}{6}(q^E - q^W). \tag{8}$$

Notice that $\tilde{x}$ and $\tilde{y}$ increase (decrease) in q^W (q^E). This has some interesting implications, to which I will return in Section 4 below.

Indifference Between Intermediated Trade and Search Market Having determined the location of the buyer and seller who are indifferent between search markets, I can now derive the expected utility from search market participation for the buyer and seller who are indifferent between search market participation and trading with an intermediary. This will then allow me to compute the reservation price of this buyer and seller for trading with the intermediary.

The expected utility of the buyer at q^W from search market participation is $V_c^W(q^W) = \frac{1}{2}v - \frac{1}{16}t - \frac{17}{24}tq^W - \frac{1}{24}tq^E$. As this buyer is indifferent between search market participation and buying from the intermediary in W at ask price a^W, which gives him a surplus of $U_c^W(q^W) = v - tq^W - a^W$, it has to be true that $U_c^W(q^W) = V_c^W(q^W)$. Solving this equation for a^W yields the reservation price of the indifferent buyer for buying from the intermediary in W, and thus the inverse demand function this intermediary faces. Let $A^W(q^W, q^E)$ denote this solution. It is

$$A^W(q^W, q^E) = \frac{v}{2} + \frac{1}{16}t + \frac{1}{24}tq^E - \frac{7}{24}tq^W. \tag{9}$$

Analogously, the seller at q^W expects utility $V_p^W(q^W) = \frac{1}{2}v - \frac{1}{16}t - \frac{17}{24}tq^W - \frac{1}{24}tq^E$ from participating in the search market in W. If he sells to the intermediary at bid price b^W, his surplus is $U_p^W(q^W) = b^W - tq^W$. He is indifferent between search and selling to the market maker if and only if $U_p^W(q^W) = V_p^W(q^W)$. Solving this equality for b^W yields the reservation price of the seller at q^W for selling to the intermediary, and thus the inverse supply function facing the intermediary. Denote this solution by $B^W(q^W, q^E)$. It is

$$B^W(q^W, q^E) = \frac{v}{2} - \frac{1}{16}t - \frac{1}{24}tq^E + \frac{7}{24}tq^W. \tag{10}$$

Given the inverse demand and supply functions (9) and (10), the profit of the market maker in W is

$$\Pi^W(q^W, q^E) = Z^W(q^W, q^E)q^W = t\left(\frac{1}{8} + \frac{1}{12}q^E - \frac{7}{12}q^W\right)q^W, \tag{11}$$

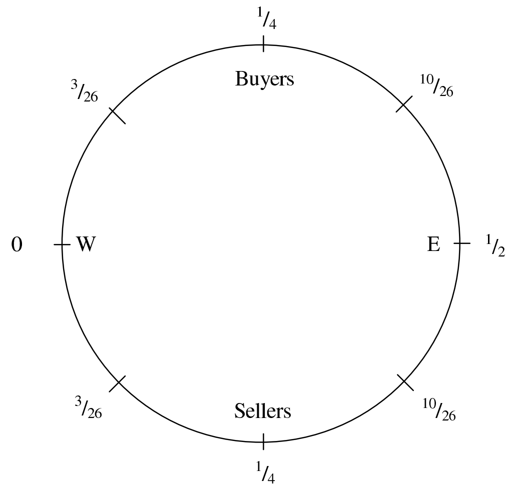

Figure 3: Equilibrium with Search and Intermediated Markets.

where $Z^W(q^W, q^E) = t\left(\frac{1}{8} + \frac{1}{12}q^E - \frac{7}{12}q^W\right)$ is the spread function the intermediary in W faces, the spread being defined as $Z^W \equiv A^W - B^W$. As in the case without search markets, the profit of an intermediary is thus independent of buyers' gross utility v. Maximizing $\Pi^W(q^W, q^E)$ over q^W, taking q^E as given, yields $0 = t\left(\frac{1}{8} + \frac{1}{12}q^E - \frac{14}{12}q^W\right)$, so that the reaction function is

$$q^W(q^E) = \frac{3}{28} + \frac{1}{14}q^E. \tag{12}$$

Notice that in the present model quantities are strategic complements, i.e., $q^W(q^E)$ increases in q^E, which has to do with the observation made above that $\tilde{x}$ and $\tilde{y}$ decrease in q^E.

By symmetry, $q^E = q^W = q^*$ will hold in equilibrium. Thus, equilibrium quantity traded by an intermediary will be $q^* = \frac{3}{26}$. Inserting $q^E = q^W = q^*$ into the inverse demand and supply functions (9) and (10) yields the equilibrium ask and bid prices

$$a^* = \frac{v}{2} + \frac{7}{208}t \quad \text{and} \quad b^* = \frac{v}{2} - \frac{7}{208}t, \tag{13}$$

so that the equilibrium spread is $z^* = t\frac{7}{104}$. Equilibrium profit is $\Pi^* = t\frac{21}{2704} \approx 0.0078t$.

Witness the substantial reduction of equilibrium profit compared to the case without search markets, where equilibrium profit is $\frac{t}{4}$, which is more than thirty times as large as $0.0078t$.

Proposition 2 *The model has a unique equilibrium with two market makers and two active search markets. In this equilibrium, market makers set* $a^* = \frac{v}{2} + \frac{7}{208}t$ *and* $b^* = \frac{v}{2} - \frac{7}{208}t$*. The buyers and sellers with locations* $x, y \in \left[0, \frac{3}{26}\right]$ *trade with the intermediary in* W*, the buyers and sellers with* $x, y \in \left(\frac{3}{26}, \frac{1}{4}\right]$ *join the search market in* W*, the buyers and sellers with* $x, y \in \left(\frac{1}{4}, \frac{10}{26}\right)$ *join the search market in* E *and the buyers and sellers with* $x, y \in \left[\frac{10}{26}, \frac{1}{2}\right]$ *trade with the intermediary in* E*.*

Figure 4 illustrates the equilibrium outcome.[13] The functions tq and $v - tq$ are the unconstrained inverse supply and demand functions the intermediary in W would face absent any competition. The solid dashed lines depict the inverse supply and demand functions an intermediary faces that are constrained by the search markets and the behavior of its competitor. Buyers and sellers with $x, y \leq \frac{3}{26}$ join the intermediary in W and those located in $\left(\frac{3}{26}, \frac{1}{4}\right]$ join the search market in W. Symmetrically, buyers and sellers located in $\left(\frac{1}{4}, \frac{10}{26}\right)$ join the search market in E and those with $x, y \geq \frac{10}{26}$ join the intermediary in E. The shaded rectangles of size $\frac{3}{26}(a^* - b^*)$ depict the intermediaries' profits in the presence of search markets Π^*. The rectangles of size $\frac{t}{4}$, labelled $\hat{\Pi}$, depict the intermediaries' equilibrium profits absent search markets.

Let me discuss the result and compare it to some of the results in the literature. First, the claim of Proposition 2 is not that there is a unique equilibrium, but that there is a unique equilibrium where the two search markets and the two market makers are active.

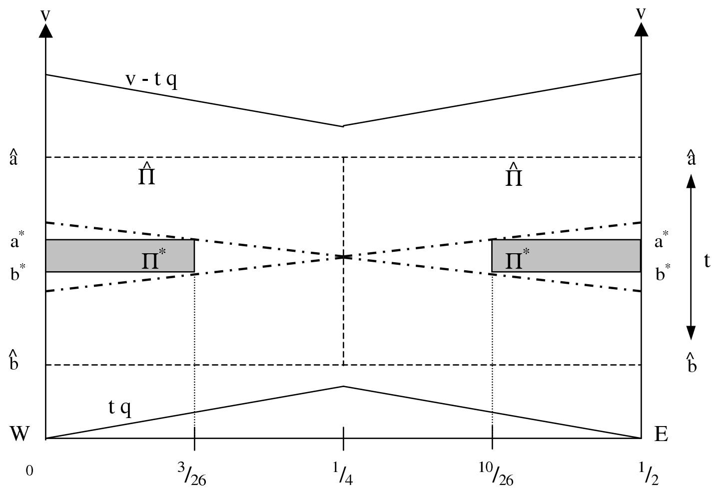

Figure 4: Equilibrium Supply and Demand Functions and Equilibrium Prices.

Nonetheless, the uniqueness of equilibrium in the presence of search markets contrasts with the continuum of equilibria in the model without search markets.[14]

Second, as in the models of Gehrig (1993) and Rust and Hall (2003), equilibrium is characterized by a partition of the sets of sellers and buyers. High valuation buyers and low cost sellers trade with intermediaries, and less efficient producers and lower valuation consumers participate in search markets.[15] This happens in the presence of competition between intermediaries who net positive profits in equilibrium.

Third, the result contrasts with the prediction of Neeman and Vulkan (2003) that centralized trade, which corresponds to intermediated trade in this model, drives out trade based on bilateral negotiations, corresponding to search markets in my model. The reason for the difference is that intermediated markets are only imperfectly competitive. If intermediaries' spreads in each market place become smaller and smaller, say because of increased competition between market makers within each market place, then the search markets would become smaller, too.[16]

Last, note that it is not an outcome of an equilibrium, not even of an imperfect one, that market makers set (a^*, b^*) and subsequently search markets remain inactive. Given that there are no search markets, (a^*, b^*) is not a best response to itself. An interesting question, not pursued any further here, is whether the equilibrium with active search markets is more plausible than (some of) the equilibria where search markets are inactive, and vice versa. To see what "implausible" means in the current context, suppose that market makers were able to offer sellers bid prices that are so high that every seller fares strictly better by selling to an intermediary than when joining a search market. If these bid prices are equilibrium prices, then it seems natural to assume that market makers would be able to coordinate on these high price equilibria. On the other hand, if no such bid prices exist, then it seems also natural to assume that buyers and sellers could coordinate on joining search markets, so that the equilibria without active search market would appear implausible. A natural conjecture is that the equilibria without search markets are implausible for moderate values of v/t, while for large values, the equilibrium with search markets is implausible.

4 Discussion

I first go through the comparative static exercise of comparing the equilibrium profit of an intermediary when there is no competitor and when there is one under the assumption that both search markets are active in both equilibria. Second, I introduce Cournot competition between market makers within each city. Third, I compare the distributions of equilibrium prices in the search markets when the number of intermediaries differs across markets.

4.1 Complementarity

So as to derive the equilibrium behavior of the intermediary in W when there is no intermediary in E, plug $q^E = 0$ into W's reaction function, which is given in (12). This yields $q^{M*} \equiv \frac{3}{28} < \frac{3}{26} = q^*$, where the superscript M stands for monopoly. Accordingly, the monopoly's equilibrium ask and bid prices are

$$a^{M*} \equiv \frac{v}{2} + \frac{t}{32} \quad \text{and} \quad b^{M*} \equiv \frac{v}{2} - \frac{t}{32}.$$

Notice that $a^{M*} < a^*$ and $b^{M*} > b^*$, i.e., the ask prices are higher, and the bid prices lower, under duopoly.[17] Because both the spread the intermediary earns and the quantity it trades are smaller under monopoly than under duopoly, its equilibrium profit will also be smaller than under duopoly. It is $\Pi^{M*} \equiv \frac{3}{448}t$.

Proposition 3 *With two active search markets, a market maker fares strictly worse under monopoly than under duopoly. Specifically,* $q^{M*} < q^*$, $a^{M*} < a^*$ *and* $b^{M*} > b^*$, *and* $\Pi^{M*} < \Pi^*$ *holds.*

A few comments are in order. First, I haven't shown yet that there is no profitable deviation for the monopoly under which it extinguishes one search market or both. The proof that no such deviation exists is in the Appendix. Second, though the result may seem surprising, the intuition for it is very clear. Consider Figure 5 for an illustration.[18] The top panel depicts the equilibrium configuration with two intermediaries and two search markets. Everything being symmetric, $\tilde{x} = \tilde{y} = \frac{1}{4}$ holds. After the exit of the intermediary in E, buyers and sellers close to E join the search market in E. Thus, the search market in E becomes more attractive from the perspective of buyers and

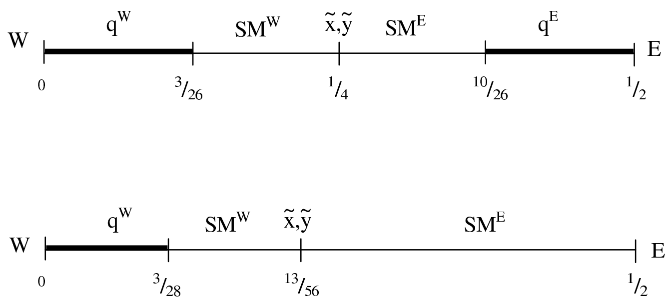

Figure 5: Complementarity Due To Search Market Interactions.

sellers close to $\frac{1}{4}$. Consequently, the locations of the buyer and seller who are indifferent between the search markets are now to the left of $\frac{1}{4}$. In equilibrium, these are at $\frac{13}{56}$, as depicted in the bottom panel. As a consequence, the least efficient producers and the lowest valuation buyers in the search market in W have now lower costs and higher net valuations. Thus, the search market in W becomes more attractive from the point of view of those buyers and sellers who are close to W and who, in equilibrium, trade with the intermediary. As the outside option of search market participation becomes more attractive, consumers' willingness to pay for intermediated trade decreases and producers' reservation prices to sell to the intermediary increase. Therefore, the profit of the intermediary in W decreases.[19]

4.2 Competition Within Each Market Place

Though there is some evidence, both theoretical and empirical, that independent market makers within a given market place may act collusively,[20] in which case the assumption of monopolistic market making within a market place is appropriate, it is important and interesting to see what happens if there is competition between market makers both within a market place and across markets. A simple way to do this is to assume that market makers in a given market place compete à la Cournot. This can be justified as a short-cut to a more complicated model where, much like in Kreps and Scheinkman (1983), market makers set capacity constraints prior to setting prices, where capacity constraints are such that trade up to capacity is costless and trading any quantity beyond is prohibitively costly.[21] Absent capacity constraints, competition between intermediaries in each market place is of the homogenous Bertrand type and yields zero profits for all intermediaries. In equilibrium, all intermediaries i who trade a positive quantity set the Walrasian prices $a_i = b_i = \frac{v}{2}$ and extinguish the search markets.

The Cournot model for market makers is naturally defined as the model in which firms simultaneously choose the quantities q_i they want to trade, facing ask and bid prices set by an auctioneer at which the demand and supply functions clear. I focus here on the symmetric Cournot equilibrium when both search markets are active, where symmetric means that all market makers in a given market place choose the same actions. Let n^k be the number of market makers active in market k with $k = E, W$ and denote by $-k$ "not k".

Proposition 4 *In the symmetric Cournot equilibrium, (i) the equilibrium profit and quantity of every market maker in k increases in n^{-k} and (ii) search markets are extinguished when both cities become perfectly competitive, i.e., when n^W and n^E become arbitrarily large. In the competitive limit, all market makers i set $a_i = b_i = \frac{v}{2}$ and thereby extinguish the search markets.*

The first result was to be expected from the discussion of complementarity. The Cournot equilibrium profit and equilibrium quantity (or capacity) of a market maker in W increases when the number of competitors in E increases. That is, all the incumbent market makers in W will optimally expand their capacities when the intermediated market in E becomes more competitive. Result (ii) can be seen as a corroboration, or application, of the result by Neeman and Vulkan (2003) that centralized trade drives out trade based on bilateral negotiations.

4.3 Price Dispersion in Search Markets

I now briefly investigate the impact of asymmetric market structures on equilibrium price dispersion. Ju, Linn and Zhu (2006) observe that in the aftermath of the exit of Enron as a market maker in the North-American natural gas market, price dispersion in the search market increased dramatically. This raises the question whether this result also obtains in the present model.

As a buyer with valuation v_c and a seller with cost v_p share the aggregate surplus evenly, the price at which they trade is given as $p = \frac{v_c+v_p}{2}$. Thus, the highest price, $\overline{p}$, occurs when the seller with the highest cost $\overline{v}_p$ is matched to the buyer with the greatest valuation $\overline{v}_c$. Similarly, the lowest price, denoted as $\underline{p}$, in the support occurs when the most efficient seller $\underline{v}_p$ active in a search market is matched to the buyer with the lowest valuation $\underline{v}_c$. Denote by $\underline{p}^k$ and $\overline{p}^k$, respectively, the lowest and highest price in the search market k with $k = E, W$ in the case of an asymmetric market structure, where there is an intermediary in W and none in E. The prices $\underline{p}$ and $\overline{p}$ denote, respectively, the lowest and highest price in either search market for the case with two intermediaries. Finally, denote by σ_p^k the variance of equilibrium prices in search market k when there is no intermediary in E and by σ_p the variance of equilibrium prices when there is an intermediary in each city.

Proposition 5 *If only the market maker in W is active, facing a search market in E and W, then the equilibrium price dispersion in the search market in W (E) is smaller (larger) than when both intermediaries are active. That is,*

$$(i)\quad \underline{p}^E < \underline{p} < \underline{p}^W < \overline{p}^W < \overline{p} < \overline{p}^E \quad \textit{and} \quad (ii)\quad \sigma_p^W < \sigma_p < \sigma_p^E.$$

5 Market Makers vs. Matchmakers

Up to now, the assumption was maintained that sellers are paid for providing the good, regardless of whether a market maker attracts any buyers at all. On the up side, this assumption of "standing ready to buy" has the advantage of eliminating coordination problems of buyers and sellers. On the down side, however, it precludes many interesting applications. For example, intermediaries in many markets do not actually buy goods to stock, but rather make payments only when the transaction is completed. For example, a job market intermediary typically only pays wages once a worker is matched to a firm. Similarly, intermediaries in housing markets do typically not buy or sell houses and apartments, but rather require a fee from buyers and/or sellers, which becomes due only when a contract is written.[22] Clearly, any customer of such an intermediary will not only be concerned about the fees it charges, but also about the number (and quality) of sellers and buyers it attracts. This is why these types of intermediaries can be called *matchmakers*, or platforms. Though there are substantial differences between matchmakers and market makers, there is also a great degree of similarity between the two types of intermediary. In particular, as I show next, every equilibrium outcome in a game with market makers is also an equilibrium outcome in an "appropriately defined" game with matchmakers. I begin by introducing the game with matchmakers, whence it should become clear what I mean by appropriately defined.

The Matchmaker Game Matchmakers charge fees $\phi_c \geq 0$ and $\phi_p \geq 0$ to consumers and producers for joining their platform. I assume that these fees are due when joining the matchmaker (or platform), i.e., before eventually being matched. This assumption is made mainly for convenience, though it is not completely innocuous.[23] I also assume that each matchmaker sets an internal transaction price of $p = \frac{v}{2}$ at which a buyer and a seller on the platform exchange the good. This is a simple way to make sure that a matchmaker generates the same gross utility for buyers and sellers as a market maker if it attracts the same buyers and sellers.[24] In case the number of agents of one type (buyers or sellers) exceeds the number of the other type, the agents on the short side are matched with probability one, and those on the long side are matched with a probability less than one.[25] Buyers and sellers are uniformly distributed on semicircles of length $\frac{1}{2}$ and have transportation costs t. Gross utility is v for every buyer, and matchmakers are located at W and E. There may or may not be search markets in W and E as well. I assume also that in addition to joining either search market and either intermediated market, all buyers and sellers may remain inactive.

Before stating and proving the proposition, it is useful to note the connection between the fees ϕ_c and ϕ_p set by matchmakers and ask and bid prices a and b set by market makers. Let $a > \frac{v}{2}$ and $b < \frac{v}{2}$ be the prices set by a market maker. Then, the fees

$$\phi_c(a) \equiv a - \frac{v}{2} \quad \text{and} \quad \phi_p(b) \equiv \frac{v}{2} - b$$

are equivalent in the sense that they generate the same utility for buyers and sellers, conditional on being matched with probability one. To see this, note that the net utility of a buyer with utility v_c from being served by the market maker is $v_c - a$, which equals the surplus of joining the matchmaker, $v_c - \phi_c - \frac{v}{2}$, if $\phi_c = \phi_c(a)$. Similarly, a seller with cost v_p expects $b - v_p$ from a market maker and $\frac{v}{2} - \phi_p - v_p$ from a matchmaker if matched with certainty. The two expressions are the same if $\phi_p = \phi_p(b)$.

Denote by (a^{**}, b^{**}) the ask and bid prices in an equilibrium of the game with market makers, and let $I^{W*} = (I_p^{W*}, I_c^{W*})$ and $I^{E*} = (I_p^{E*}, I_c^{E*})$ be the sets of sellers and buyers joining the intermediary in W and E in this equilibrium. Also, let $S^{W*} = (S_p^{W*}, S_c^{W*})$ and $S^{E*} = (S_p^{E*}, S_c^{E*})$ be the sets of sellers and buyers joining the search market W and E in equilibrium. Note that the sets S^{W*} and S^{E*} can be empty. So, $a^{**} = a^*$ and $b^{**} = b^*$ if the search markets are non-empty and $a^{**} = \hat{a}$ and $b^{**} = \hat{b}$ otherwise, where a^* and b^* are given in Proposition 2 and $\hat{a}$ and $\hat{b}$ are as in (2).

Proposition 6 *The matchmaker game has a subgame perfect equilibrium, where both matchmakers set*

$$\phi_c = \phi_c(a^{**}) \quad \text{and} \quad \phi_p = \phi_p(b^{**}),$$

and where on the equilibrium path the buyers and sellers in I^{W*} and I^{E*} join the matchmaker in W and E and the sellers and buyers in S^{W*} and S^{E*} join the search markets in W and E.

Let me briefly comment on this result. The proof is straightforward and uses the fact that not joining a platform is always a best response if no one else joins it. Thus, whenever a matchmaker deviates from a "prescribed" behavior, all buyers and sellers can punish the matchmaker by not joining it. Anticipating this, every matchmaker optimally chooses not to deviate.[26]

To the best of my knowledge, this is the first model where platforms compete with one another and, on top of that, with simultaneously open search markets.[27] For many applications analyzed in the literature, e.g., platforms that match men and women and business-to-business platforms, the outside options for potential platform customers obviously exist. The usually implicit assumption that these outside options do not depend on the behavior of the platforms is certainly critical. The present model provides a first step into the direction of modelling this kind of interactions.

6 Conclusions

This paper analyzes competition between market makers who are horizontally differentiated and who face competition from simultaneously active search markets. In equilibrium, market makers net positive profits. This contrasts with competition between homogenous intermediaries. Moreover, with active search markets, an intermediary nets a larger profit and trades a larger quantity under duopoly than under monopoly. The intuition is that a larger quantity traded by the competitor in, say, E, makes the search market in E less attractive from the point of view of buyers and sellers who are indifferent between search markets. Consequently, they will join the search market in W, which in turn renders the search market in W less attractive from the point of view of buyers and sellers trading with the intermediary. Thus buyers' willingness to pay for intermediated trade in W increase and sellers' reservation prices to sell to the intermediary in W decrease. This complementarity suggests a potential for contagion among market makers in the presence of search markets. If one market maker goes bankrupt, the other one should be worse off, too.

There are many potentially fruitful paths for further research. As there are multiple equilibria in the matchmaker game, an interesting question is which of these would be selected in an evolutionary game set-up. A second and related question is whether some of the equilibria in the market maker game are more plausible than others, in the sense that either the coalition of buyers and sellers or the coalition of intermediaries can coordinate to block (some of) them. Consequently, the equilibria that can be blocked would appear implausible. Third, as the model provides a channel through which one market maker's bankruptcy can harm the other market maker, future work could include explicitly modelling uncertainty and the possibility of bankruptcy. Last, alternative locations of search and intermediated markets, and the interplay of market makers with matchmakers, can be studied, possibly extending the model to one with endogenous location choice.

Appendix

A Proofs

Proof of Lemma 1: Let $F_p^k(y)$ be the non-degenerate equilibrium distribution of sellers in the search market in k, $k = E, W$, and let $\gamma_c^k \leq 1$ be the probability that a buyer is matched in market k. Then,

$$V_c^W(x') = \frac{\gamma_c^W}{2} \int_0^{\frac{1}{2}} [v - tx' - ty] dF_p^W(y) \tag{14}$$

and

$$V_c^E(x') = \frac{\gamma_c^E}{2} \int_0^{\frac{1}{2}} \left[v - t\left(\frac{1}{2} - x'\right) - t\left(\frac{1}{2} - y\right) \right] dF_p^E(y). \tag{15}$$

Note that because of the continuum assumption, whether a particular buyer joins market k has no effect on γ_c^k. Therefore, $\frac{\partial V_c^W(x')}{\partial x'} < 0$ and $\frac{\partial V_c^E(x')}{\partial x'} > 0$ follows for any non-degenerate distributions F_p^W and F_p^E. Consequently, if $V_c^W(x') \geq V_c^E(x')$ holds, then $V_c^W(x) > V_c^E(x)$ holds for $x < x'$, and similarly, if $V_c^W(x') \leq V_c^E(x')$ holds, then $V_c^W(x) < V_c^E(x)$ holds for $x > x'$. The proof for sellers is completely symmetric and is therefore omitted. ■

Proof of Proposition 1: I first prove a lemma, from which the proposition immediately follows. Call the search market k balanced if the number of buyers and sellers in k is the same, with $k = E, W$.[28]

Lemma 2 *In any equilibrium, both search markets are balanced (i.e., the number of sellers in k equals the number of buyers in k) and equal in size.*

Proof of Lemma 2: The proof consists of two parts. Part I shows that there are no equilibria with balanced search markets that are of different size. Part II then shows that there are no equilibria with unbalanced search markets.

Part I: Notice first that Lemma 1 implies that in any perfect equilibrium, the buyer and seller at 0 will join the search market in W and the buyer and seller at $\frac{1}{2}$ will join the one in E. Therefore, $\underline{v}_p^k = 0$ and $\overline{v}_c^k = v$ for $k = E, W$.

Next, assume without loss of generality that the larger search market is in W. Let $q \geq \frac{1}{4}$ be the number of buyers joining W in equilibrium. Since by hypothesis search markets are balanced, q is also the number of sellers joining W, and all buyers and sellers will be matched in either market. Then, the buyer at location q must get at least the level of utility from the search market in W as from the one in E. That is,

$$V_c^W(q) = \frac{1}{2}\left[v - tq - \frac{1}{2}tq\right] \geq \frac{1}{2}\left[v - t\left(\frac{1}{2} - q\right) - \frac{1}{2}t\left(\frac{1}{2} - q\right)\right] = V_c^E(q),$$

where $tq = \overline{v}_p^W$ and $t\left(\frac{1}{2} - q\right) = \overline{v}_p^E$. Solving for q yields $q \leq \frac{1}{4}$, implying together with the assumption $q \geq \frac{1}{4}$ that $q = \frac{1}{4}$. Thus, both markets must be of equal size.

Part II: Without loss of generality, assume $\tilde{x} < \tilde{y}$, so that buyers are rationed in market E and sellers in market W, and assume that this is an equilibrium. When joining the search market in W, buyer $\tilde{x}$ gets utility $V_c^W(\tilde{x}) = \frac{1}{2}\left[v - t\tilde{x} - \frac{1}{2}t\tilde{y}\right]$, while when joining the market in E he gets $V_c^E(\tilde{x}) = \frac{\gamma_c^E}{2}\left[v - t\left(\frac{1}{2} - \tilde{x}\right) - \frac{1}{2}t\left(\frac{1}{2} - \tilde{y}\right)\right]$. Since $\gamma_c^E < 1$, it follows that $V_c^E(\tilde{x}) < \frac{1}{2}\left[v - t\left(\frac{1}{2} - \tilde{x}\right) - \frac{1}{2}t\left(\frac{1}{2} - \tilde{y}\right)\right]$ holds. In equilibrium, $\tilde{x}$ must be indifferent between the two markets, so $\frac{1}{2}\left[v - t\tilde{x} - \frac{1}{2}t\tilde{y}\right] < \frac{1}{2}\left[v - t\left(\frac{1}{2} - \tilde{x}\right) - \frac{1}{2}t\left(\frac{1}{2} - \tilde{y}\right)\right]$ must hold. Re-arranging and simplifying yields $\tilde{x} > \frac{3}{8} - \frac{1}{2}\tilde{y}$. On the other hand, the seller at $\tilde{y}$ gets utility $V_p^W(\tilde{y}) = \frac{\gamma_p^W}{2}\left[v - \frac{1}{2}t\tilde{x} - t\tilde{y}\right]$ when joining the search market in W and utility $V_p^E(\tilde{y}) = \frac{1}{2}\left[v - \frac{1}{2}t\left(\frac{1}{2} - \tilde{x}\right) - t\left(\frac{1}{2} - \tilde{y}\right)\right]$ when going to E. Since $\gamma_p^W < 1$, $V_p^W(\tilde{y}) < \frac{1}{2}\left[v - \frac{1}{2}t\tilde{x} - t\tilde{y}\right]$ holds. In equilibrium, $\tilde{y}$ must be indifferent between the two markets. A necessary condition for this is $\frac{1}{2}\left[v - \frac{1}{2}t\tilde{x} - t\tilde{y}\right] > \frac{1}{2}\left[v - \frac{1}{2}t\left(\frac{1}{2} - \tilde{x}\right) - t\left(\frac{1}{2} - \tilde{y}\right)\right]$. Re-arranging and simplifying yields $\frac{3}{4} - 2\tilde{y} > \tilde{x}$. Taken together, the equilibrium conditions are thus

$$\frac{3}{4} - 2\tilde{y} > \tilde{x} > \frac{3}{8} - \frac{1}{2}\tilde{y} \tag{16}$$

and by assumption

$$\tilde{y} > \tilde{x}. \tag{17}$$

The final step is to show that conditions (16), (17), and $\tilde{x} \neq \frac{1}{4}$ are not compatible. To see this, assume first $\tilde{x} < \frac{1}{4}$. The second inequality in (16) can then be satisfied only if $\tilde{y} > \frac{1}{4}$. However, the conditions in (16) require also $\frac{3}{4} - 2\tilde{y} > \frac{3}{8} - \frac{1}{2}\tilde{y} \quad \Leftrightarrow \quad \tilde{y} < \frac{1}{4}$, which is the desired contradiction. On the other hand, if $\tilde{x} > \frac{1}{4}$, the first inequality in (16) requires $\tilde{y} < \frac{1}{4}$, which contradicts (17). Hence, there is no equilibrium with unbalanced search markets. ■

So as to complete the proof of the proposition, consider now the constraints in the second part of the proof of Lemma 2 and replace all strict inequalities with weak inequalities. The only case when all constraints are satisfied is when $\tilde{x} = \tilde{y} = \frac{1}{4}$.■

Proof of Proposition 2:

Existence. I first show that the strategies mentioned in the proposition constitute an equilibrium. It has already been shown in the text that given that search markets exist where all buyers and sellers behave as stated, market makers' prices are mutually best responses. It has also been shown that given these prices and given the behavior of all other buyers and sellers, every buyer and seller is best off playing the strategy assigned to him in the proposition. What remains to be shown in order to prove existence is therefore that no market maker has an incentive to unilaterally deviate and to extinguish the search markets.

Consider the market maker in W who can "extinguish" the search market by attracting all buyers and sellers who are not attracted by the intermediary in E. That is, intermediary W could set prices $\tilde{b}^W$ and $\tilde{a}^W$ such that

$$\tilde{b}^W - \frac{10}{26}t = \overbrace{\frac{1}{2}v - \frac{7}{208}t}^{=b^*} - \frac{3}{26}t \quad \text{and} \quad v - \tilde{a}^W - \frac{10}{26}t = v - \overbrace{\left(\frac{1}{2}v + \frac{7}{208}t\right)}^{=a^*} - \frac{3}{26}t,$$

where the right-hand side of both equations is the net utility of the seller and buyer at $\frac{10}{26}$ of patronizing the market maker in E. Solving for $\tilde{b}^W$ and $\tilde{a}^W$ reveals that $\tilde{b}^W = \frac{1}{2}v + \frac{49}{208}t > \frac{1}{2}v - \frac{49}{208}t = \tilde{a}^W$. In other words, the bid price required by this deviation exceeds the ask price. Therefore, this deviation cannot be profitable.

However, there is a priori no reason why the intermediary should contend itself with symmetric strategies when attempting to extinguish the much disliked search market. Similar to the model of Stahl (1988), it is in principle enough to buy the whole stock in order to corner the market. Depending on the elasticity of the demand function, it may then be in its best interest to sell less than it bought. I am going to show now that even this asymmetric deviation strategy is not profitable.

So as to see this, note first that the buyer with location $q < \frac{10}{26}$ is indifferent between buying from the intermediary in W at a^W and the intermediary in E at a^* if and only if $v - a^W - tq = v - a^* - t\left(\frac{1}{2} - q\right)$, which after replacing a^* by $\frac{1}{2}v + \frac{7}{208}t$ and rearranging is seen to imply $\tilde{A}^W(q) = \frac{1}{2}v + \frac{111}{208}t - 2tq$ as the inverse demand function. The revenue maximizing quantity $\tilde{q}^* \equiv \arg\max_q A^W(q)q$ is therefore $\tilde{q}^* = \frac{104v+111t}{832t}$. The maximal revenue is $\frac{(104v+111t)^2}{346112t}$. On the other hand, the expenditure required to buy the quantity $\frac{10}{26}$ is $\tilde{b}^W \cdot \frac{10}{26} = \frac{5}{26}v + \frac{245}{2704}t$. The deviation has already been shown not to pay for $\tilde{q}^* = \frac{10}{26}$, so only the case with $\tilde{q}^* < \frac{10}{26}$ needs to be considered further. But $\tilde{q}^* < \frac{10}{26}$ holds if and only if $v < \frac{209}{104}t$, whereas the deviation is profitable if and only if $\frac{(104v+111t)^2}{346112t} - \left(\frac{5}{26}v + \frac{245}{2704}t\right) \geq \frac{27}{2704}t$. This in turn requires v to be larger than $4.4t$, which contradicts $v < \frac{209}{104}t$. Thus, there is no feasible, profitable deviation.

Uniqueness. The proof of uniqueness consists of three parts. I first show that for given symmetric ask and bid prices $\overline{a}$ and $\overline{b}$ that are the same in E and W, where symmetry means that for any spread $z \geq 0$

$$\overline{a} = \frac{v+z}{2} \quad \text{and} \quad \overline{b} = \frac{v-z}{2}$$

holds, there is a unique quantity $q^*(z)$ traded by each intermediary. (Note that the equilibrium prices a^* and b^* are symmetric.) Second, I show that there are no equilibria where ask and bid prices are symmetric in E and W, but where $a^W \neq a^E$ (and $b^W \neq b^E$). Third, I show that there are no equilibria where an intermediary sets asymmetric prices.

Claim I: For any spread z that is symmetric around $\frac{v}{2}$, there is a unique quantity $q^*(z)$ of sellers and buyers joining each intermediary.

Note: The concern here is that in principle there may be a coordination problem between sellers and buyers because if more (high valuation) buyers are active in the search market, search markets are more attractive for sellers. I will show that because of the market maker's commitment to buy any alternative candidate equilibrium unravels.

Proof: For analytical ease, I assume that if there is rationing at an intermediary, the efficient rationing rule applies. That is, buyers and sellers who are closer to an intermediary have priority over agents who are further away. Buyers and sellers who are rationed can go back to the search market.

Assume $q^W = q^E = q^*$ and invert the function $Z(.)$ given in equation (11) to get quantity $q^*(z)$ as a function of the spread z (replacing Z^W by z). It is easy to see that $q^*(z) = \frac{1}{4} - 2\frac{z}{t}$.

Step 1a: Given $\bar{b} = \frac{v-z}{2} > 0$, there is a unique $y_1 \in (0, q^*(z)]$ such that all $y \leq y_1$ and all $y \geq \frac{1}{2} - y_1$ join the intermediaries in W and E even if all buyers are active in the search market. The remainder of the argument applies repeatedly the same two steps.

Proof: Solve $V_p^W(y_1) = \frac{1}{2}\left[\frac{1}{2}(v + v - t\frac{1}{4}) - ty_1\right] = \bar{b} - ty_1$ for y_1 to get $y_1 = \frac{1}{8} - \frac{z}{t} = \frac{1}{2}q^*(z)$. Note that the assumption $\tilde{x} = \frac{1}{4}$ has been implicitly used. Since initially, i.e., before the set of sellers in $[0, y_1]$ and $[\frac{1}{2} - y_1, \frac{1}{2}]$ joined the intermediaries in W and E, the two search markets are symmetric, the assumption is indeed correct. Moreover, the search markets being symmetric after step 1a, the assumption that $\tilde{y} = \frac{1}{4}$ is correct and can be used in step 1b. For the same reasons, $\tilde{x} = \tilde{y} = \frac{1}{4}$ will hold in any subsequent step.

Step 1b: Given $\overline{a}$ and that the y_1 most efficient sellers leave the search markets, there is a $x_1 \in (0, q^*(z)]$ such that all buyers with $x \leq x_1$ and all $x \geq \frac{1}{2} - x_1$ join the intermediaries in W and E.

Proof: Here the efficient rationing rule comes to play a role. Solve $V_c^W(x_1') = \frac{1}{2}\left[v - tx_1' - \frac{1}{2}(ty_1 + t\frac{1}{4})\right] = v - tx_1' - \overline{a}$ for x_1' to get $x_1' = \frac{1}{8} - \frac{z}{t} + \frac{1}{4}q^*(z) = \frac{3}{4}q^*(z) > y_1$. However, since only the $y_1 < x_1'$ closest buyers will be served, only the y_1 closest buyers will leave the search market. So, let $x_1 = y_1$.

Step 2a: Given steps 1a and 1b, there is a $y_2 \in (y_1, q^*(z)]$ such that all $y \leq y_2$ and all $y \geq \frac{1}{2} - y_2$ join the intermediaries in W and E even if all remaining buyers are active in the search market.

Proof: Solve $V_p^W(y_2) = \frac{1}{2}\left[\frac{1}{2}(v - tx_1 + v - t\frac{1}{4}) - ty_2\right] = \overline{b} - ty_2$ for y_2 to get $y_2 = \frac{3}{16} - \frac{3}{2}\frac{z}{t} = \frac{3}{4}q^*(z)$.

Step 2b: Apply the same reasoning as in step 1b to establish that $x_2 = y_2$.

Step k: In general, after the k-th step, the buyers and sellers attracted by an intermediary will be $y_k = \sum_{i=1}^{k}\left(\frac{1}{2}\right)^i q^*(z)$. Let k go to infinity and use the formula for a geometric sum to see that $y_\infty \equiv \sum_{i=1}^{\infty}\left(\frac{1}{2}\right)^i q^*(z) = q^*(z)$. Thus, there is a unique equilibrium with active search markets given symmetric prices $(\overline{a}, \overline{b})$. This completes the proof of claim I. ■

Claim II: There are no equilibria where spreads are symmetric but different in E and W. That is, there are no equilibria with $z^W \neq z^E$, z^k being such that $a^k = \frac{v+z^k}{2}$ and $b^k = \frac{v-z^k}{2}$, $k = E, W$.

Proof: Consider the spread function $Z^W(q^W, q^E) = t\left(\frac{1}{8} + \frac{1}{12}q^E - \frac{7}{12}q^W\right)$ the intermediary in W faces and the corresponding function the intermediary in E faces, which is $Z^E(q^E, q^W) = t\left(\frac{1}{8} + \frac{1}{12}q^W - \frac{7}{12}q^E\right)$. The two first order conditions $0 = Z^{k'}(q^k, k^{-k})q^k + Z^k(q^k, k^{-k})$ for $k = E, W$ have a unique solution, with $q^k = q^{-k}$. Accordingly, $z^W = z^E$ will hold. ■

Claim III: There are no equilibria with asymmetric spread(s).

Proof: Let $y(b, a)$ and $x(a, b)$ be the sellers and buyers joining the intermediary in W who sets price a and b. Assume first that its prices a and b are such that $y(b, a) < x(a, b)$. Since the rationed buyers (who are located in $[y(b, a), x(a, b)]$) will join the search market, increasing a until $x(a, b) = y(b, a)$ will unambiguously increase the profit of the intermediary.

So, assume that $y(b, a) > x(a, b)$. Ruling out this case is not straightforward because decreasing b and thereby decreasing y will make search market participation more attractive for buyers, thus reducing their willingness to pay. Put differently, one reason why an intermediary might choose prices a and b to induce $y(b, a) > x(a, b)$ is that this increases buyers' reservation prices to buy from the intermediary because search market participation is less attractive. However, as I will show now, such a policy will never be optimal because it will be in the interest of the intermediary to sell all the quantity it buys.

Denote by $\hat{A}^W(x)$ the inverse demand function facing the intermediary in W when inducing an unbalanced search market in W. Clearly, a necessary condition for the policy to be profitable is that it results in an outward shift of this inverse demand function, i.e., $\hat{A}^W(x) > A^W(q^W, q^E)$ must hold for $x = q^W$, where $A^W(q^W, q^E)$ is as defined in (9). Denote $q_0 \equiv \arg\max_{q^W} A^W(q^W, q^E)q^W = \frac{6}{7}\frac{v}{t} + \frac{3}{28} + \frac{1}{14}q^E$. Since $\frac{v}{t} > \frac{3}{2}$, $q_0 > \frac{1}{2}$ follows (which of course is not feasible, but that is immaterial for the present argument). In other words, the intermediary would like to sell more than its stock if there were no costs of acquiring stock (and neglecting any other constraints). Now, because $\hat{A}^W(x) > A^W(q^W, q^E)$, $x_0 \equiv \arg\max_x \tilde{A}^W(x)x > q_0$ follows. Since $y(b, a) > x(a, b)$ holds by hypothesis, it will be both possible and desirable to sell more than $x(a, b)$ by adjusting prices to a', b' such that $y(b', a') = x(a', b')$ holds. Thus, the strategy (a, b) such that $y(b, a) > x(a, b)$ cannot be optimal.

The final thing to show is that $y(b, a) = x(a, b)$ can only be achieved by symmetric prices $a = \frac{v+z}{2}$ and $b = \frac{v-z}{2}$. But the seller at location q^W will be indifferent between selling to W at b^W and joining the search market in W if and only if $b^W = B^W(q^W, q^E) = \frac{v}{2} - \frac{1}{16}t - \frac{1}{24}tq^E + \frac{7}{24}tq^W$, where $B^W(q^W, q^E)$ is the inverse supply function from equation (10). Similarly, given ask price a^W the buyer at q^W is indifferent to buy from the intermediary and participating in the search market if and only if $a^W = A^W(q^W, q^E) = \frac{v}{2} + \frac{1}{16}t + \frac{1}{24}tq^E - \frac{7}{24}tq^W$, where $A^W(q^W, q^E)$ is the inverse demand function in (9). Clearly, $a^W = \frac{v+z}{2}$ and $b^W = \frac{v-z}{2}$ with $z = \frac{1}{8}t + \frac{1}{12}tq^E - \frac{7}{12}tq^W$. Thus, given that the other intermediary attracts the same number of buyers and sellers (i.e., q^E), an intermediary can attract the same number of buyers and sellers q^W if and only if it sets symmetric prices. ■

Proof of Proposition 3: So as to show that there is no profitable deviation for the monopolistic intermediary, I first show that it is not profitable for the monopolistic intermediary in W to extinguish only the search market in W while keeping the one in E alive. The issue here is that if the quantity q^W traded by the intermediary in

W is large enough, then no agent with location $\tilde{x} > q^W$ will be indifferent between the two search markets, but will rather prefer search market E to the search market W. This is easiest to see by inspection of (8) when setting $q^E = 0$. For $\tilde{x} \geq q^W$ to hold, $\frac{1}{4} - \frac{1}{6}q^W \geq q^W \Leftrightarrow q^W \leq \frac{3}{14}$. For larger q^W's, the agents at q^W will be indifferent between joining the intermediary in W and the search market in E.[29] Consequently, the reservation prices for trading the quantity $\tilde{q}$ with the intermediary in W will be an a^W and b^W such that $v - a^W - t\tilde{q} = V_c^E(\tilde{q})$ and $b^W - t\tilde{q} = V_p^E(\tilde{q})$, where $V_c^E(\tilde{q})$ and $V_c^E(\tilde{q})$ are as defined in the previous footnote. Solving for a^W and b^W yields the inverse demand and supply functions $A^W(\tilde{q}) = \frac{v}{2} + \frac{3}{8}t - \frac{7}{4}t\tilde{q}$ and $B^W(\tilde{q}) = \frac{v}{2} - \frac{3}{8}t + \frac{7}{4}t\tilde{q}$, so that the intermediary's profit is $\Pi^W(\tilde{q}) = \frac{3}{4}t(3 - 14\tilde{q})\tilde{q}$, which is maximized at $\tilde{q}^* = \frac{3}{28}$. However, this violates the restriction that $\tilde{q}$ must be larger than $\frac{3}{14}$ for these inverse demand and supply functions to be valid in the first place. Consequently, the optimal quantity will be as small as necessary, i.e., will equal $\frac{3}{14}$. Inserting this value into the profit function reveals immediately that the profit will be zero. Hence, this deviation does not pay.

Second, the intermediary may want to take over the whole market by extinguishing the search market in E, too. However, so as to attract the buyer and seller at $\frac{1}{2}$, these agents must be offered an ask price below $\frac{v}{2}$ and a bid price above $\frac{v}{2}$, where $\frac{v}{2}$ is the price at which they would trade in the search market in E if only these two agents join the search market in E. Clearly, this deviation entails a negative profit and is therefore not profitable. ■

Proof of Proposition 4: Denote by Q^k the aggregate quantity of all market makers in k and by q_i^k firm i's quantity and by q_{-i}^k the aggregate quantity of all firms other than i in k with $k = E, W$. So $Q^k \equiv q_i^k + q_{-i}^k$. Part (i): Without loss of generality, focus on the market in W. Market maker i's problem is to maximize the analogue of equation (11) for the Cournot model, i.e., to choose the q_i^W that maximizes $t\left(\frac{1}{8}+\frac{1}{12}Q^E-\frac{7}{12}q_i^W-\frac{7}{12}q_{-i}^W\right)q_i^W$. The first order condition is $0=t\left(\frac{1}{8}+\frac{1}{12}Q^E-\frac{7}{6}q_i^W-\frac{7}{12}q_{-i}^W\right)$. Imposing symmetry, i.e., $q_{-i}^W=(n^W-1)1q_i^W$, one gets $q_i^W=\frac{3+2Q^E}{14(n^W+1)}$ and thus $Q^W=\frac{n^W(3+2Q^E)}{14(n^W+1)}$. Similarly, $Q^E=\frac{n^E(3+2Q^W)}{14(n^E+1)}$ can be derived. Solving this system of two equations, one obtains $Q^{W*}=\frac{3}{2}\frac{n^W(8n^E+7)}{48n^En^W+49+49n^W+49n^E}$ and $Q^{E*}=\frac{3}{2}\frac{n^E(8n^W+7)}{48n^Wn^E+49+49n^E+49n^W}$ as equilibrium quantities (or capacities), which increase in n^W and n^E. The equilibrium profit of a market maker in W is thus $\frac{21}{16}\frac{8(n^E+7)^2}{(48n^En^W+49+49n^W+49n^E)^2}$, which increases in n^E. Part (ii): $\lim_{n^W,n^E\to\infty}Q^{W*}=\frac{1}{4}$. Thus, in the limit intermediated markets drive out search markets. ■

Proof of Proposition 5: I derive first the distribution and variance of equilibrium prices in a search market for given $\underline{v}_c, \underline{v}_p, \overline{v}_c$ and $\overline{v}_p$. Then I derive the values of $\underline{v}_c, \underline{v}_p, \overline{v}_c$ and $\overline{v}_p$ for the case when there are two market makers and when there is a market maker in W and none in E.

Let w and r be two independent random variables that are uniformly distributed on $[\underline{w},\underline{w}+\theta]$ and $[\underline{r},\underline{r}+\theta]$, respectively, where $\theta>0$. Also, let $\Sigma\equiv w+r$ and denote by $f(\Sigma)$ the density of Σ. Then,

$$f(\Sigma)=\begin{cases}\frac{1}{\theta^2}\left(\Sigma-\underline{w}-\underline{r}\right) & \text{for} \quad \underline{w}+\underline{r}\leq\Sigma\leq\underline{w}+\underline{r}+\theta\\ \frac{1}{\theta^2}\left(\underline{w}+\underline{r}+2\theta-\Sigma\right) & \text{for} \quad \underline{w}+\underline{r}+\theta<\Sigma\leq\underline{w}+\underline{r}+2\theta\end{cases}$$

and zero else. The mean of Σ is $\underline{w}+\underline{r}+\theta$ and its variance is $\frac{\theta^2}{6}$, which obviously increases in θ. Now $p\equiv\frac{v_c+v_p}{2}\in[\underline{p},\overline{p}]$ is the sum of the independent random variables $w+r$ with $w\equiv v_c/2$ and $r\equiv v_p/2$, which are uniformly distributed on $[\frac{\underline{v}_c}{2},\frac{\overline{v}_c}{2}]$ and $[\frac{\underline{v}_p}{2},\frac{\overline{v}_p}{2}]$. Since search markets are balanced in equilibrium, $\frac{\overline{v}_c}{2}-\frac{\underline{v}_c}{2}=\frac{\overline{v}_p}{2}-\frac{\underline{v}_p}{2}$. Let $\theta\equiv\overline{v}_c-\underline{v}_c=\overline{v}_p-\underline{v}_p$.

Then, the difference between the highest price, $\frac{\overline{v}_c+\overline{v}_p}{2}$, and the lowest price, $\frac{\underline{v}_c+\underline{v}_p}{2}$, is θ. Thus, (ii) follows once (i) is shown.

When two market makers are active, each of them trades the quantity $q = \frac{3}{26}$ and $\tilde{x} = \tilde{y} = \frac{1}{4}$. Thus, $\overline{v}_p = \frac{1}{4}t$ and $\overline{v}_c = v - \frac{3}{26}t$, so that $\overline{p} = \frac{v}{2} + \frac{7}{104}t$. As for the lowest price, $\underline{v}_p = \frac{3}{26}t$ and $\underline{v}_c = v - \frac{1}{4}t$, so that $\underline{p} = \frac{v}{2} - \frac{7}{104}t$. When there is but one market maker, the two search markets differ. Recall that $\tilde{x} = \frac{13}{56}$. Therefore, $\overline{v}_c^E = v$ and $\overline{v}_p^E = t\frac{15}{56}$. This implies $\overline{p}^E = \frac{v}{2} + \frac{15}{112}t$. The lowest price in E is given by $\underline{p}^E = \frac{v}{2} - \frac{15}{112}t$, since $\underline{v}_p^E = 0$ and $\underline{v}_c^E = v - t\frac{15}{56}$. In the search market in W, $\overline{v}_p^W = \frac{13}{56}t$, $\overline{v}_c^W = v - \frac{3}{28}t$, $\underline{v}_p^W = \frac{3}{28}t$ and $\underline{v}_c^W = v - \frac{13}{56}t$. Consequently, $\overline{p}^W = \frac{v}{2} + \frac{7}{112}t$ and $\underline{p}^W = \frac{v}{2} - \frac{7}{112}t$ follows. Therefore, (i) is true. ■

Proof of Proposition 6: Note first that in any equilibrium, market makers make positive profits. If buyers and sellers behave as stated in the proposition, then matchmakers will thus make positive profits when setting $\phi_c(a^{**})$ and $\phi_p(b^{**})$. Second, from the fact that the strategies of the buyers and sellers are an equilibrium in the game with market makers, it follows that the actions of buyers and sellers in the proposition are optimal in the game with matchmakers, given that all the other buyers and sellers behave as stated. What therefore remains to be shown is that a matchmaker cannot increase its profits given the strategies played by buyers and sellers and the other matchmaker. To see that buyers and sellers can deter any deviation by a matchmaker, note that not joining any matchmaker if no one else joins a matchmaker is a best response for every buyer and seller. Thus, if buyers and sellers join the matchmakers if and only if these set $\phi_c = \phi_c(a^{**})$ and $\phi_p = \phi_p(b^{**})$, and otherwise remain inactive or go to the search markets, deviation does not pay for matchmakers either. ■

Notes

[1]For a model of market making at stock markets, see, e.g., Gehrig (1993); for fish markets, see Graddy (1995, 2006).

[2]According to the definition of the U.S. Securities and Exchange Commission (SEC) (http://www.sec.gov/answers/mktmaker.htm.), "[a] *market maker* is a firm that stands ready to buy and sell a particular stock on a regular and continuous basis at a publicly quoted price."

[3]See the discussion paper version (Loertscher, 2005) for a complete treatment of the cases with $\frac{v}{t} < \frac{3}{2}$. For the model with search markets, the results do not change as long as $\frac{v}{t} > 1$. Without search markets, the market makers' equilibrium prices would be different for $\frac{v}{t} \in \left(1, \frac{3}{2}\right)$. These prices would be such that the indifferent buyer and seller derive zero net utility, which is completely analogous to the standard Hotelling-Salop type of model for this parameter range.

[4]On the other hand, an intermediary is not obliged to serve all buyers. If quantity demanded exceeds its stock, some buyers will be rationed.

[5]This maintains a property of models with homogenous market makers; see Gehrig (1993), Fingleton (1997) and Rust and Hall (2003). It contrasts with Yavas (1994), where low valuation buyers and high cost sellers join the intermediary. In Yavas' model, an intermediary does not set ask and bid prices but charges a percentage of the surplus generated by a match, which enables the intermediary to price discriminate. Consequently, those buyers and sellers who expect to generate a high surplus gain less by joining the intermediary.

[6]Note that the connection to Salop's model is somewhat spurious as the circle assumption is used merely as a simple device that prevents trade by buyers and sellers in locations other than W and E.

[7]A more detailed treatment can be found in the discussion paper version or in Spulber (1999, Ch.3).

[8]I assume that in case a buyer is rationed by an intermediary he can join the other intermediary at no cost in excess of the cost he would have incurred had he joined the other intermediary at the outset. This amounts to assuming that the travel cost are incurred only if the good is actually bought.

[9]The results generalize straightforwardly to the case where a buyer grasps the share $\alpha \in (0, 1)$ when bargaining in the search market and a seller gets the share $1 - \alpha$ of the surplus generated by the match.

The only difference will be that the equilibrium ask and bid prices will depend negatively on α.

[10]Using the terminology of Spulber (2006), all possible matches are viable for $\frac{v}{t} > 1$.

[11]I make a slight abuse of notation by using $V_i(.), i = c, p$ to denote both the expected utility from search market participation as a function of the net valuation (e.g., v_c) *and* as a function of the location (e.g., x). Though the two things are clearly connected, e.g. for a buyer at x who joins the search market in W $v_c \equiv v - tx$, the two functions are not exactly the same.

[12]To see this, differentiate both sides of (5) with respect to v_c. The derivative of the left-hand side is one, while the right-hand side increases by less as v_c increases. Therefore, if for a given $a, \tilde{v}_c - a = V_c(\tilde{v}_c)$ holds for some $\tilde{v}_c$, then $v_c - a \gtreqless V_c(v_c) \Leftrightarrow v_c \gtreqless \tilde{v}_c$, whence $\tilde{v}_c = \overline{v}_c$ follows.

[13]Notice that here the half-circles have been collapsed into straight lines. So the horizontal axes depicts the locations both of buyers and sellers, who can, however, only trade with one another in W or E.

[14]This point is similar to Yavas (1995), who observes that in a model with endogenous search intensities the presence of a broker can reduce the set of equilibria and may even induce a unique equilibrium.

[15]Similar equilibrium behavior obtains in the model of Fingleton (1997).

[16]See Proposition 4 below for a precise statement under Cournot competition within a market place.

[17]Chen and Riordan (2006) derive the conditions under which in a Hotelling model of product market competition prices are higher under duopoly than under monopoly. Their paper also contains references to papers providing some empirical evidence consistent with price increasing competition.

[18]As in Figure 4, the half-circles have been collapsed into straight lines. The horizontal axes depicts the locations both of buyers and sellers, who can, however, only trade with one another in W or E.

[19]There is some similarity with the Hotelling model of product market competition. Chen and Riordan (2007) analyze a generalized Hotelling model, the spokes model, where for some parameter ranges, profits increase when the number of firms increases.

[20]See, e.g., Graddy (1995) or Dutta and Madhavan (1997).

[21]If one assumes efficient rationing Cournot actions are an equilibrium outcome of the intermediation game under otherwise fairly mild assumptions, all of which are satisfied here; see Loertscher (2007) for

details. Efficient rationing is not implausible if agents are served on a first come first serve basis and if agents who are closer to an intermediary reach the intermediary before others do.

[22]An additional fee may also be charged to the customer to be admitted to the database of the intermediary.

[23]If fees are only due in case a match occurs, then not joining a platform may be weakly dominated by joining it, whereas this is not true when fees are charged upon joining a platform.

[24]Alternatively, and equivalently, platforms could use uniform random matching of buyers and sellers, who share the gains from trade evenly. In the presence of search markets, platforms would then simply induce a separation of random matching markets.

[25]For example, if there are 5 buyers and 3 sellers, each buyer is matched with probability 0.6.

[26]This is not surprising, of course. It is clear, though, that the harsh punishment strategy I make use of in the proof is only sufficient.

[27]See Gehrig (1993) for an early model in which a monopoly platform competes with a search market.

[28]I am grateful to an anonymous referee whose suggestions helped shorten and clarify the proof.

[29]An alternative way to see this is as follows. Let $\tilde{q}$ be the location of the buyer and seller who are the only agents in the search market in W and who would consequently be the agents with the lowest valuation and the highest cost in the search market in E. Then, their utility from the search market in W is $V_c^W(\tilde{q}) = \frac{v}{2} - t\tilde{q} = V_p^W(\tilde{q})$, while their expected utility from participating in E is $V_c^E(\tilde{q}) = V_p^E(\tilde{q}) = \frac{v}{2} - \frac{3}{8}t + \frac{3}{4}t\tilde{q}$, where the fact has been used that the most efficient seller in E has cost equal to zero and the highest valuation buyer a valuation of v. Notice that these utilities increase in $\tilde{q}$ because the search market in E becomes more attractive the closer to $\frac{1}{2}$ the marginal buyers and sellers are. It is easy to see that $V_c^W(\tilde{q}) \geq V_c^E(\tilde{q}) \Leftrightarrow \tilde{q} \leq \frac{3}{14}$ and $V_p^W(\tilde{q}) \geq V_p^E(\tilde{q}) \Leftrightarrow \tilde{q} \leq \frac{3}{14}$.

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