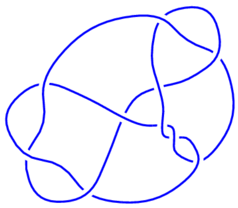
10_{142} $(K10n_{30})$



[1]

Arc Sequences

$$\rightarrow\downarrow\rightarrow\downarrow\rightarrow\downarrow\rightarrow\downarrow\rightarrow\downarrow\rightarrow\downarrow\rightarrow\downarrow\rightarrow\downarrow\rightarrow\downarrow\rightarrow\downarrow\rightarrow$$

8 6 9 8 2 3 1 4 5 2

Solving Sequence

$$2{,}5 \xrightarrow[c_5]{} 6 \xrightarrow[c_2]{} 3{,}8 \xrightarrow[c_1]{} 1 \xrightarrow[c_4]{} 4 \xrightarrow[c_8]{} 9 \xrightarrow[c_7]{} 7 \xrightarrow[c_{10}]{} 10 \twoheadrightarrow c_3, c_6, c_9$$

Representation Ideals

$$I = \bigcap_{i=1}^{4} I_i^u$$

$$I_1^u = \langle u - 1,\ a,\ b + 1\rangle$$
$$I_2^u = \langle a^2 + 2,\ u + 1,\ b - 1\rangle$$
$$I_3^u = \langle u^4 + u^3 - 2u^2 + 3,\ -u^3 + b + 2u - 1,\ 2u^3 - u^2 + 3a - 4u + 3\rangle$$
$$I_4^u = \langle u^6 + u^5 - 5u^4 - 4u^3 + 5u^2 - u - 1,\ b - u,\ -u^4 - u^3 + 4u^2 + 2a + 5u - 1\rangle$$

There are 4 irreducible components with 13 representations.

[1]The knot diagram image is adapter from "C. Livingston and A. H. Moore, KnotInfo: Table of Knot Invariants, http://www.indiana.edu/ knotinfo"

I. $I^u_1 = \langle u-1,\ a,\ b+1\rangle$

(i) Arc colorings

$$a_2 = \begin{pmatrix}1\\0\end{pmatrix}$$
$$a_5 = \begin{pmatrix}0\\1\end{pmatrix}$$
$$a_6 = \begin{pmatrix}-1\\1\end{pmatrix}$$
$$a_3 = \begin{pmatrix}0\\1\end{pmatrix}$$
$$a_8 = \begin{pmatrix}0\\-1\end{pmatrix}$$
$$a_1 = \begin{pmatrix}1\\-1\end{pmatrix}$$
$$a_4 = \begin{pmatrix}0\\1\end{pmatrix}$$
$$a_9 = \begin{pmatrix}0\\-1\end{pmatrix}$$
$$a_7 = \begin{pmatrix}-1\\0\end{pmatrix}$$
$$a_{10} = \begin{pmatrix}0\\-1\end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= -12$

(iv) Complex Volumes and Cusp Shapes

Solution to I_1^u	$\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$	Cusp shape
$u = 1.00000$ $a = 0$ $b = -1.00000$	-3.28987	-12.0000

II. $I_2^u = \langle a^2 + 2,\ u + 1,\ b - 1 \rangle$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$a_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} a \\ 1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} -a + 1 \\ -1 \end{pmatrix}$$
$$a_4 = \begin{pmatrix} 2 \\ -a - 1 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} -a \\ a - 1 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_{10} = \begin{pmatrix} -a \\ -1 \end{pmatrix}$$

(ii) Obstruction class $= 1$

(iii) Cusp Shapes $= -12$

(iv) Complex Volumes and Cusp Shapes

Solution to I_2^u	$\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.00000$ $a = \quad - 1.41421I$ $b = \quad 1.00000$	1.64493	−12.0000
$u = -1.00000$ $a = \quad 1.41421I$ $b = \quad 1.00000$	1.64493	−12.0000

III. $\boldsymbol{I_3^u = \langle u^4 + u^3 - 2u^2 + 3,\ -u^3 + b + 2u - 1,\ 2u^3 - u^2 + 3a - 4u + 3\rangle}$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_6 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_3 = \begin{pmatrix} -u^2 + 1 \\ u^2 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -\frac{2}{3}u^3 + \frac{1}{3}u^2 + \frac{4}{3}u - 1 \\ u^3 - 2u + 1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} \frac{2}{3}u^3 - \frac{1}{3}u^2 - \frac{1}{3}u + 3 \\ -u^3 + u^2 + u - 4 \end{pmatrix}$$
$$a_4 = \begin{pmatrix} -\frac{1}{3}u^3 + \frac{2}{3}u^2 + \frac{2}{3}u - 1 \\ u^3 - u^2 - u + 2 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} -\frac{1}{3}u^3 + \frac{2}{3}u^2 + \frac{2}{3}u - 1 \\ -1 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} u^3 - 2u \\ -u^3 + u \end{pmatrix}$$
$$a_{10} = \begin{pmatrix} -\frac{1}{3}u^3 + \frac{2}{3}u^2 + \frac{2}{3}u - 1 \\ -u^3 + u^2 + u - 4 \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= 4u^3 - 4u - 6$

(iv) Complex Volumes and Cusp Shapes

Solution to I_3^u	$\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$	Cusp shape
$u=-1.45326-0.16311I$ $a=-0.273719+0.626639I$ $b=\quad 0.953264-0.702911I$	$-3.28987+2.02988I$	$-12.00000-3.46410I$
$u=-1.45326+0.16311I$ $a=-0.273719-0.626639I$ $b=\quad 0.953264+0.702911I$	$-3.28987-2.02988I$	$-12.00000+3.46410I$
$u=\quad 0.953264-0.702911I$ $a=\quad 0.773719-0.337964I$ $b=-1.45326-0.16311I$	$-3.28987+2.02988I$	$-12.00000-3.46410I$
$u=\quad 0.953264+0.702911I$ $a=\quad 0.773719+0.337964I$ $b=-1.45326+0.16311I$	$-3.28987-2.02988I$	$-12.00000+3.46410I$

IV.

$$\boldsymbol{I^u_4 = \langle u^6 + u^5 - 5u^4 - 4u^3 + 5u^2 - u - 1,\ b - u,\ -u^4 - u^3 + 4u^2 + 2a + 5u - 1\rangle}$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_6 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_3 = \begin{pmatrix} -u^2 + 1 \\ u^2 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} \frac{1}{2}u^4 + \frac{1}{2}u^3 - 2u^2 - \frac{5}{2}u + \frac{1}{2} \\ u \end{pmatrix}$$
$$a_1 = \begin{pmatrix} -\frac{1}{2}u^5 - \frac{1}{2}u^4 + \cdots - \frac{1}{2}u + 1 \\ -u^2 \end{pmatrix}$$
$$a_4 = \begin{pmatrix} -\frac{1}{2}u^5 - u^4 + \cdots - 2u - \frac{3}{2} \\ \frac{1}{2}u^4 - \frac{1}{2}u^3 - 2u^2 + \frac{3}{2}u + \frac{1}{2} \end{pmatrix}$$
$$a_9 = \begin{pmatrix} -\frac{1}{2}u^5 - \frac{1}{2}u^4 + \cdots - \frac{1}{2}u + 1 \\ \frac{1}{2}u^5 + \frac{1}{2}u^4 - 2u^3 - \frac{3}{2}u^2 + \frac{1}{2}u \end{pmatrix}$$
$$a_7 = \begin{pmatrix} u^3 - 2u \\ -u^3 + u \end{pmatrix}$$
$$a_{10} = \begin{pmatrix} -\frac{1}{2}u^5 - \frac{1}{2}u^4 + \cdots - \frac{1}{2}u + 1 \\ -u^2 \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= -u^4 - 3u^3 + 4u^2 + 11u - 13$

(iv) Complex Volumes and Cusp Shapes

Solution to I_4^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.85126 - 0.30576I$ $a = 0.464021 + 0.716665I$ $b = -1.85126 - 0.30576I$	$-13.0621 - 6.7708I$	$-12.38492 + 2.96218I$
$u = -1.85126 + 0.30576I$ $a = 0.464021 - 0.716665I$ $b = -1.85126 + 0.30576I$	$-13.0621 + 6.7708I$	$-12.38492 - 2.96218I$
$u = -0.338910$ $a = 1.10469$ $b = -0.338910$	-0.610583	-16.1650
$u = 0.526900 - 0.379519I$ $a = -1.19618 + 1.56453I$ $b = 0.526900 - 0.379519I$	$3.26038 + 1.42716I$	$-6.28345 - 4.88332I$
$u = 0.526900 + 0.379519I$ $a = -1.19618 - 1.56453I$ $b = 0.526900 + 0.379519I$	$3.26038 - 1.42716I$	$-6.28345 + 4.88332I$
$u = 1.98762$ $a = -0.640373$ $b = 1.98762$	-17.6195	-14.4983

V. u-Polynomials

Crossings	u-Polynomials at each crossings
c_1	$(u-1)^2(u+1)(u^4+u^3-2u^2+3)(u^6+u^5+\cdots-u-1)$
c_2	$(u-1)(u+1)^2(u^4+u^3-2u^2+3)(u^6+u^5+\cdots-u-1)$
c_3, c_4, c_8	$u(u^2+2)(u^2-u+1)^2(u^6+3u^5+7u^4+10u^3+10u^2+8u+2)$
c_5, c_6	$(u-1)^3(u^4+u^3-2u^2+3)(u^6+u^5-5u^4-4u^3+5u^2-u-1)$
c_7	$(u+1)^3(u^4+u^3-2u^2+3)(u^6+u^5-5u^4-4u^3+5u^2-u-1)$
c_9	$u(u^2+2)(u^2-u+1)^2(u^6+3u^5-11u^4-32u^3-2u^2-16u+10)$
c_{10}	$(u-1)^2(u+1)(u^4+5u^3+10u^2+12u+9)$ $(u^6+11u^5+43u^4+66u^3+27u^2+11u+1)$

VI. Riley Polynomials

Crossings	Riley Polynomials at each crossings
c_1, c_2, c_5 c_6, c_7	$(y-1)^3(y^4-5y^3+10y^2-12y+9)$ $(y^6-11y^5+43y^4-66y^3+27y^2-11y+1)$
c_3, c_4, c_8	$y(y+2)^2(y^2+y+1)^2(y^6+5y^5+9y^4-4y^3-32y^2-24y+4)$
c_9	$y(y+2)^2(y^2+y+1)^2$ $(y^6-31y^5+309y^4-864y^3-1240y^2-296y+100)$
c_{10}	$(y-1)^3(y^4-5y^3-2y^2+36y+81)$ $(y^6-35y^5+451y^4-2274y^3-637y^2-67y+1)$