

Phyx 320

Modern Physics

March 1, 2021

Reading: 38.5 – 38.7

Homework #5 and Reading Reflection Tuesday 11:59 pm

Matter Waves

Both light and matter are waves

De Broglie wavelength:

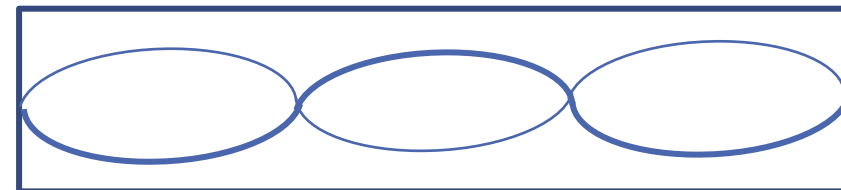
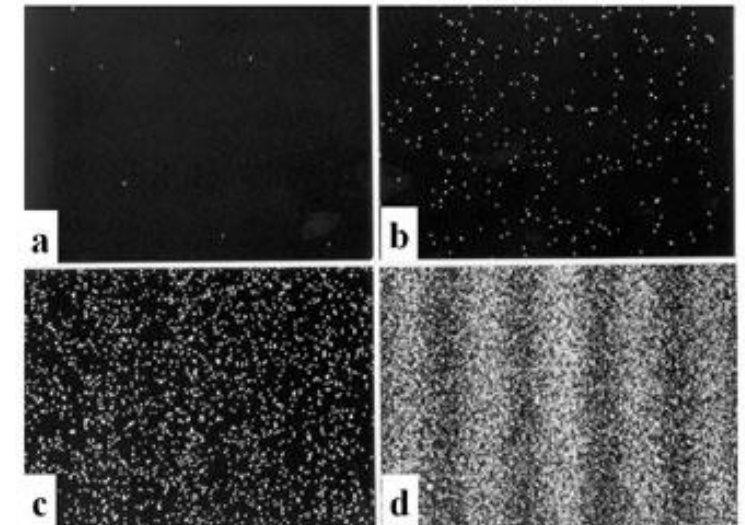
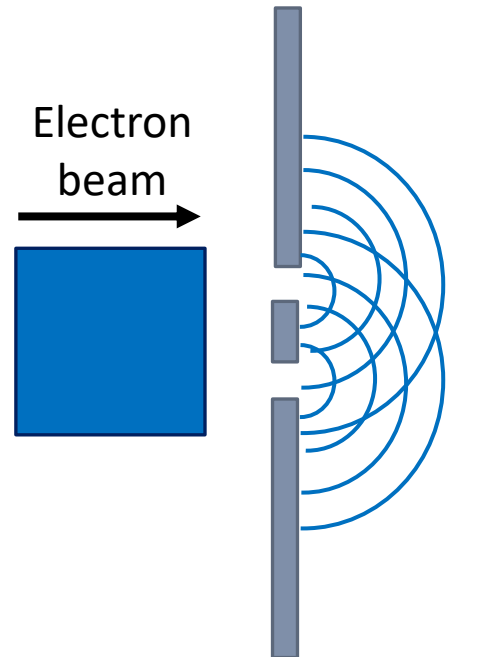
$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Wave nature of matter causes particle in box to have quantized energy

$$E = \frac{n^2 h^2}{8mL^2}$$

Quantum number:

$$n = (\text{number of nodes}) - 1$$



Bohr's Atomic Model

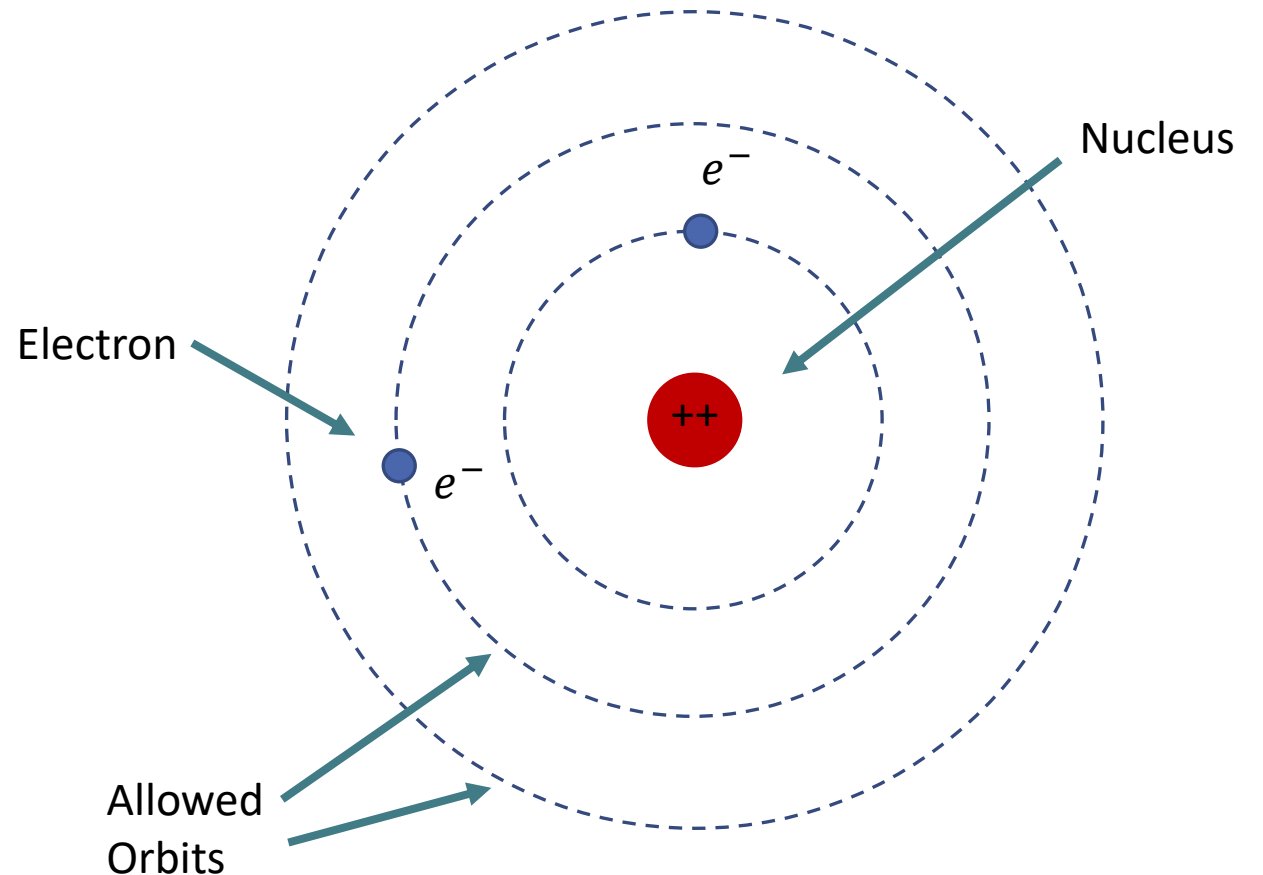
It was understood that atoms were made of a positively charged nucleus surrounded by negatively charged electrons

But why didn't the electrons fall into the nucleus? Why is matter stable?

Bohr solved this by introducing a new model of the atom

Electrons only allowed at specific orbits

The arrangement of orbits is called a stationary state



Bohr's Atomic Model

Each stationary state has a distinct, quantized energy

Energy determined with quantum number: n

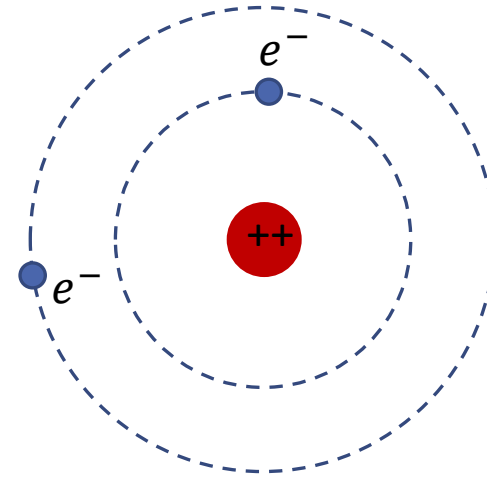
Lowest energy state ($n = 1$) called the ground state

All other higher energy states called excited states

Ground state is stable (persists indefinitely)

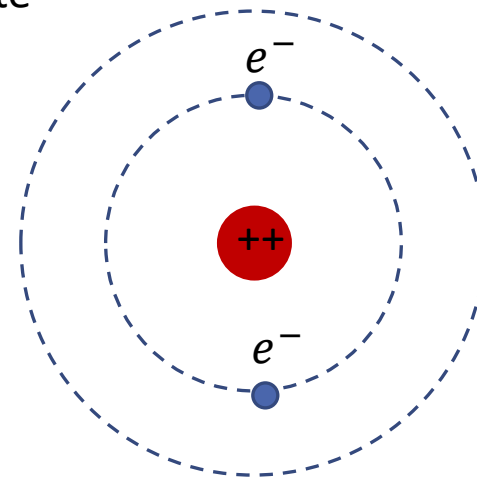
Excited State

$$E_2 > E_1$$



Ground State

$$E_1$$



Bohr's Atomic Model

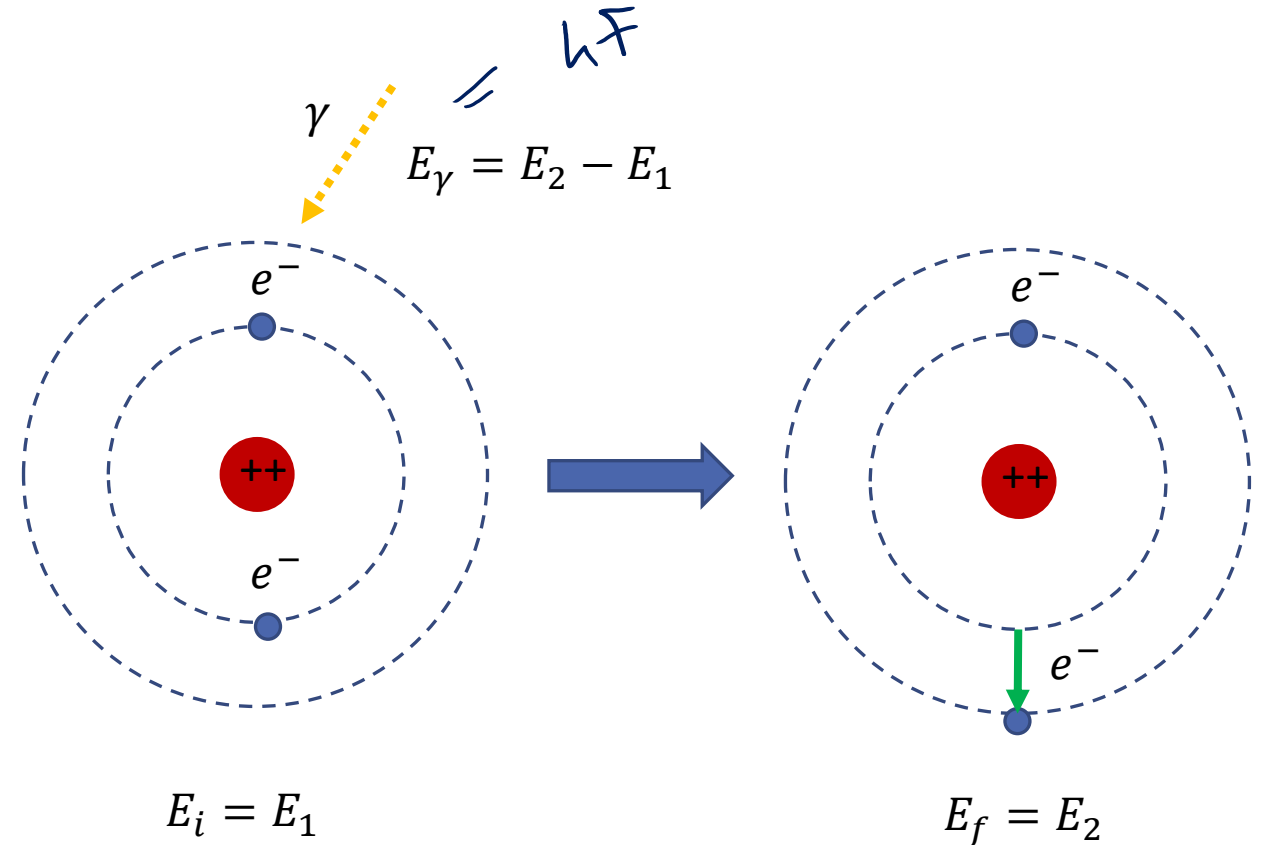
Atoms can transition between states by interacting with a photon

Absorbing a photon moves the atom to higher energy state (excites)

Emitting a photon moves the atom to a lower energy state (decays)

Photon **must** be exact energy of the difference between the two states

Ground state can't emit photon because there are no lower energy states allowed

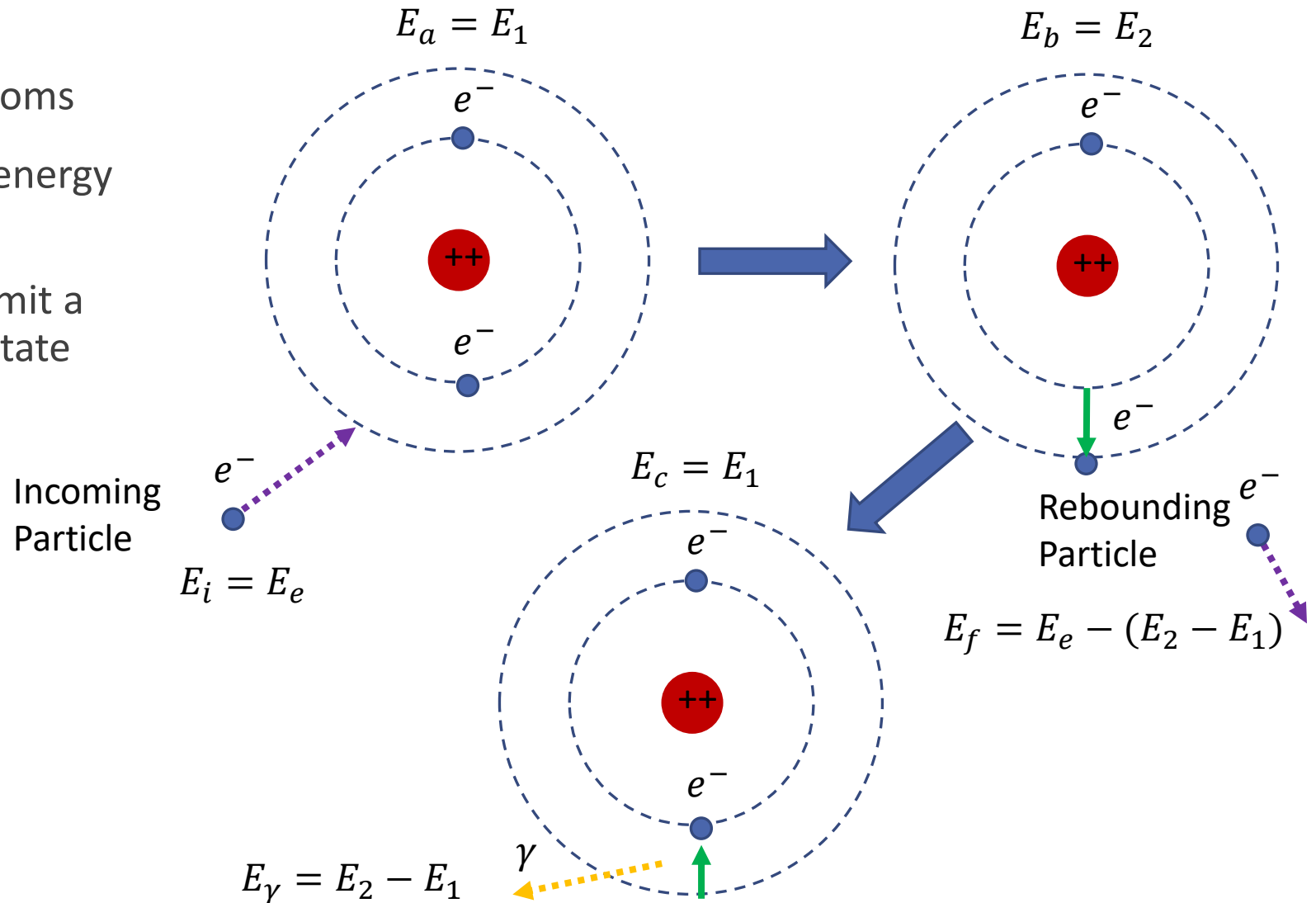


Bohr's Atomic Model

Other particles can also excite atoms

Some of the incoming particle's energy gets transferred to an electron

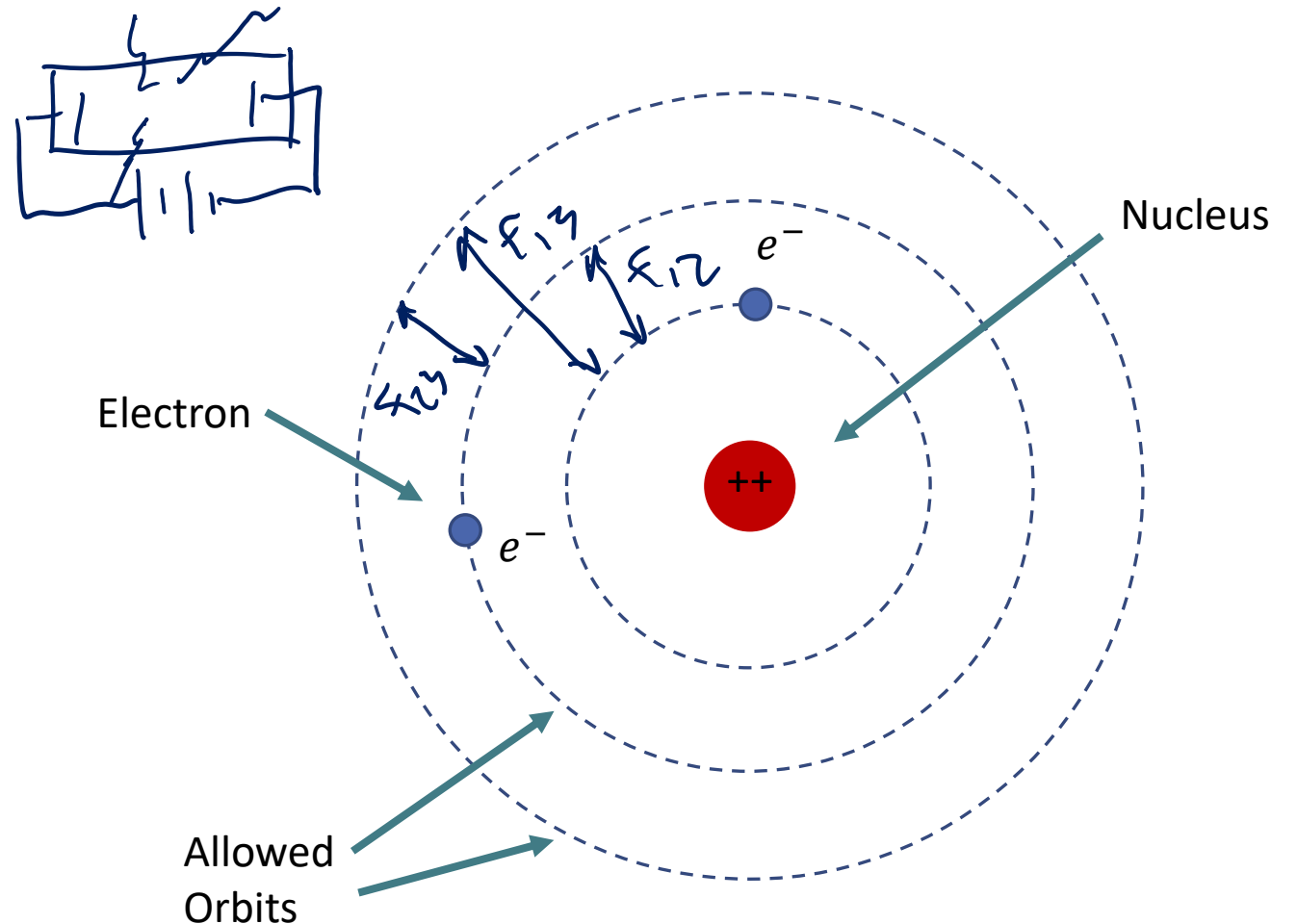
At a later time, this could then emit a photon to get back into ground state



Bohr's Atomic Model

Implications:

- Matter is stable since ground state is lowest energy so can't decay
- Atoms absorb and emit discrete spectra
$$f_{\text{photon}} = \frac{\Delta E_{\text{atom}}}{h}$$
- Emission spectra can be produced by collisions
Gas discharge tube = electrons from cathode strike gas atoms and excite them
- Absorption spectra don't include all emission wavelengths
Emission: $3 \rightarrow 2, 3 \rightarrow 1$
Absorption: $1 \rightarrow 2, 1 \rightarrow 3$
- Each element has a unique spectrum
Different elements have different number of protons = different energies



Energy Level Diagrams

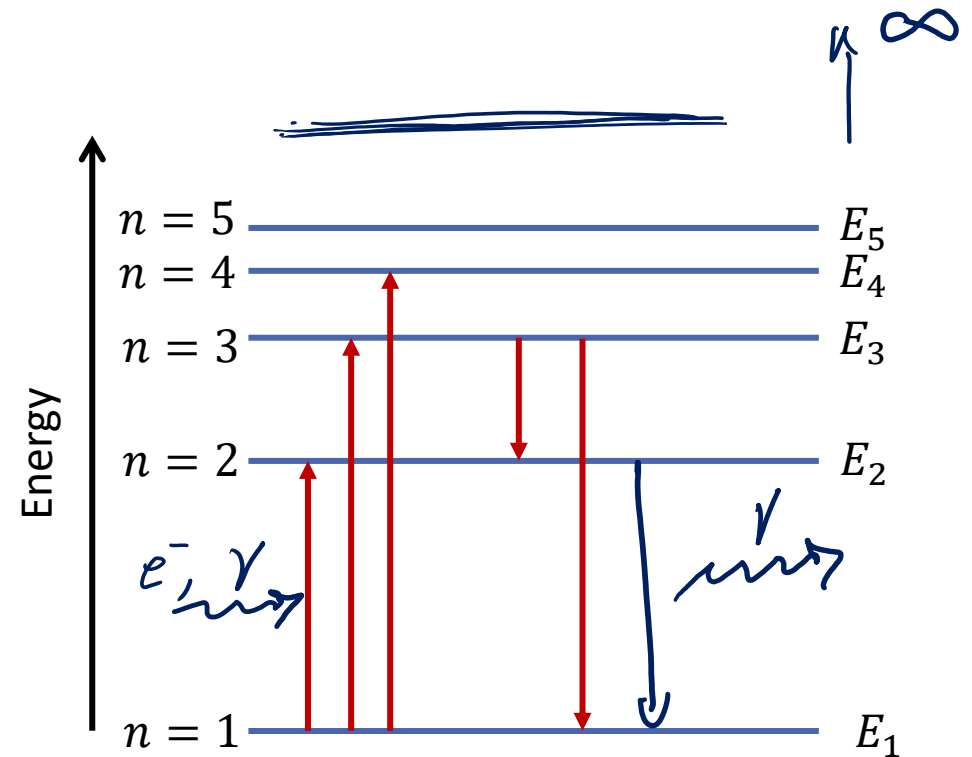
Transitions can be described using energy-level diagram

Each line corresponds to different energy states

Labeled by quantum number: n

Arrow indicate changes in energy state

- Collision of another particle
- Emission of photon
- Absorption of photon



Bohr Hydrogen

Let's apply the Bohr model to hydrogen

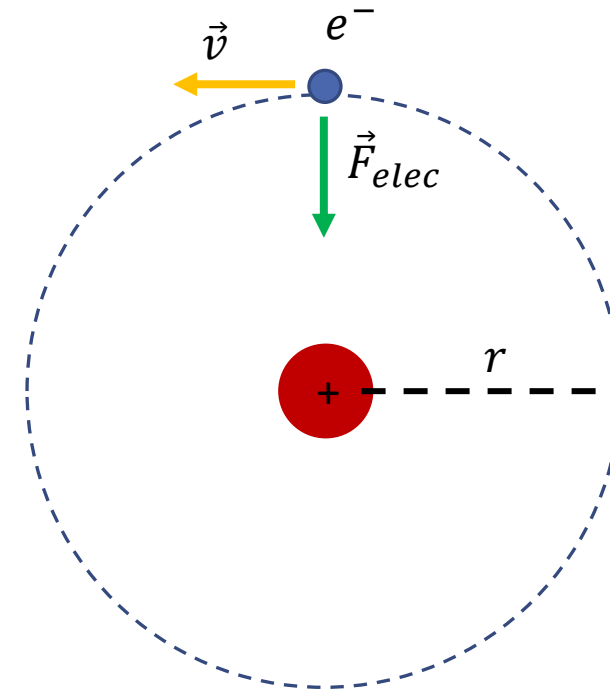
First what does it look like classically

$$E = K + U$$

$$K = \frac{1}{2} m v^2$$

$$U = \frac{q_e q_p}{4\pi\epsilon_0 r} = \frac{-e^2}{4\pi\epsilon_0 r}$$

$$E = \frac{1}{2} m v^2 - \frac{e^2}{4\pi\epsilon_0 r}$$



Bohr Hydrogen

Let's apply the Bohr model to hydrogen

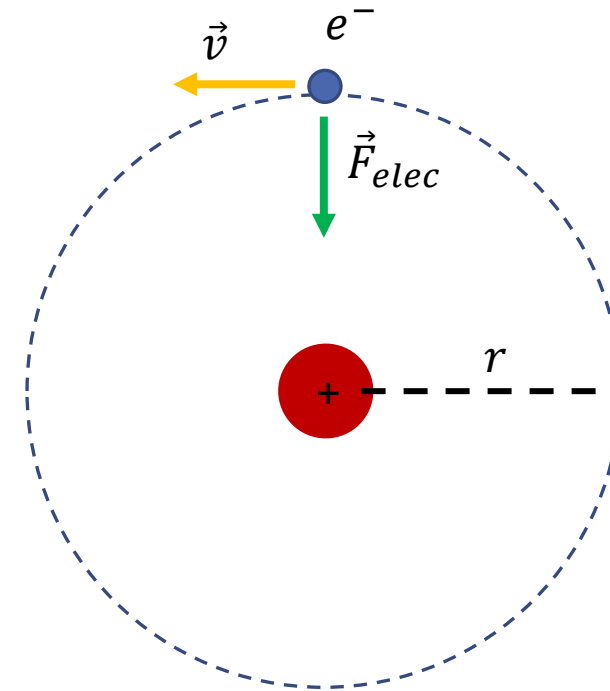
First what does it look like classically

$$\vec{F} = \frac{e^2}{4\pi\epsilon_0 r^2} (-\hat{r})$$

Circular Motion

$$a = \frac{v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$v^2 = \frac{e^2}{4\pi\epsilon_0 r}$$



Bohr Hydrogen

But we know that electrons are also a matter wave

Circumference of orbit must be integer multiple of wavelength

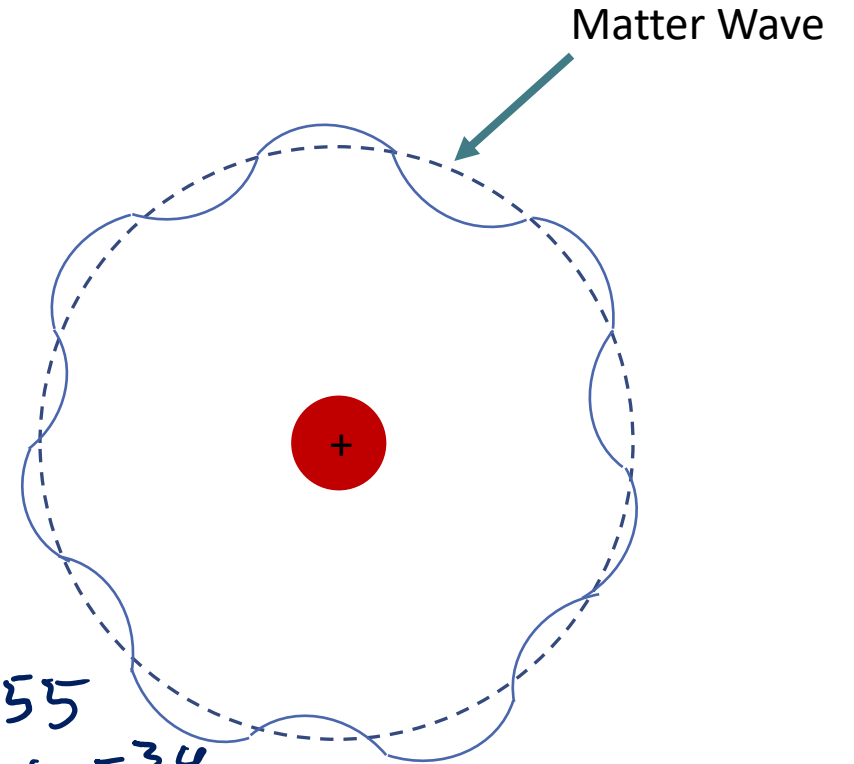
$$2\pi r = n\lambda \quad n = 1, 2, 3, \dots$$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$2\pi r = \frac{nh}{mv}$$

$$v = \frac{nh}{2\pi m r} = \frac{nh}{m r}$$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ J}\cdot\text{s}$$



Bohr Hydrogen

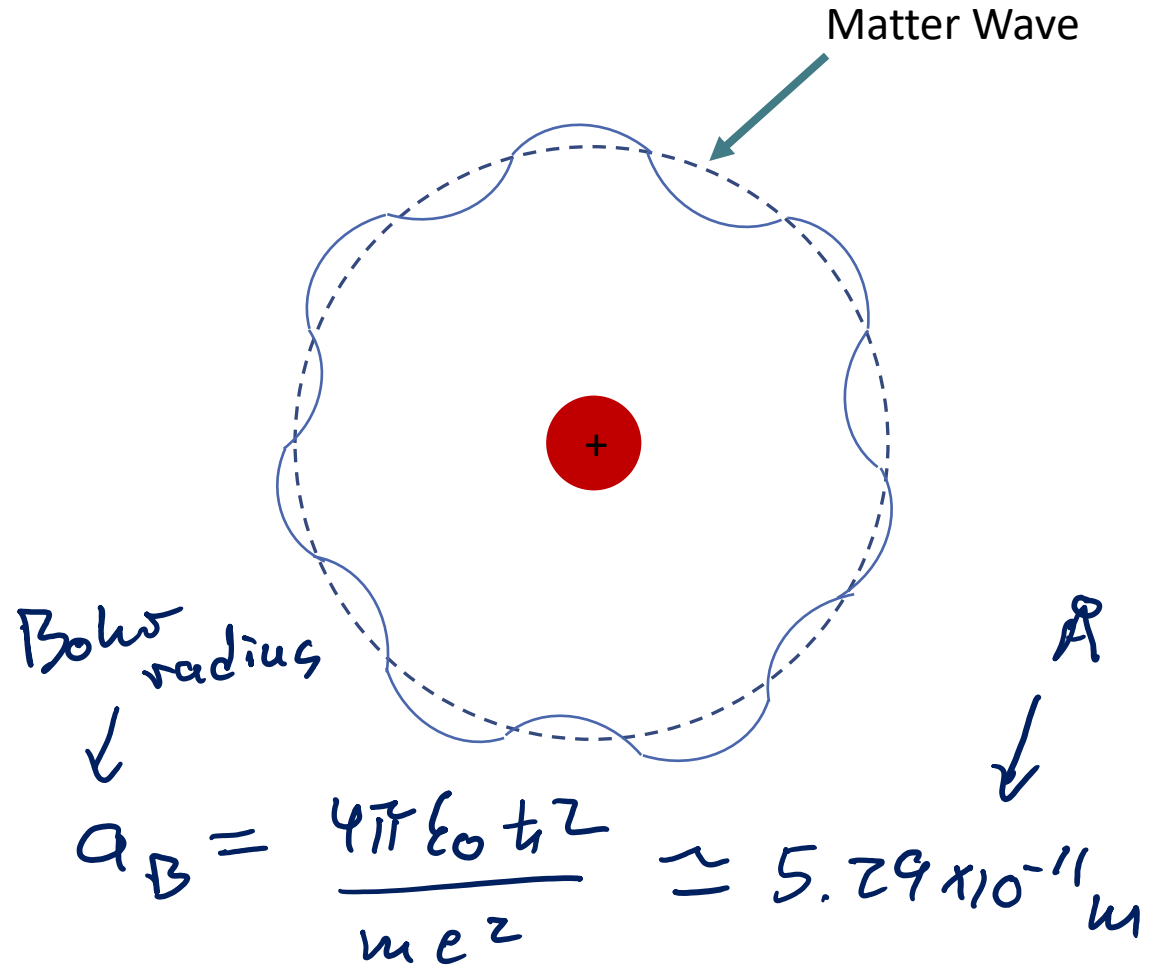
But we know that electrons are also a matter wave

Circumference of orbit must be integer multiple of wavelength

$$v^2 = \frac{e^2}{4\pi\epsilon_0 m r} = \frac{n^2 h^2}{m^2 r^2}$$

$$r = \frac{4\pi\epsilon_0 n^2 h^2}{m e^2}$$

$r = a_B n^2$ radius
 is discrete
 $n = 1, 2, 3 \dots$



Bohr Hydrogen

Radii of electron orbit in hydrogen is quantized

$$r_n = a_B n^2 \quad n = 1, 2, 3 \dots$$

Defined Bohr radius

$$a_B = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.0529 \text{ nm}$$

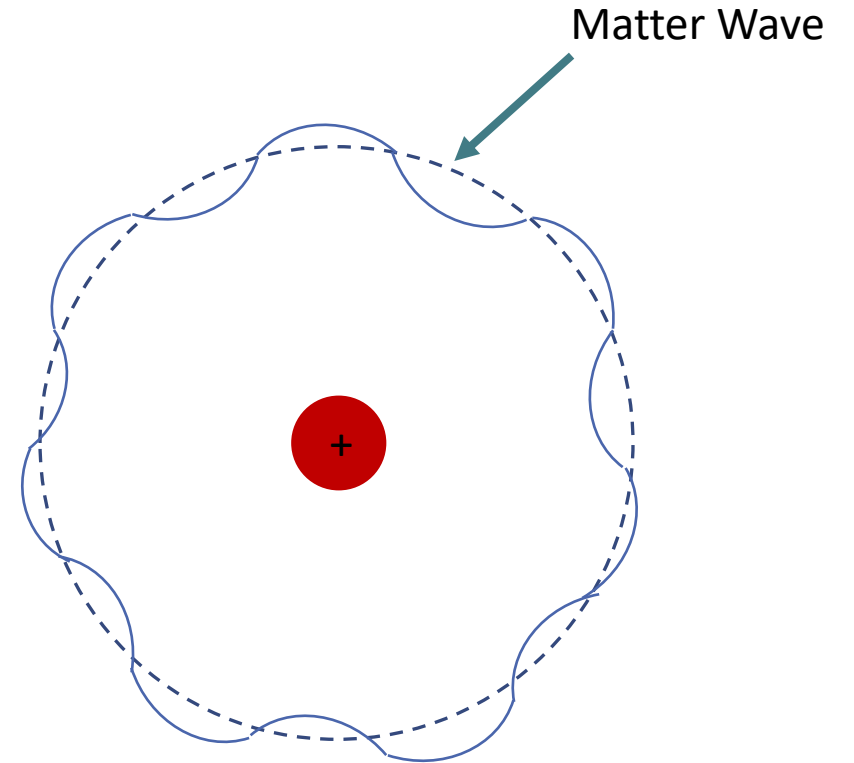
Examples of electron radius:

$$r_1 = 1a_B = 0.053 \text{ nm}$$

$$r_2 = 4a_B = 0.212 \text{ nm}$$

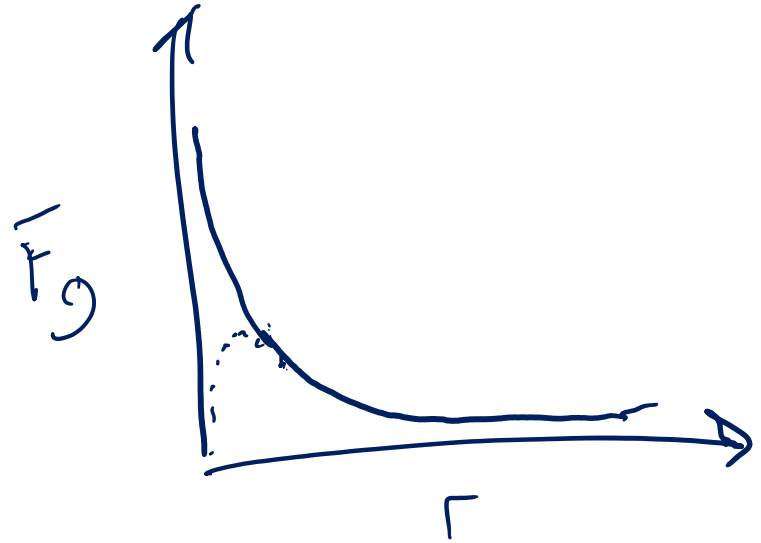
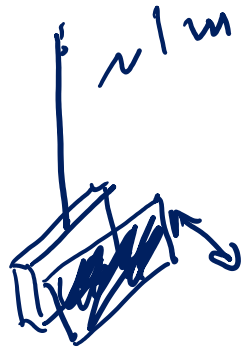
$$r_3 = 9a_B = 0.476 \text{ nm}$$

Hydrogen atoms at other radii **can not** exist



Homework Questions

$$F_g = \frac{G m_1 m_2}{r^2}$$



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