A symmetric *n* by *n* matrix *D*, where *n* is the number of vertices in the graph. The element $d_{x,y}$ is the length of a shortest path between *x* and *y*; if there is no such path $d_{x,y}$ = infinity. It can be derived from powers of *A*
$d_{x,y} = \min\{n \mid A^n[x,y] \neq 0\}.$

Problems in graph theory

Enumeration

There is a large literature on graphical enumeration: the problem of counting graphs meeting specified conditions. Some of this work is found in Harary and Palmer (1973).

Subgraphs, induced subgraphs, and minors

A common problem, called the subgraph isomorphism problem, is finding a fixed graph as a subgraph in a given graph. One reason to be interested in such a question is that many graph properties are *hereditary* for subgraphs, which means that a graph has the property if and only if all subgraphs have it too. Unfortunately, finding maximal subgraphs of a certain kind is often an NP-complete problem.

- Finding the largest complete graph is called the clique problem (NP-complete).

A similar problem is finding induced subgraphs in a given graph. Again, some important graph properties are hereditary with respect to induced subgraphs, which means that a graph has a property if and only if all induced subgraphs also have it. Finding maximal induced subgraphs of a certain kind is also often NP-complete. For example,

- Finding the largest edgeless induced subgraph, or independent set, called the independent set problem (NP-complete).

Still another such problem, the *minor containment problem*, is to find a fixed graph as a minor of a given graph. A minor or **subcontraction** of a graph is any graph obtained by taking a subgraph and contracting some (or no) edges. Many graph properties are hereditary for minors, which means that a graph has a property if and only if all minors have it too. A famous example:

- A graph is planar if it contains as a minor neither the complete bipartite graph $K_{3,3}$(See the Three-cottage problem) nor the complete graph K_5.

Another class of problems has to do with the extent to which various species and generalizations of graphs are determined by their *point-deleted subgraphs*, for example:

- The reconstruction conjecture.

Graph coloring

Many problems have to do with various ways of coloring graphs, for example:

- The four-color theorem
- The strong perfect graph theorem
- The Erdős–Faber–Lovász conjecture (unsolved)
- The total coloring conjecture (unsolved)
- The list coloring conjecture (unsolved)
- The Hadwiger conjecture (graph theory) (unsolved).