# Five-Dimensional Flow in Solid Fuel Rocket Engines 

by

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This thesis entitled:
Five-Dimensional Flow in Solid Fuel Rocket Engines
written by I. B. Scriptor
has been approved for the Department of Rocket Science

Ed Visor

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Date $\qquad$

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

Scriptor, I. B. (Ph.D., Rocket Science)
Five-Dimensional Flow in Solid Fuel Rocket Engines
Thesis directed by Prof. Ed Visor

Solid fuel rocket engines are one of the most reliable and efficient propulsion systems used to lift payloads into orbit, in terms of $\lambda=(\square+\diamond) \psi$. Used throughout the astrodynamics community, the theory of the flow within the motor chamber is in fact a black art which defies all attempts at analysis.

The present work (no exception to the statement above) contains a theoretical and numerical approach to the flow of the gases within the motor chamber. The shape of the chamber and original fuel configuration, and the patterns of combustion and flow/expulsion of gases, are modelled by a system of thirty fourth-degree differential equations.

$$
f_{i}^{(34)}(x, y, z, t)=\sum_{j=0}^{33} a_{i j} f_{i}^{(\mathrm{j})}(x, y, z, t)
$$

Acceptable numerical solutions would require one thousand pentium processors working day and night for $10^{11.2}$ years.

## Dedication

To all of the fluffy kitties.

## Acknowledgements

Here's where you acknowledge folks who helped.

## Contents

## Chapter

1 Introduction 1
1.1 Lists in thesis class . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2

2 Mathematical Formulation 6
2.1 Conditions for Catastrophic Combustion . . . . . . . . . . . . . . . . . . 9
2.2 More Boundary Conditions . . . . . . . . . . . . . . . . . . . . . . . . . 11
2.2.1 Just meaningless text to test lines per page . . . . . . . . . . . . 12
2.2.2 This is a subsection . . . . . . . . . . . . . . . . . . . . . . . . . 16
2.2.3 This is another subsection . . . . . . . . . . . . . . . . . . . . . . 16
2.3 The End . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 17

Bibliography

## Appendix

A Objective Symptoms 19

B Ode to Spot 21

## Figures

## Figure

1.1 A IATEX "figure", using $\backslash$ framebox . . . . . . . . . . . . . . . . . . . . . 2
1.2 A figure imported from a PostScript file . . . . . . . . . . . . . . . . . . 3
2.1 Cutting up a triangular pyramid . . . . . . . . . . . . . . . . . . . . . 8
2.2 A Sideways Figure . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10

## Tables

## Table

1.1 Example of a table with its own footnotes ..... 5
2.1 Yet another tabular table ..... 9
2.2 Table from a PostScript file ..... 11
2.3 A sideways table with \tabular ..... 13
2.4 A sideways table in a PostScript file ..... 14

## Chapter 1

## Introduction

This sample document illustrates how to use the thesis class, originally written by John P. Weiss and updated by Bruce Fast. The necessary file, thesis.cls, is on all the main computer systems of C.U.Boulder, and can be downloaded from the web site http://www.Colorado.EDU/ITS/docs/latex/ThesisClass/. Some requirements of the Graduate School are written into that file; page size, line spacing, appropriate placement of captions for tables and figures, etc. Other tasks of conforming to the requirements are left to other existing $\mathrm{EAT}_{\mathrm{E}} \mathrm{X}$ packages. For example, a common problem is to insert graphics - figures and tables - into the body of the thesis. For this one should use the epsfig package, which is part of the standard $\mathrm{T}_{\mathrm{E}} \mathrm{X}$ distribution. Likewise, the Grad School specs say that a large table may be displayed in landscape mode at reduced size, but its caption must also be in rotated position, in the same font and size as the normal text in the body of the thesis. To accomplish this, the user must invoke the rotating package, the use of which is illustrated in this document. See Table 2.4 and Fig. 2.2.

More illustrations of the use of \label\{\} and $\backslash$ ref $\}$ : see $\S 2.2 .1$ and $\S 2.2 .3 .2$. You might enjoy $\S 2$ more than $\S 1$. And undoubtedly you will like $\S 2.3$ better than either §2.1, Table 2.2, or equation (2.5).

Figure 1.2 shows something or other; the image is from a PostScript file which is imported into this document using the epsfig package (the command ./images/edebb1a94fd6a28a058bd9bdf976b886_208_448_737_1567.jpg)undefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefined

Figure 1.1: A IATEX "figure", using $\backslash$ framebox
appears near the very top of the $\mathrm{LATEX}_{\mathrm{E}}$ file) and the $\backslash \operatorname{epsfig}\{\ldots\}$ or $\backslash \operatorname{epsfbox}\{\ldots\}$ command.

Question: What are the issues in studying this subject?
A major goal in studying solid fuel rocket motors is to create a model of the dynamics of a motor chamber. This involves two major goals: the combustion zone and the acoustic zone. Figure 1.2 shows this. The combustion zone consists of the thin layer above the solid fuel where the gasification of the fuel takes places. The zone is very reactive and highly turbulent. The acoustic-vortical zone is the volume of gas above the combustion zone. Within this zone, the gas is non-reactive and contains acoustic waves and vorticity.

### 1.1 Lists in thesis class

In thesis class (for Colorado University), lists are defined so that nested lists will be numbered or marked appropriately. First, an itemized (non-enumerated) list prefaces each item with a bullet. Nested itemized list use asterisks, then dashes, then dots. These lists are typed between the \begin\{itemize\} and \end\{itemize\} commands. }

- This is "itemized" item A.
- This is "itemized" item B.
- This is "itemized" item C.
* This is "itemized" subitem A.


Model Rocket Motor

Fuel and Combustion Zone not studied


Figure 1.2: The combustion zone in a rocket motor chamber is thin relative to the radius of the chamber, etc. How did this image get into the thesis? I used the \epsfig or \epsfbox command, which is defined in the epsfig package. (The "sub-captions" are part of the PostScript image.)

- This is "itemized" subsubitem A.
- This is "itemized" subsubsubitem A.
- This is "itemized" subsubitem B.
* This is "itemized" subitem B.
- This is "itemized" item D.

Enumerated lists use the commands \begin\{enumerate\} and \end\{enumerate\}, } and nested enumerations appear like this.
(1) This is "enumerated" item A.
(2) This is "enumerated" item B.
(3) This is "enumerated" item C.
(a) This is "enumerated" subitem A.
(i) This is "enumerated" subsubitem A.
(i.a) This is "enumerated" subsubsubitem A.
(ii) This is "enumerated" subsubitem B.
(b) This is "enumerated" subitem B.
(4) This is "enumerated" item D.

The work presented here ${ }^{1}$ is an extension of Lao[6] and Lao et al.[7] The driving frequency is on the order of the inverse of the axial acoustic time scale, $t_{A}^{\prime}=L^{\prime} / C_{0}^{\prime}$, where $L^{\prime}$ is the length of the cylinder and $C_{0}^{\prime}$ is the reference speed of sound. ${ }^{2}$ Radial and azimuthal velocities ${ }^{5}$ are found to vanish exponentially fast in the downstream direction, as suggested by Table 1.1.

[^0]Table 1.1: Here is an example of a table with its own footnotes. Don't use the $\backslash$ footnote macro if you don't want the footnotes at the bottom of the page. Also, note that in a thesis the caption goes above a table, unlike figures.

| wave form | $S$ <br> $(\mathrm{kVA})$ | $P$ <br> $(\mathrm{~kW})$ | $Q^{*}$ <br> $(\mathrm{kVAr})$ | $D^{\dagger}$ <br> $(\mathrm{kVAd})$ |
| :--- | :---: | :---: | :---: | :---: |
| Fig. 1.1 | 25.87 | 25.83 | 1.3 | $\approx 0$ |
| Fig. 1.2a | 25.48 | 25.00 | -2.82 | 4.03 |
| Fig. 1.2b | 25.11 | 18.02 | -9.75 | 14.52 |
| Table 2 | 24.98 | 22.26 | 9.19 | 6.64 |
| Fig. 2.4 | 23.48 | 15.00 | 6.59 | 16.82 |
| Fig. 2.1 | 24.64 | 22.81 | -0.44 | 9.3 |
| Fig. 2.2 | 23.03 | 18.01 | 3.36 | 13.95 |

*kVAr means reactive power.
${ }^{\dagger} \mathrm{kVAd}$ means distortion power.

These results provide an analytical explanation of those found from computational analysis by Fabnis et al.[4] The non-axisymmetric flow near the endwall contains crosssectional velocity patterns that include flow across the cylinder axis. A viscous boundary layer adjacent to the sidewall and near the endwall is studied to find the transition between the transient core flow and the no-slip condition on the sidewall. It is found, as in Lao et al.[7], that the azimuthal component of the vorticity is proportional to the inverse of the Mach number. In addition, the axial component of the vorticity driven by the non-axisymmetric boundary condition at the endwall is also found to be proportional to the the inverse of the Mach number.

## Chapter 2

## Mathematical Formulation

The objective of this fake thesis document is to demonstrate a multitude of $\mathrm{EAT}_{\mathrm{E}} \mathrm{X}$ features as well as features specific to the thesis class. We start by giving one short formula, and one big hairy multi-line formula (one of the non-dimensional Navier-Stokes equations):

$$
\begin{equation*}
A=\pi r^{2} \tag{2.1}
\end{equation*}
$$

$$
\rho\left[\frac{D V_{r}}{D t}-M \epsilon^{2} \frac{V_{\theta}^{2}}{r}\right]=-\frac{\delta^{2}}{\gamma M} \frac{\partial P}{\partial r}+\frac{M \delta^{2}}{R e}\left\{2 \frac{\partial}{\partial r}\left[\mu\left(\frac{\partial V_{r}}{\partial r}-\frac{1}{3} \nabla \cdot \overline{\mathbf{V}}\right)\right]\right.
$$

$$
+\frac{1}{r} \frac{\partial}{\partial \theta}\left[\mu\left(\frac{1}{r} \frac{\partial V_{r}}{\partial \theta}+\epsilon \frac{\partial V_{\theta}}{\partial r}-\epsilon \frac{V_{\theta}}{r}\right)\right]
$$

$$
+\frac{\partial}{\partial z}\left[\mu\left(\frac{1}{\delta^{2}} \frac{\partial V_{r}}{\partial z}+\frac{\partial V_{z}}{\partial r}\right)\right]
$$

$$
\begin{equation*}
\left.+2 \frac{\mu}{r}\left[\frac{\partial V_{r}}{\partial r}-\frac{\epsilon}{r} \frac{\partial V_{\theta}}{\partial \theta}-\frac{V_{r}}{r}\right]\right\} \tag{2.2}
\end{equation*}
$$

The latter equation is non-dimensionalized using the following definitions:

$$
r=\frac{r^{\prime}}{R^{\prime}}, \quad z=\frac{z^{\prime}}{L^{\prime}}, \quad t=\frac{t^{\prime}}{t_{a}^{\prime}}, \quad \kappa=\frac{\kappa^{\prime}}{\kappa_{0}^{\prime}}, \quad \mu=\frac{\mu^{\prime}}{\mu_{0}^{\prime}}, \quad C_{V}=\frac{C_{V}^{\prime}}{C_{V 0}^{\prime}},
$$

where $P_{0}^{\prime}$ is the initial static pressure in the cylinder, and $\rho_{0}^{\prime}$ and $T_{0}^{\prime}$ are the density and temperature of the fluid being injected from the sidewall. The aspect ratio is given by $\delta=\frac{L^{\prime}}{R^{\prime}}$, where $\delta \gg 1$. The induced characteristic axial velocity and the characteristic
endwall velocity disturbance $V_{z 0}^{\prime}$ is defined with respect to the injection reference sidewall velocity, $V_{r 0}^{\prime}$ by overall mass conservation, $\frac{V_{00}^{\prime}}{V_{r 0}^{\prime}}=\delta$. The size of the initially unknown reference azimuthal velocity $V_{\theta 0}^{\prime}$ is related to $V_{r 0}^{\prime}$ by $\frac{V_{\theta 0}^{\prime}}{V_{r 0}^{\prime}}=\epsilon$. Later, it is shown that $\epsilon=1$.

The time is non-dimensionalized using the axial acoustic time scale, $t_{a}^{\prime}=\frac{L^{\prime}}{C_{0}^{\prime}}$, where $C_{0}^{\prime}=\left(\gamma \mathcal{R}^{\prime} T_{0}^{\prime}\right)^{\frac{1}{2}}$ is the speed of sound ${ }^{1}, \mathcal{R}^{\prime}$ is the gas constant, and $\gamma$ is the ratio of specific heats. Also the Reynolds number, Wrenchl number, and Mock number are defined as

$$
R e=\frac{\rho^{\prime} V_{z_{0}}^{\prime} L^{\prime}}{\mu_{0}^{\prime}}, \quad W r=\frac{\mu_{0}^{\prime} C_{p_{0}}^{\prime}}{\kappa_{0}^{\prime}}, \quad M=\binom{V_{z_{0}}^{\prime}}{C_{0}^{\prime}} \cdot\left(\begin{array}{cc}
8 a & z_{0}-\rho \\
2 & z_{0}-\mu
\end{array}\right)\binom{W r}{p-7}
$$

where $R e \ll 1, M \gg 1$, and $W r=O(1)$.
Here is an example of using the macros \singlespacing and \doublespacing:
This paragraph was preceded by the command \singlespacing. The Mock number is chosen as a small parameter to model the small magnitude found in a typical rocket motor chamber, as opposed to the rocket nozzle where larger values are possible ${ }^{2}$. The aspect ratio, $\delta$, is taken to be a large parameter, because many chambers have aspect ratios between 15 and 50 . Now the command "\doublespacing":

The Grad School specifications allow for single spacing everywhere in the body of the thesis, except in quotations of four or more lines (Dole and Abramson[3]), table/figure captions, chapter headings, footnotes and entries in the Contents and Bibliography (with double spaces between entries).

And now, here is an example of using the macros \begin\{singlespace\} and } \end\{singlespace\}; another way to get single-spacing. }

Two cases are studied in the present work which differ only in the boundary conditions. Each different boundary condition model a different source of instability. The boundary of the first case consists of a steady, axisymmetric sidewall radial velocity boundary and a time-dependent, non-axisymmetric endwall axial velocity boundary. The second case is studied with a fixed impermeable axial velocity along the endwall and a combination axisymmetric steady and non-axisymmetric unsteady radial velocity along the sidewall.

[^1]

Figure 2.1: A triangular pyramid may be cut up as shown, to yield one top pyramid (with one-eighth the volume of the full pyramid), three bottom corner pyramids (which, when joined, are congruent to the top pyramid), three prisms along the bottom edges (the area of whose bottom faces total $B / 2$ ) and the large central prism (volume $=$ $(B / 4)(h / 2)=B h / 8)$. The image, from PostScript file "pyr.ps", was read in using the \epsffile command, from the epsfig package.

Table 2.1: This is a table constructed with $\mathrm{ATEX}_{\mathrm{E}}$ commands in the tabular environment.

| n | $n^{2}$ | $n^{3}$ | $n^{4}$ | $n^{7}$ | $n^{13}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 4 | 8 | 16 | 128 | 8192 |
| 3 | 9 | 27 | 81 | 2187 | 1594323 |
| 4 | 16 | 64 | 256 | 16384 | 67108864 |
| 5 | 25 | 125 | 625 | 78125 | 1220703125 |
| 6 | 36 | 216 | 1296 | 279936 | 13060694016 |
| 7 | 49 | 343 | 2401 | 823543 | 96889010407 |

Now on to other items. The following table is created using $\mathrm{LATEX}_{\mathrm{E}} \mathrm{macros}$, i.e., \begin\{tabular\}... \end\{tabular\}. }

However, sometimes you want to use a table produced by some other software, such as Excel. If the table is saved to a PostScript file, then it can be displayed using the \epsfig macro inside a table environment:

### 2.1 Conditions for Catastrophic Combustion

Initially, a steady flow is generated by the sidewall injection, $V_{r}=-V_{r w s}(z)$. The subscript $s r w$ is used to mean that there is a steady radial wall velocity. The sign is negative due to the injection toward centerline. At $t=0^{+}$, the endwall begins oscillating with the non-dimensionalized sinusoidal axial velocity, $V_{z}=\widetilde{F}_{r w}(r, \theta, t)$, for $\omega=O(1)$. Figure 2.2 conforms to these thesis specs: "Figures are placed immediately after their first mentions ... Figure captions appear below figures and are typed in the same style and size as the text. Captions should fit within the standard margins and are not reduced if the figures are reduced . . Figures may be printed broadside, with the top toward the left margin; the caption then appears beneath the figure and is typed from bottom to top of the page within the standard margins..."

Some of the boundary conditions are:


Figure 2.2: If a figure must appear sideways in a thesis (for greater detail) then the caption must still appear below the figure - i.e., along the right edge of the page. By including the rotating package at the top of the $\mathrm{L}_{\mathrm{A}} \mathrm{T}_{\mathrm{E}} \mathrm{X}$ file, you can get this rotated figure by using the \begin\{sidewaysfigure\} ... \end\{sidewaysfigure\} environment. }

Table 2.2: This table wasn't constructed with LATEX $_{\mathrm{E}} \mathrm{X}$ commands, but resides in a PostScript file (tableD.ps) created by some other software.

| $\mathbf{n}$ | $\mathbf{n}^{\mathbf{2}}$ | $\mathbf{n}^{\mathbf{3}}$ | $\mathbf{n}^{\mathbf{4}}$ | $\mathbf{n}^{\mathbf{7}}$ | $\mathbf{n}^{\mathbf{1 3}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 4 | 8 | 16 | 128 | 8192 |
| 3 | 9 | 27 | 81 | 2187 | 1594323 |
| 4 | 16 | 64 | 256 | 16384 | 67108864 |
| 5 | 25 | 125 | 625 | 78125 | 1220703125 |
| 6 | 36 | 216 | 1296 | 279936 | 13060694016 |
| 7 | 49 | 343 | 2401 | 823543 | 96889010407 |

$$
\begin{array}{ll}
z=0 ; & V_{z}= \begin{cases}0, & t \leq 0 \\
\widetilde{F}_{z w}(r, \theta, t), & t>0\end{cases} \\
z=0 ; & V_{\theta}=V_{r}=0 \\
r=0 ; & P, \rho, T, V_{r}, V_{\theta}, V_{z} \text { finite }, \\
r=1 ; & V_{r}=F_{r w s}(z) \\
r=1 ; & V_{z}=V_{\theta}=0 \tag{2.7}
\end{array}
$$

and solutions must be periodic in $\theta$.

### 2.2 More Boundary Conditions

Initially, a steady flow is generated by the sidewall injection, $V_{r}=-V_{r w s}(z)$. The sign is negative due to the injection toward centerline ${ }^{3}$. At $t=0^{+}$, the endwall begins oscillating with the non-dimensionalized sinusoidal axial velocity, $V_{z}=\widetilde{F}_{r w}(r, \theta, t)$, for $\omega=O(1)$. The frequency condition chosen represents the first few axial acoustic modes observed in high aspect ratio chambers ${ }^{4}$.

The full boundary conditions include:

[^2]\[

$$
\begin{array}{ll}
z=0 ; & V_{\theta}=V_{r}=0 \\
r=1 ; & V_{r}= \begin{cases}F_{r w s}(z), & t<0 \\
F_{r w s}(z)+\widetilde{F}_{r w}(z, \theta, t) & t \geq 0\end{cases} \\
r=1 ; & V_{z}=V_{\theta}=0 \tag{2.10}
\end{array}
$$
\]

and solutions must be periodic in $\theta$.
If you don't believe this stuff, check out Mulick[9] and Baylor[1].
The following two tables, and their respective captions, are turned sideways. They use the sidewaystable environment defined in the rotating package. The first uses the IATEX tabular environment, and would be used when the normal $\mathrm{IAT}_{\mathrm{E}} \mathrm{X}$ table is a bit too wide to fit the width of the page, but fits the page when rotated. The second table is actually from a PostScript file, perhaps produced by some other software. It is read in using the \epsfig command, and is needed when the table is very large and must be shrunk to fit the page. Inside the \epsfig command one can specify width=8.75in, which will scale the (rotated) PostScript image to nearly fill the maximum amount of vertical space on a thesis page.

Tables are placed immediately after their first mention in the text ... Tables that will not fit within the required margins may be typed in smaller type or may be reduced; they also may be printed broadside with the top toward the left margin ... Table titles are typed above the tables in the same style and type size as the text. Table titles should fit within the standard margins and are not reduced if the table is reduced. Table footnotes are typed immediately beneath the table and have no relation to text footnotes.

### 2.2.1 Just meaningless text to test lines per page

According to the Grad School specs. there should be $24-27$ lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be
Table 2.3: This sideways table is constructed using the \tabular environment. This would only be necessary for tables so wide that they don't fit the normal width, but not so wide that they would also exceed 8.75 ", the usable height of a thesis page, using the usual $\mathrm{EA}_{\mathrm{E}} \mathrm{X}$ font in the table. Notice that this table uses the same font style and table (nearest the left edge of the page) as it should in a C.U. thesis.

| n | $n^{2}$ | $n^{3}$ | $n^{4}$ | $n^{7}$ | $n^{13}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 4 | 8 | 16 | 128 | 8192 |
| 3 | 9 | 27 | 81 | 2187 | 1594323 |
| 4 | 16 | 64 | 256 | 16384 | 67108864 |
| 5 | 25 | 125 | 625 | 78125 | 1220703125 |
| 6 | 36 | 216 | 1296 | 279936 | 13060694016 |
| 7 | 49 | 343 | 2401 | 823543 | 96889010407 |
| 100 | 10000 | 1000000 | 100000000 | 100000000000000 | 100000000000000000000000000 |
| 1000 | 1002001 | 1003003001 | 1004006004001 | 1007021035035021007001 | 1013078286716288717717287715286078013001 |

Table 2.4: This table is actually from a PostScript file. If it is just too tiny to read in the normal orientation, where the width is limited to dewaysatawidth(verticallength)ofupto8.75".Thecontentsofthetableshowthatithasbeenreducedinsize;however,thecaptionappearsinthecorrectplaceabovethetable(leftedgeofpage)inthesamefontstyle/sizeasinthebodyofthethesis.Thecaptionalsoappearsinthelistoftablesatthefrontofthethesis.Thisconstructusesthesidewaystableenvironmentandthe$\backslashepsfig$command,whicharedefinedintherotatingandepsfigpackages,respectively.Thesepackagesarereadin(with\usepackage)atthetopofthe$\mathrm{IAT}_{\mathrm{E}}\mathrm{X}$file.undefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefinedundefined

| $\mathrm{n}=$ | 2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{n}^{2}$ | 4 | 9 | 25 | 49 | 121 | 169 | 289 | 361 | 529 | 841 |
| $\mathrm{n}^{3}$ | 8 | 27 | 125 | 343 | 1331 | 2197 | 4913 | 6859 | 12167 | 24389 |
| $\mathrm{n}^{4}$ | 16 | 81 | 625 | 2401 | 14641 | 28561 | 83521 | 130321 | 279841 | 707281 |
| $\mathrm{n}^{5}$ | 32 | 243 | 3125 | 16807 | 161051 | 371293 | 1419857 | 2476099 | 6436343 | 20511149 |
| $\mathrm{n}^{6}$ | 64 | 729 | 15625 | 117649 | 1771561 | 4826809 | 24137569 | 47045881 | 148035889 | 594823321 |
| $\mathrm{n}^{7}$ | 128 | 2187 | 78125 | 823543 | 19487171 | 62748517 | 410338673 | 893871739 | 3404825447 | 17249876309 |
| $\mathrm{n}^{8}$ | 256 | 6561 | 390625 | 5764801 | 214358881 | 815730721 | 6975757441 | 16983563041 | 78310985281 | 500246412961 |
| $\mathrm{n}^{9}$ | 512 | 19683 | 1953125 | 40353607 | 2357947691 | 10604499373 | 118587876497 | 322687697779 | 1801152661463 | 14507145975869 |
| $\mathrm{n}^{10}$ | 1024 | 59049 | 9765625 | 282475249 | 25937424601 | 137858491849 | 2015993900449 | 6131066257801 | 41426511213649 | 420707233300201 |
| $\mathrm{n}^{11}$ | 2048 | 177147 | 48828125 | 1977326743 | 285311670611 | 1792160394037 | 34271896307633 | 116490258898219 | 952809757913927 | 12200509765705829 |

24-27 lines of print per page of a thesis. This should be true whether the font size is 10 , 11, or 12. Count them up; does this document conform? According to the Grad School specs. there should be $24-27$ lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be 24-27 lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be 24-27 lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12. Count them up; does this document conform? According to the Grad School specs. there should be $24-27$ lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be 24-27 lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be 24-27 lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12. Count them up; does this document conform? According to the Grad School specs. there should be $24-27$ lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be 24-27 lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be 24-27 lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12. Count them up; does this document conform? According to the Grad School specs. there should be $24-27$ lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be 24-27 lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does
this document conform? According to the Grad School specs. there should be $24-27$ lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12. Count them up; does this document conform? According to the Grad School specs. there should be $24-27$ lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be $24-27$ lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be 24-27 lines of print per page of a thesis. This should be true whether the font size is 10,11, or 12. Count them up; does this document conform? According to the Grad School specs. there should be $24-27$ lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be $24-27$ lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12 . Count them up; does this document conform? According to the Grad School specs. there should be 24-27 lines of print per page of a thesis. This should be true whether the font size is 10,11 , or 12. Count them up; does this document conform?

What is it? This is a labelled paragraph. The heading of the paragraph is emphasized. This is a labelled paragraph. The heading of the paragraph is emphasized.

### 2.2.2 This is a subsection

This is a subsection. Filler filler filler filler filler filler filler filler. Filler filler filler filler filler filler filler filler.

### 2.2.3 This is another subsection

This is another subsection. Filler filler filler filler filler filler filler filler. Filler filler filler filler filler filler filler filler.

This is paragraph number 2. It used a $\backslash$ paragraph\{\} header, which are always inlined (with extra space) and boldfaced.

This is the third paragraph of the subsection. Filler filler filler filler filler filler filler filler. Filler filler filler filler filler filler filler filler.

### 2.2.3.1 This is a subsubsection (1)

This is the first paragraph of the subsubsection. Whether it is numbered or inlined depends on the option selected at the beginning of the thesis.

By default, a \subsubsection heading is numbered and set off on a separate line, left-justified.

However. Using the inlineh4 option, subsubsection headers are inlined. And using the nonumh 4 option suppresses numbering of the subsubsections. Together they make subsubsection headings just the same as paragraph headings.

### 2.2.3.2 This is another subsubsection (2)

Once again, whether its heading is numbered and/or inlined depends on the class options chosen at the start.

There is no "subsubsubsection" entity, and "subparagraph" gets no special treatment in thesis class.

### 2.3 The End

Finally, this is the end. The bibliography starts on the next page.

## Bibliography

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## Appendix A

## Objective Symptoms

Appendices follow the same page-numbering rules as regular chapters. The first page of a multi-page appendix is not numbered.

Are they slow learners or is it a REAL problem? These are classic findings in the hopelessly computer challenged.
(1) Can't copy from hard drive to disk.
(2) Can't eject disks.
(3) The word "disk" has thousands of meanings to them. None are correct.
(4) Saving a document in any form is a concept totally unexplainable to them.
(5) Desktop covered with Untitled Folders - look again, untitled folders are everywhere.
(6) "Lost" documents found often in the Apple Menu.
(7) Trash always full. Claim they don't know how to place things in trash.
(8) Mysterious things happen to their documents or computer when they are not present. AKA "computer victims".
(9) Highlighting $=$ deleting. Dragging $=$ Oblivion.
(10) Selecting, double-clicking a problem? They will always say their mouse is broken.
(11) Their double- click mechanics wants you to send them to a neurologist.
(12) Computer always on due to fear of having to restart it.
(13) Have never read their QuickMail - will say "I prefer a phone call".
(14) Have magical beliefs about what computers do.
(15) Describes some flaky way computers could REALLY help them, but is not yet available.
(16) Constantly saying they need more "memory".
(17) Requests gizmos and gadgets, i.e., "mouse leash" or "disk cozy".
(18) Avoids eye contact when talking about computers.

## Appendix B

## Ode to Spot

(Data, Stardate 1403827) (A one-page chapter - page must be numbered!) Throughout the ages, from Keats to Giorchamo, poets have composed "odes" to individuals who have had a profound effect upon their lives. In keeping with that tradition I have written my next poem ... in honor of my cat. I call it... Ode... to Spot. (Shot of Geordi and Worf in audience, looking mystified at each other.)

Felus cattus, is your taxonomic nomenclature an endothermic quadruped, carnivorous by nature?
Your visual, olfactory, and auditory senses
contribute to your hunting skills, and natural defenses.
I find myself intrigued by your sub-vocal oscillations, a singular development of cat communications that obviates your basic hedonistic predilection for a rhythmic stroking of your fur to demonstrate affection. A tail is quite essential for your acrobatic talents; you would not be so agile if you lacked its counterbalance. And when not being utilized to aid in locomotion, It often serves to illustrate the state of your emotion.
(Commander Riker begins to applaud, until a glance from Counselor Troi brings him to a halt.) Commander Riker, you have anticipated my denouement. However, the sentiment is appreciated. I will continue.

O Spot, the complex levels of behavior you display connote a fairly well-developed cognitive array.
And though you are not sentient, Spot, and do not comprehend I nonetheless consider you a true and valued friend.


[^0]:    ${ }^{1}$ Footnotes are handled neatly by ${ }^{\mathrm{A}} \mathrm{T}_{\mathrm{E}} \mathrm{X}$.
    ${ }^{2}$ Remember the traditional method of calculating the distance of lightning? See the flash, count seconds until you hear the thunder, divide by five, that's the number of miles. That assumes $C_{0}=\frac{1 m i}{5 s}$.
    ${ }^{5}$ gratuitous footnote

[^1]:    ${ }^{1}$ In air at $1 \mathrm{~atm} ., \frac{1 m i}{5 \mathrm{~s}}$.
    ${ }^{2}$ Not just possible, desirable!

[^2]:    ${ }^{3}$ This convention was suggested by Goddard and Smythe.
    ${ }^{4}$ Toy rockets, the kind you used to shoot off with your dad in the park, typically have only two significant modes.

