

which is, nevertheless, not a strong P-point. These results answer the questions of M. Canjar and C. Laflamme.

## 1. PRELIMINARIES

We use standard notation,  $\omega^\omega$  denotes all functions from  $\omega$  to  $\omega$ ,  $[\omega]^\omega$  denotes all infinite subsets of  $\omega$  while  $[X]^{<\omega}$  denotes all finite subsets of  $X$ . We write  $A \subseteq^* B$  if  $|A \setminus B| < \omega$  and  $f \leq^* g$  if  $|\{n : f(n) > g(n)\}| < \omega$ . The cardinal number  $\mathfrak{b}$  denotes the least cardinality of an unbounded subset of  $(\omega^\omega, \leq^*)$  and  $\mathfrak{d}$  denotes the least cardinality of a dominating (cofinal) subset of  $(\omega^\omega, \leq^*)$ .  $\chi(\mathcal{U})$  denotes the *character* of  $\mathcal{U}$ , i.e. the least cardinality of a basis of the (ultra)filter  $\mathcal{U}$ .

**1.1. Definition** ([10]). A nonprincipal ultrafilter  $\mathcal{U}$  is a *P-point* if for any sequence  $\langle X_n : n < \omega \rangle \subseteq \mathcal{U}$  there is an  $X \in \mathcal{U}$  such that  $(\forall n < \omega)(X \subseteq^* X_n)$ .

**1.2. Definition** ([9]). An ultrafilter  $\mathcal{U}$  is *rapid* if the family  $\{e_X : X \in \mathcal{U}\}$  of increasing enumerations of sets in  $\mathcal{U}$  is a dominating family of functions in  $(\omega^\omega, \leq^*)$ .

**1.3. Definition.** Let  $\mathcal{U}, \mathcal{V}$  be ultrafilters on  $\omega$ .

(i) (Rudin-Keisler ordering, [4])  $\mathcal{U} \leq_{RK} \mathcal{V}$  if there is a function  $f : \omega \rightarrow \omega$  such that  $\mathcal{U} = f_*(\mathcal{V}) = \{A \subseteq \omega : f^{-1}[A] \in \mathcal{V}\}$ . In this situation we also say that  $\mathcal{U}$  is an RK-predecessor of  $\mathcal{V}$ .

(ii) (Rudin-Blass ordering, [6])  $\mathcal{U} \leq_{RB} \mathcal{V}$  if  $\mathcal{U} \leq_{RK} \mathcal{V}$  and the function witnessing this can be chosen to be finite-to-one. As above we say that  $\mathcal{U}$  is an RB-predecessor of  $\mathcal{V}$ .

**1.4. Definition** ([8]). *Mathias forcing* is the partial order where conditions are pairs  $(a, X)$  with  $a \in [\omega]^{<\omega}$  and  $X \in [\omega]^\omega$  ordered as  $(a, X) \leq (b, Y)$  if  $b \sqsubseteq a$ ,  $X \subseteq Y$  and  $a \setminus b \subseteq Y$ . Given an ultrafilter  $\mathcal{U}$ , *relativized Mathias forcing*  $\mathbb{M}_{\mathcal{U}}$  is the subset of Mathias forcing consisting of conditions whose second coordinate is in  $\mathcal{U}$ .

**1.5. Remark.** Mathias forcing can be written as an iteration  $\mathbb{M} = \mathcal{P}(\omega)/\text{fin}^* \mathbb{M}_{\dot{G}}$ , where  $\dot{G}$  is a name for the generic ultrafilter added by the first forcing. It is easy to verify that the generic real for relativized Mathias forcing  $\mathbb{M}_{\mathcal{U}}$ , which is the union of the first coordinates of conditions in the generic filter, is a pseudointersection of  $\mathcal{U}$ .

## 2. CHARACTERIZATION OF CANJAR ULTRAFILTERS

**2.1. Definition.** A *Canjar ultrafilter* is an ultrafilter on  $\omega$  such that  $\mathbb{M}_{\mathcal{U}}$  does not add dominating reals.

The following observation will motivate the definition of a strong P-point.

**2.2. Observation.** *An ultrafilter  $\mathcal{U}$  is a P-point if and only if for any descending sequence of sets  $\langle X_n : n < \omega \rangle$  from  $\mathcal{U}$  there is an interval partition  $\langle I_n : n < \omega \rangle$  of  $\omega$  such that*

$$X = \bigcup_{n < \omega} (I_n \cap X_n) \in \mathcal{U}.$$



This is an annotation with an Appearance Stream:  $2^{\aleph_0} = \aleph_1$ .