Fair Voting Systems

Mathematics and Citizenship, in honor of the USA Midterm Elections

Gerrymandering

In many voting systems, such as our parliamentary election system, each "district" elects one person. Gerrymandering is the process of deliberately choosing district boundaries in order to affect the election results. It is rife in the USA.

Activities (15–20 minutes)

- Take 8 yellow votes, 10 red votes, and 12 blue votes (30 in total), and divide them into 6 constituencies of 5 votes each. Each constituency is won by the party with the most votes. (Try to avoid ties!) Can you arrange for the blue party to win an absolute majority of seats? The red party to win an absolute majority? For it to be a three-way tie?
- Explore different numbers of votes, different numbers of constituencies, and so on; is there usually a way to adjust the groupings to affect the overall outcome of the vote?
- Investigation at home (optional): To what extent is the British parliamentary election system affected (afflicted?) by gerrymandering?

Fair voting systems

We'll explore systems of electing one person to a role (e.g., a parliamentary seat, a school council rep, or the like).

Every system involves the voters *ranking* the candidates in order of preference. Candidates can also be ranked as equally liked.

For example, some voting systems allow voters to vote "1" for their favorite candidate, "2" for their next favorite, and so on. Then voting $A = 2$, $B = 1$, $C = 4$, $D = 3$ ranks the candidates as $B > A > D > C$.

Others only allow voters to vote for one candidate. Then if someone votes for C, this is the same as giving them the ranking $C = 1$, $A = B = D = 2$, that is, C is preferred to A, B, and D, who are all joint equal.

The voting system then considers all of the ballots and uses them to decide an overall ranking of the candidates.

As an example, our parliamentary system simply counts the number of votes for each candidate, and then ranks them according to this.

Activity: Design some voting systems

Design at least three other voting systems. You want to make them as “fair” as possible. (If you are stuck, ask for ideas!)

Try out the parliamentary system and each of your voting systems on the ballot papers your group has. Who wins the election for different schemes? (If voters are only allowed to vote for their favorite candidate, take this as the “1” vote on the ballot papers.)

Do your voting systems work even if people are allowed to give some candidates equal rankings (say, A = 3, B = 1, C = 2, D = 2)?

How fair are your voting systems?

Fair voting systems

There are a couple of things we would like from voting systems:

(a) If every voter ranks candidate A at least as high as candidate B, then the overall ranking also has candidate A as least as high as candidate B.

(b) If a candidate walks out of the election, they are scrubbed off every ballot paper. We could then scrub them off the overall ranking or we could recalculate the election results using the modified ballot papers. The overall result should be the same for both methods.

(The second condition is designed to avoid “tactical voting”. This is where a voter votes for a candidate they don’t really want to prevent an even less-desirable candidate from winning.)

Are any of your voting systems “fair” with this definition of fairness?

If not, can you design a “fair” voting system?