Homework 1. Matrix Differentiation

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1.a

$$\begin{aligned} Df(t)[h] &= -f(t)\langle (A - tI_n)^{-1}, I_n\rangle h \\ D^2 f(t)[h_1, h_2] &= -f(t)\left(\langle (A - tI_n)^{-2}, I_n\rangle - \langle (A - tI_n)^{-1}, I_n\rangle^2\right) h_1 h_2 \end{aligned}$$

1.b

$$\begin{aligned} Df(t)[h] &= -f(t)\left(A + tI_n\right)^{-1} h \\ D^2 f(t)[h_1, h_2] &= 2f(t)\left(A - tI_n\right)^{-2} h_1 h_2 \end{aligned}$$

2.a

$$\begin{aligned} \nabla f(x) &= 2\left(xx^T - A\right)x \\ \nabla^2 f(x) &= 2\left(x^T x I_n + 2xx^T - A\right) \end{aligned}$$

2.b

$$\begin{aligned} \nabla f(x) &= 2f(x)\left(\ln\langle x, x\rangle + 1\right)x \\ \nabla^2 f(x) &= 4f(x)\left(2\left(\left(\ln\langle x, x\rangle + 1\right)^2 + \frac{2}{\langle x, x\rangle}\right)xx^T + \left(\ln\langle x, x\rangle + 1\right)I_n\right) \end{aligned}$$

2.c

$$\begin{aligned} \nabla f(x) &= p\|Ax - b\|^{p-2} A^T (Ax - b) \\ \nabla^2 f(x) &= p\left((p-2)\|Ax - b\|^{p-4} A^T\left(Ax - b\right)\left(Ax - b\right)^T A + \|Ax - b\|^{p-2} A^T A\right) \end{aligned}$$