Constraints on the Mass-Radius Relation for Neutron Stars

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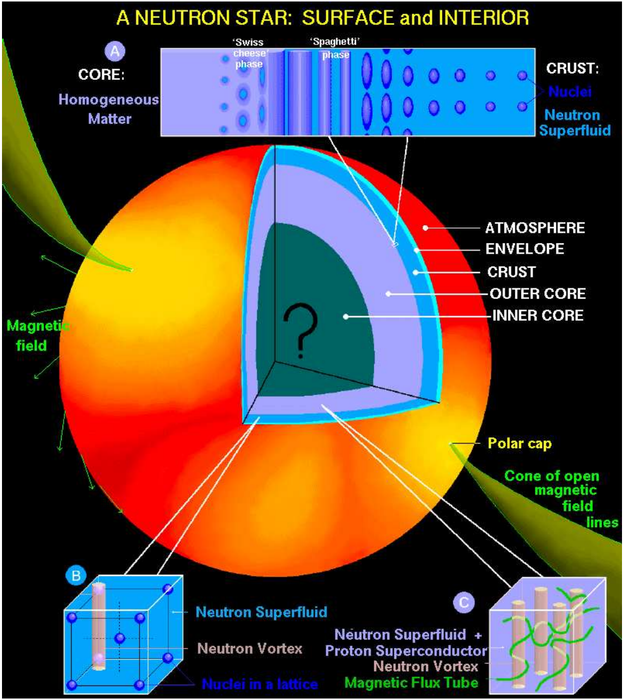
Latest Results from the Neutron-Star Laboratory
Amsterdam, 6–10 May 2013

Important Questions

- How Does the Structure of Neutron Stars Depend On the Nucleon-Nucleon Interaction?
  - The Neutron Star Maximum Mass and Causality
  - The Neutron Star Radius and the Nuclear Symmetry Energy
  - Does Exotic Matter (Hyperons, Kaons/Pions, Deconfined Quarks) Exist in Neutron Star Interiors?
- How Do Nuclear Experiments Constrain the Nuclear Symmetry Energy and Neutron Star Radii?
  - Binding Energies
  - Heavy ion Collisions
  - Neutron Skin Thicknesses
  - Dipole Polarizabilities
  - Giant (and Pygmy) Dipole Resonances
  - Pure Neutron Matter
- What Astrophysical Constraints Exist?
  - Nuclear Mass Measurements
  - Photospheric Radius Expansion Bursts
  - Thermal Emission from Isolated and Quiescent Binary Sources
  - Pulse Modeling of X-ray Bursts, QPOs, etc.

Dany Page, UNAM

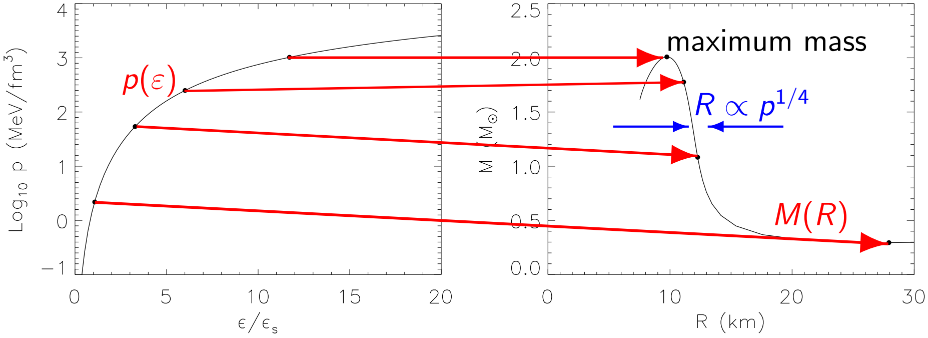


Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^4}\frac{(mc^2 + 4\pi pr^3)(\varepsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$

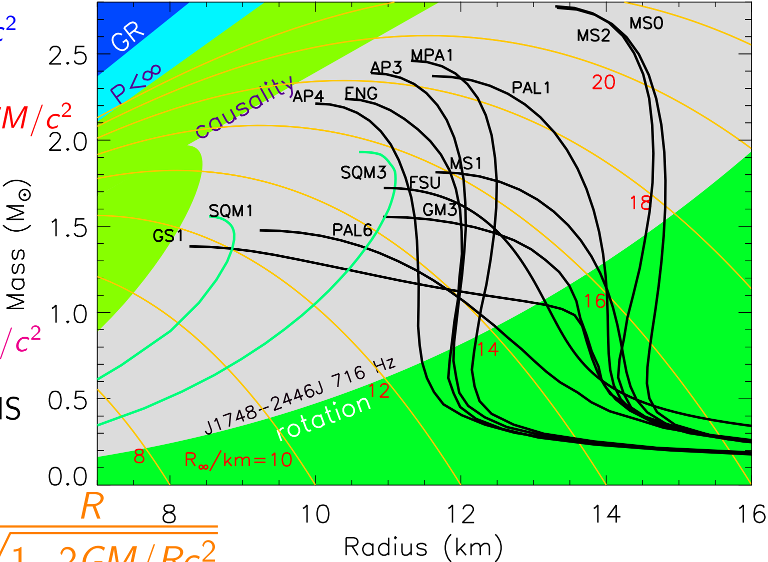


Equation of State ← Observations

Mass-Radius Diagram and Theoretical Constraints

GR:
$R > 2GM/c^2$

$P < \infty$:
$R > (9/4)GM/c^2$



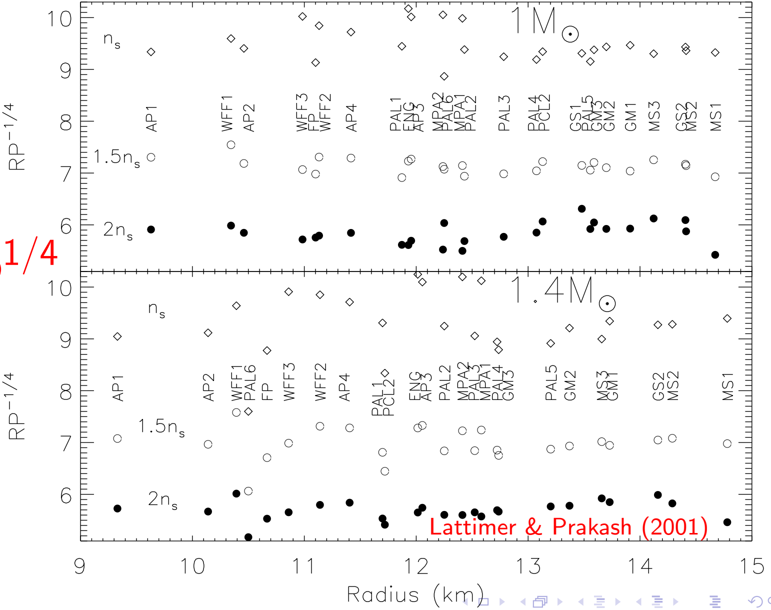
causality:
$R \gtrsim 2.9GM/c^2$

— normal NS
— SQS

— $R_\infty = \dfrac{R}{\sqrt{1-2GM/Rc^2}}$

The Radius – Pressure Correlation

$R \propto p^{1/4}$



Neutron Star Structure

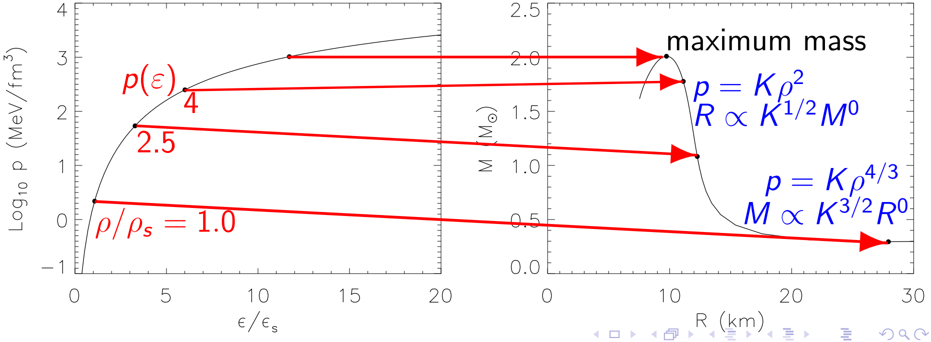
Newtonian Gravity:

$$\frac{dp}{dr} = -\frac{Gm\rho}{r^2}; \qquad \frac{dm}{dr} = 4\pi\rho r^2; \qquad \rho c^2 = \varepsilon$$

Newtonian Polytrope:

$$p = K\rho^\gamma; \qquad M \propto K^{1/(2-\gamma)} R^{(4-3\gamma)/(2-\gamma)}$$

$$\rho < \rho_s:\ \gamma \simeq \tfrac{4}{3}; \qquad \rho > \rho_s:\ \gamma \simeq 2$$



Nuclear Symmetry Energy

Defined as the difference between energies of pure neutron matter ($x = 0$) and symmetric ($x = 1/2$) nuclear matter.

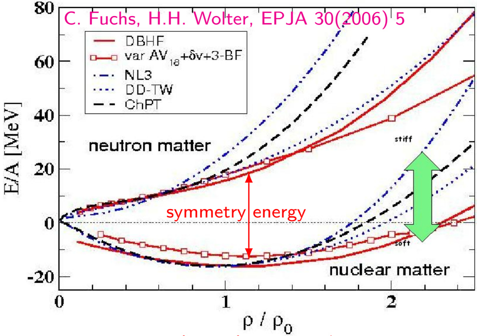
$$S(\rho) = E(\rho, x = 0) - E(\rho, x = 1/2)$$

Expanding around saturation density (ρ_s) and symmetric matter ($x = 1/2$)

$$E(\rho, x) = E(\rho, 1/2) + (1-2x)^2 S_2(\rho) + \ldots$$

$$S_2(\rho) = \mathbf{S_v} + \frac{\mathbf{L}}{3}\frac{\rho - \rho_s}{\rho_s} + \ldots$$

$$\mathbf{S_v} \simeq 31 \text{ MeV}, \mathbf{L} \simeq 50 \text{ MeV}$$



Connections to neutron matter:

$$E(\rho_s, 0) \approx S_v + E(\rho_s, 1/2) = S_v - B, \qquad p(\rho_s, 0) = L\rho_s/3$$

Neutron star matter (in beta equilibrium):

$$\frac{\partial(E + E_e)}{\partial x} = 0, \quad p(\rho_s, x_\beta) \simeq \frac{L\rho_s}{3}\left[1 - \left(\frac{LS_v}{\hbar c}\right)^3 \frac{4 - 3S_v/L}{3\pi^2\rho_s}\right]$$

Nuclear Binding Energies

$E_{sym}(N,Z) = I^2(S_v A - S_s A^{2/3})$

$\chi^2 = \sum_i (E_{ex,i} - E_{sym,i})^2/\mathcal{N}$

$\chi_{vv} = \frac{2}{\mathcal{N}} \sum_i I_i^4 A_i^2$

$\chi_{ss} = \frac{2}{\mathcal{N}} \sum_i I_i^4 A_i^{4/3}$

$\chi_{vs} = \frac{2}{\mathcal{N}} \sum_i I_i^4 A_i^{5/3}$

$\sigma_{S_v} = \sqrt{\frac{2\chi_{ss}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}}$

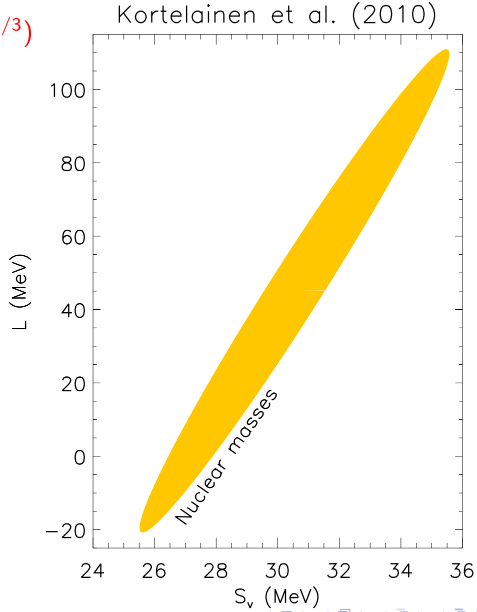
$\sigma_{S_s} = \sqrt{\frac{2\chi_{vv}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}}$

$\alpha = \frac{1}{2}\tan^{-1}\frac{2\chi_{vs}}{\chi_{vv} - \chi_{ss}}$

$r_{vs} = -\frac{\chi_{vs}}{\sqrt{\chi_{vv}\chi_{ss}}}$

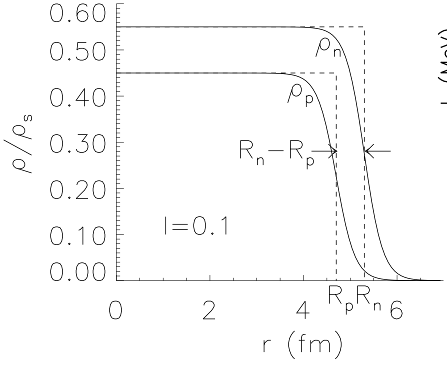
Liquid Droplet Model

$E_{sym}(N,Z) = \frac{S_v A I^2}{1 + (S_s/S_v)A^{-1/3}}$

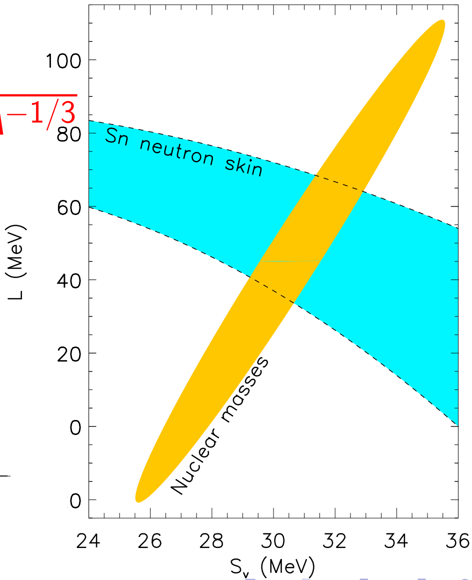


Neutron Skin Thickness

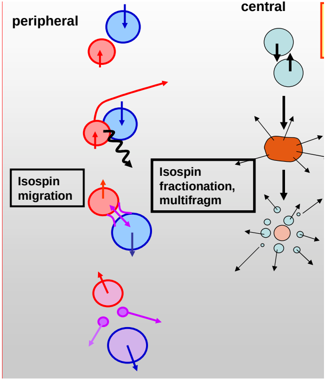
$$\frac{R_n - R_p}{r_o} \simeq \sqrt{\frac{4}{15}}\frac{S_s I}{S_v + S_s A^{-1/3}}$$



Chen, Ko, Li & Xu (2010)

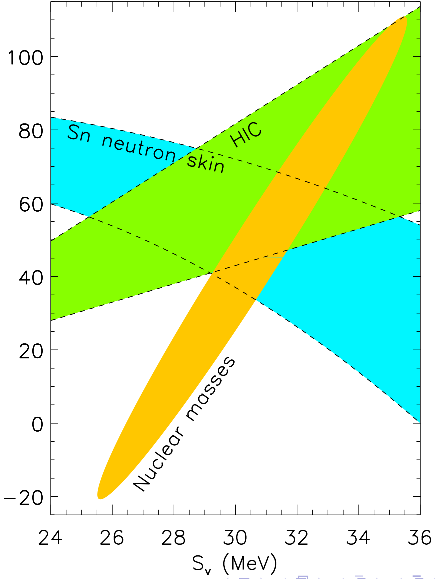


Heavy Ion Collisions

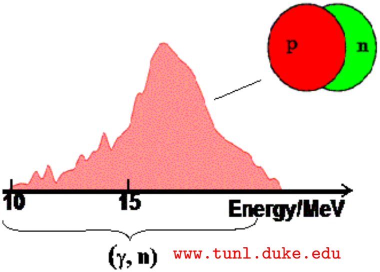


Wolter, NuSYM11

Tsang et al. (2009)

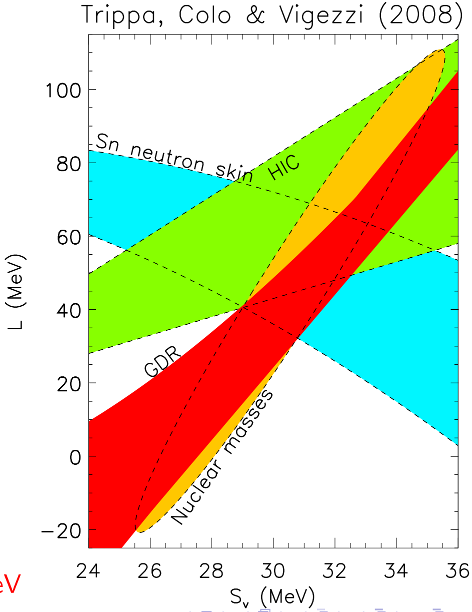


Giant Dipole Resonances



$$E_{-1} \propto \sqrt{\frac{S_v}{1+\frac{5S_s}{3S_v}A^{-1/3}}}$$

23.3 MeV$< S_2(0.1\ \mathrm{fm}^{-3}) <$24.9 MeV



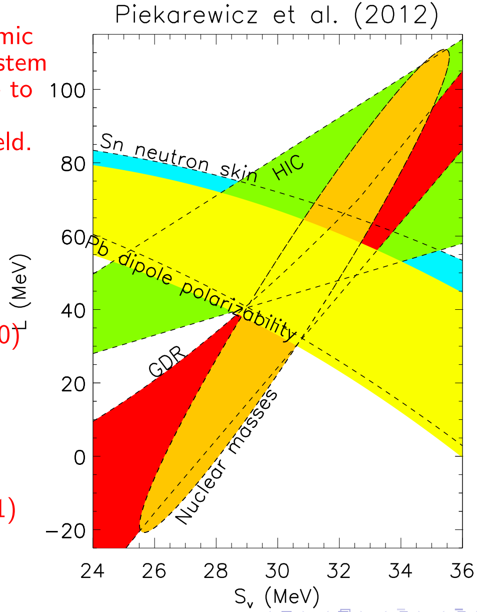
Dipole Polarizability

The linear response, or dynamic polarizability, of a nuclear system excited from its ground state to an excited state, due to an external oscillating dipolar field.

α_D and $R_n - R_p$ in ^{208}Pb
are 98% correlated

Reinhard & Nazawericz (2010)

Data from Tamii et al. (2011)

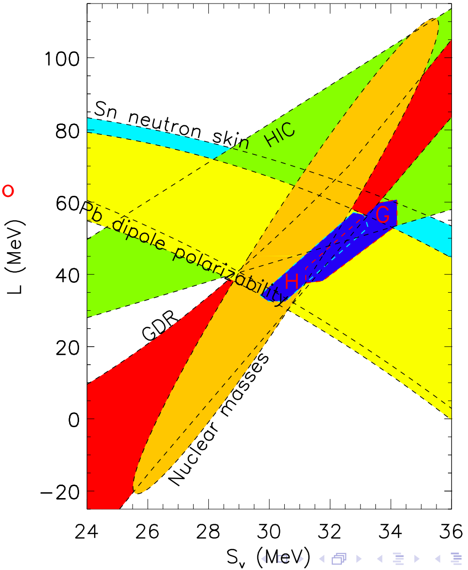


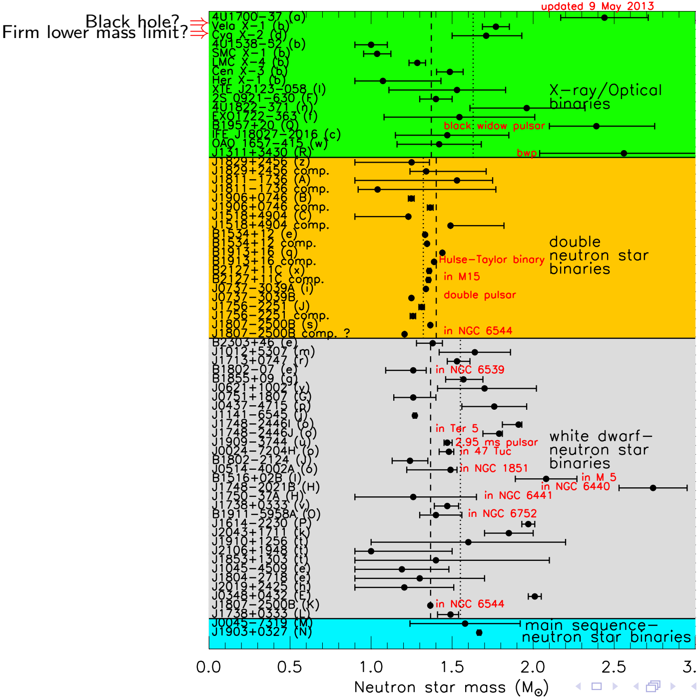
Theoretical Neutron Matter Calculations

H&S: Chiral Lagrangian

GC&R: Quantum Monte Carlo

Gandolfi, Carlson & Reddy (2011);
Hebeler & Schwenk (2011)





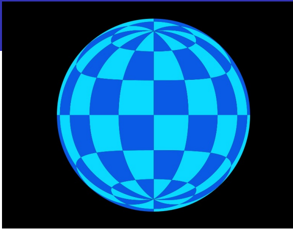
Although simple average mass of w.d. companions is 0.23 $M_\odot$ larger, weighted average is 0.04 $M_\odot$ smaller

Demorest et al. 2010

Antoniadis et al. 2013

Champion et al. 2008

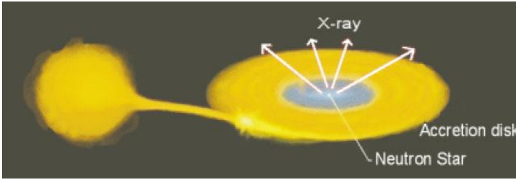
Simultaneous Mass/Radius Measurements



- The measurement of flux and temperature yields an apparent angular size (pseudo-BB):

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

- Observational uncertainties include distance, interstellar absorption (UV and X-rays), atmospheric composition

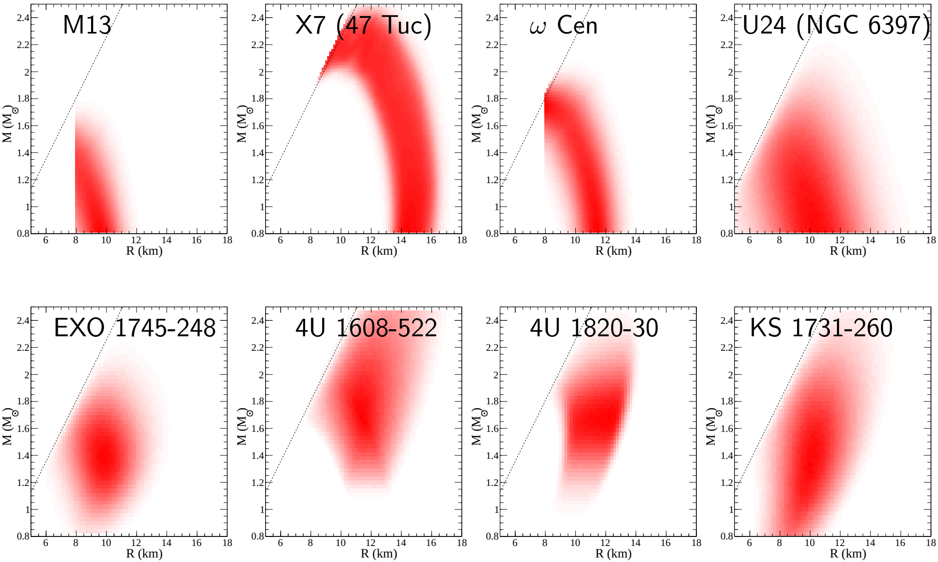


Best chances for accurate radius measurement:

- Nearby isolated neutron stars with parallax (uncertain atmosphere)
- Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmospheres)
- Bursting sources with peak fluxes close to Eddington limit (where gravity balances radiation pressure)

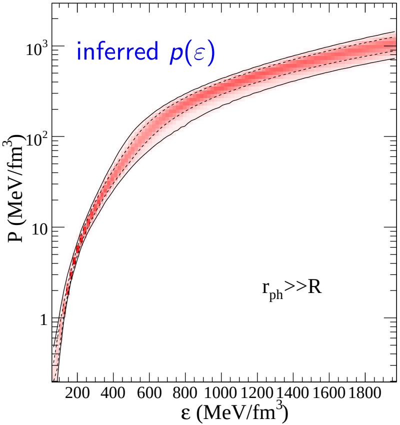
$$F_{Edd} = \frac{cGM}{\kappa D^2}$$

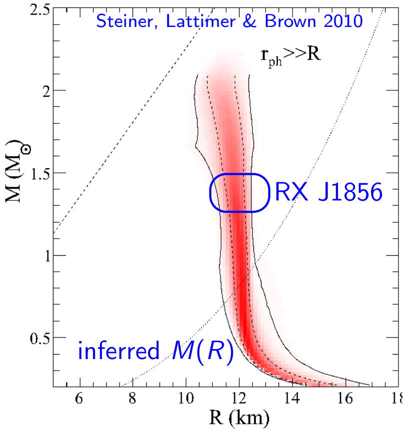
$M - R$ Probability Estimates



Bayesian TOV Inversion

- $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- Polytropic EOS: $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1; $\varepsilon > \varepsilon_2$: n_2
- EOS parameters $K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$ uniformly distributed
- $M_{max} \geq 1.97$ M$_\odot$, causality enforced
- All stars equally weighted





Astronomical Observations

$11.0 < R_{1.4}/\mathrm{km} < 12.3$ (69%)
Steiner et al. (2010)

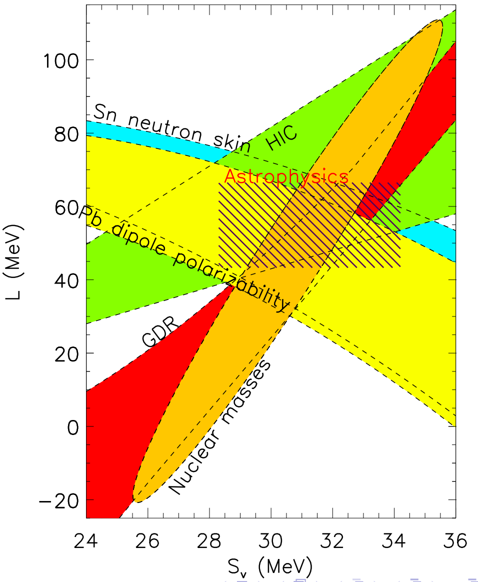
$10.6 < R_{1.4}/\mathrm{km} < 12.5$ (95%)
Steiner et al. (2013)

$10.8 < R_{1.4}/\mathrm{km} < 12.3$ (69%)
Gandolfi et al. (2012)

$9.7 < R_{1.4}/\mathrm{km} < 13.9$ (100%)
Hebeler et al. (2010)

$10.0 < R_{1.4}/\mathrm{km} < 13.7$ (100%)
Hebeler et al. (2013)

Steiner, Lattimer & Brown (2010)



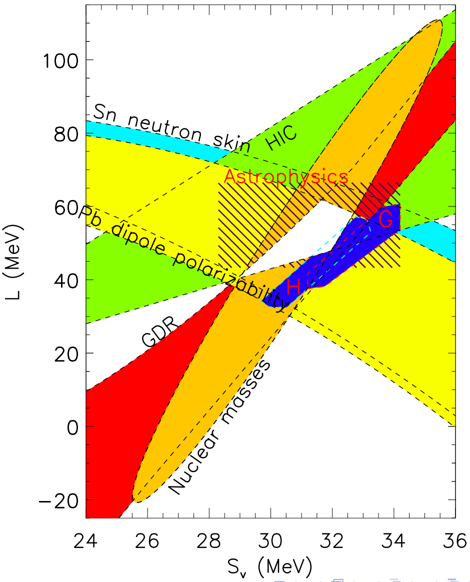
qLMXB Analysis (Guillot et al. 2013)

Source	D (kpc)	$N_{H,21}$	$N^{\dagger}_{H,21}$	M ($M_\odot$)	R (km)
M28	5.5 ± 0.3	2.52	1.89	$1.25^{+0.54}_{-0.63}$	$10.5^{+2.0}_{-2.9}$
NGC 6397	2.02 ± 0.18	0.96	1.4	$0.84^{+0.30}_{-0.28}$	$6.6^{+1.2}_{-1.1}$
M13	6.5 ± 0.6	0.08	0.145	$1.27^{0.71}_{-0.63}$	$10.2^{+3.7}_{-2.8}$
ω Cen	4.8 ± 0.3	1.82	1.04	$1.78^{+1.03}_{-1.07}$	$23.6^{+7.6}_{-7.1}$
NGC 6304	6.22 ± 0.6	3.46	2.66	$1.16^{0.96}_{-0.56}$	$9.6^{+4.9}_{-3.4}$
optimized with fixed R				0.86 – 2.42	$9.1^{+1.3}_{-1.5}$
optimized assuming crust EOS + causality‡				0.78 – 1.19	10.4 ± 0.9
optimized assuming $N^{\dagger}_{H,21}$ + crust EOS + causality + H/He‡				0.96 – 1.39	11.8 ± 0.9

† Dickey & Lockman (1990)

‡ Lattimer & Steiner (2013)

Combined Constraints



Consistency with Neutron Matter and Heavy-Ion Collisions