Modelling rate-dependent symmetric and asymmetric hysteresis loops of smart actuators

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Abstract: Smart material actuators invariably exhibit hysteresis that may be either symmetric or asymmetric depending upon the actuation principle. Moreover, the shape of hysteresis loop depends on the rate of change of the input. In this study, a generalised rate-dependent Prandtl-Ishlinskii model is proposed to characterise both the symmetric and asymmetric input-output hysteresis effects of smart material-based actuators. The model is realised upon formulation and integration of a generalised rate-dependent play operator. The validity of the generalised model is demonstrated by comparing its displacement responses with the measured symmetric and asymmetric responses obtained for piezoceramic and magnetostrictive actuators under different input frequencies in the 1–200 Hz and 10–100 Hz ranges, respectively. The results suggest that the proposed rate-dependent Prandtl-Ishlinskii model can effectively characterise the symmetric as well asymmetric hysteresis properties of the smart material actuators over a wide range of input frequencies.

Keywords: smart actuators; rate dependent hysteresis; Prandtl-Ishlinskii; play operator.

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1 Introduction

Hysteresis is a non-linear phenomenon that appears in various systems, including smart actuators and ferromagnetic materials. The non-differentiable and often unknown hysteresis properties of actuators are known to cause inaccuracies and oscillations in the system responses that may even lead to instability of the closed loop system (Tao and Kokotovic, 1995). Considerable efforts have been made to characterise the hysteresis properties of actuators for effective controller design. Smart material-based actuators may exhibit symmetric or asymmetric hysteresis property under increasing and decreasing inputs. For example, magentostrictive actuators, widely used for micropostioning applications, exhibit strongly asymmetric and rate-dependent hysteresis, primarily attributed to magnetostriction properties of the material (Tan and Baras, 2004; William and Smith, 2007). A large number of piezoceramic actuators, on the other hand, generally yield symmetric but strongly rate-dependent hysteresis properties (Yu et al., 2002; Al Janaideh et al., 2007a, 2007b). A number of physical and phenomological models have been developed to describe both symmetric and asymmetric hysteresis of smart actuators and materials (William and Smith, 2007; Mayergoyz, 2003; Brokate and Sprekels, 1996; Krasnoselskii and Pokrovskii, 1989; Galinaities, 1999; Su et al., 2005; Song et al., 2005; Mrad and Hu, 2002; Smith and Ounaies, 2000; Smith, 2005; Ge and Jouaneh, 1995). The phenomological models, such as Preisach model (Mayergoyz, 2003), Krasnosel'skii-Pokrovskii operator (Krasnoselskii and Pokrovskii, 1989) and Prandtl-Ishlinskii model (Brokate and Sprekels, 1996) have been widely used to describe hysteresis properties of smart actuators and materials. These models have also been applied to design controllers for compensating the hysteresis effects (Galinaities, 1999; Su et al., 2005; Song et al., 2005). The models, however, have been mostly applied to describe rate-independent hysteresis effects, assuming negligible effect of the rate of input.

A few studies have experimentally characterised the output-input relationships of different actuators under varying inputs. These clearly showed dependenency of the actuator displacement on the rate of input, while the area bounded by the hysteresis loops also increased under increasing input frequency (William and Simth, 2007, Al Janaideh et al., 2007a, 2007b; Ge and Jouaneh). The data reported for various piezoceramic actuators under excitation of varying magnitudes and frequencies suggest nearly symmetric major as well as minor hysteresis loops, which are strongly dependent upon the rate of input. Ge and Jouaneh (1995) performed laboratory measurements to characterise the hysteresis properties of a P–178.20 piezoceramic actuator under a constant amplitude harmonic voltage at two distinct frequencies (0.1 and 100 Hz). The study concluded symmetric hysteresis loops and only small influence of excitation rate on the output displacement, while higher hysteresis under the 100 Hz input was clearly evident. Yu et al. (2002) showed that hysteresis in a PZT bimorph actuator is rate-independent only up to 10 Hz, but strongly dependent on the input frequency above 10 Hz. The study further showed that actuator displacement amplitude decreases with increasing input frequency. Such dependency of the output displacement on the rate of input has also been observed for many other smart material actuators. The hysteresis properties of a 100 layers piezoceramic actuator, characterised by Mrad and Hu (2002) under varying excitation frequencies, showed that the width of the hysteresis loop increases to 38.6% of the measured displacement amplitude at 800 Hz, compared to only 15% at very low frequencies. Ang et al. (2003) showed that the static position error of a TS–H5–104 piezoceramic actuator due to its hysteresis is in the order of 15% and this error tends to increase with increase in frequency of the command voltage. These actuators showed that the area bounded by the hysteresis loops increases under higher frequency excitations. Moreover, the piezoceramic actuators generally exhibit symmetric major and minor hysteresis loops.

Unlike the piezoceramic actuators, magnetostrictive actuators exhibit highly asymmetric hysteresis property about the input or the output axis. On the basis of laboratory measurements, it has been further shown that hysteresis in magnetostrictive actuators is strongly rate-dependent beyond certain frequencies (Tan and Baras, 2004; William and Simth, 2007). The eddy current losses and the dynamics of the actuator are considered among the primary sources of the rate-dependent hysteresis in magnetostrictive actuators (Tan and Baras, 2004). Tan and Baras (2004) performed laboratory measurements to characterise the hysteresis properties of a magnetostrictive actuator under sinusoidal input currents of constant amplitude (0.8A) with a bias of 0.1A in the 10–300 Hz frequency range. The reported data revealed rate-dependent and asymmetric hysteresis loops between the input current and the output displacement, particularly under excitations above 10 Hz. In a similar manner, the hysteresis characteristics of a magnetostrictive actuator measured under excitation in the 100–500 Hz range showed increase in the hysteresis with increasing excitation frequency (William and Simth, 2007).

A number of dynamic density functions have been proposed to predict rate-dependent behaviour of smart actuators, when integrated to the classical phenomenological models. Mayergoyz (1988) proposed a dynamic Preisach model by adding the time rate of the output in the density function to predict rate-dependent hysteresis properties. Alternatively, Yu et al. (2002) proposed and integrated a density function in time rate of the input to the Preisach model to predict dynamic hysteresis of a piezoceramic actuator. Mrad and Hu (2002) applied a dynamic density function to construct a dynamic Preisach model to characterise rate-dependent hysteresis of a piezoceramic actuator. The validation of the model was shown using the measured displacement of a piezoceramic actuator in the 0.1–800 Hz frequency range. A dynamical model coupled with the Preisach operator was proposed by Tan and Baras (2004) in an attempt to characterise and compensate for the rate-dependent hysteresis effects in a magnetostrictive actuator over a wide frequency range. Smith (2005) presented a homogenised energy model using Preisach model to characterise the rate-dependent hysteresis in a magnetostrictive actuator over a wide frequency range (1–2 kHz).

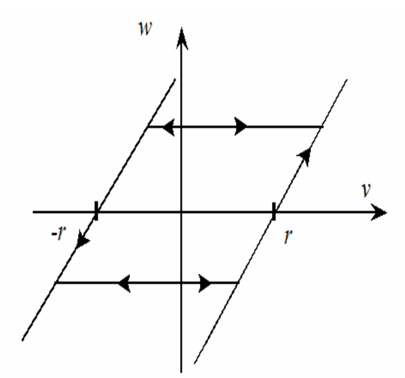
The application of the Preisach model for a real time inverse controller design, however, may be limited, since the model is not analytically invertible. Alternatively, Prandtl-Ishlinskii model has also been explored, which is a subclass of the Preisach model and is analytically invertible, although the model is also rate-independent. Ang et al. (2003) proposed a dynamic function and dead zone operator for the Prandtl-Ishlinskii model in an attempt to characterise the rate-dependent hysteresis in piezoceramic actuators. The validity of the model was demonstrated for a harmonic input at 10 Hz, and a complex harmonic input comprising 5, 20 and 35 Hz components. The study concluded that the proposed dynamic model reduced the peak error by more than 50%, when compared to that attained from the rate-independent hysteresis model. A recent study proposed a rate-dependent Prandtl-Ishlinskii play operator in conjunction with a dynamic density function to characterise rate-dependent hysteresis of a piezoceramic actuator over a wide frequency range (Al Janaideh et al., 2007a, 2007b).

Owing to the symmetric nature of the play operator, the above dynamic Prandtl-Ishlinskii model could predict only symmetric hysteresis properties. Alternatively, the classical generalised play operator, described in Visitin (1994) could be applied to characterise asymmetric hysteresis properties. The combination of generalised play operator and a density function could further enhance the symmetric as well as asymmetric hysteresis prediction abilities of the Prandtl-Ishlinskii model. In this study, a generalised rate-dependent play operator is proposed to describe the symmetric as well as asymmetric hysteresis properties as a function of the rate of input. This operator is formulated and integrated with a density function to accomplish the generalised rate-dependent Prandtl-Ishlinskii model. The validity of the proposed model is demonstrated using measured data acquired for piezoceramic and magnetostrictive actuators, which show symmetric and asymmetric rate-dependent hysteresis, respectively.

2 Play operator based Prandtl-Ishlinskii model

Prandtl-Ishlinskii model is a phenomenological hysteresis model that integrates play and stop operators with a density function to characterise hysteresis non-linearities. The play operator is continuous and rate-independent hysteresis operator, relating the output and input as shown in Figure 1 (Brokate and Sprekels, 1996). The play operator has been described by the motion of a piston within a cylinder of length $2r$, where the instantaneous position of center of the piston is represented by coordinate v and cylinder position by w (Brokate and Sprekels, 1996).

Figure 1 Play hysteresis operator



Analytically, let $C_m[0,t_E]$ represent the space of piecewise monotone continuous functions. For any input $v(t)\in C_m[0,t_E]$, the play operator is defined by (Brokate and Sprekels, 1996)

$$F_r[v](0) = f_r\left(v(0),0\right) = w(0),$$

$$F_r[v](t) = f_r\left(v(t),F_r[v](t_i)\right); \quad \text{for } t_i < t \le t_{i+1} \text{ and } 0 \le i \le N-1 \tag{1}$$

where, $f_r(v,w) = \max\left(v-r,\min(v+r,w)\right)$.

The argument of the operator is presented in square brackets to show the functional dependence, since it maps a function to another function. The play operator is characterised by input v and the threshold r. In the above formulation, $0=t_0<t_1<.....<t_N=t_E$ are partitions in $[0,t_E]$ such that the function v is monotone on each of the sub-intervals $[t,t_{i+1}]$. These, however, can be extended to space $C[0,t_E]$ of continuous functions. The Prandtl-Ishlinskii model utilises the play operator $F_r[v](t)$ to describe relationship between the output y_{Pr} and input v with a positive density function $p(r)$ and a constant q, such that (Brokate and Sprekels, 1996)

$$y_{Pr} = qv(t) + \int_0^R p(r)F_r[v](t)dr . \tag{2}$$

The density function $p(r)$ is identified from the experimental data. Prandtl-Ishlinskii model with the density function maps $C[t_o,\infty)$ into $C[t_o,\infty)$. In other words, Lipschitz continuous inputs will yield Lipschitz continuous outputs (Krasnoselskii and Pokrovskii, 1989).

The classical Prandtl-Ishlinskii model can characterise only rate-independent and symmetric hysteresis behaviour. A recent study has presented formulations of the rate-dependent play operator in order to develop a rate-dependent Prandtl-Ishlinskii model. The proposed operator comprised a dynamic threshold as a function time rate of input $\bar{r} = r(\dot{v})$, such that (Al Janaideh et al., 2007a, 2007b)

$$F_{\bar{r}}\left(v(t)\right) = f_{\bar{r}}\left(v(t),F_{\bar{r}}\left(v(t_i)\right)\right); \text{ for } t_i < t \le t_{i+1} \text{ and } 0 \le i \le N-1 \tag{3}$$

where

$$f_{\bar{r}}(v,w) = \max\left(v-\bar{r},\min(v+\bar{r},w)\right);$$

and $F_{\bar{r}}\left(v(0)\right) = f_{\bar{r}}\left(v(0),0\right) = w(0)$.

A dynamic Prandtl-Ishlinskii model was subsequently derived by integrating the rate-dependent play hysteresis operator together with a density function $p(\bar{r})$ and the constant q, such that the model output is expressed as

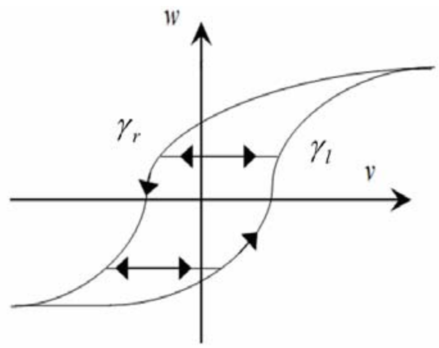
$$y_{P\bar{r}}(t) = qv(t) + \int_0^R p(\bar{r})F_{\bar{r}}[v](t)d\bar{r} . \tag{4}$$

3 Generalised play operator-based Prandtl-Ishlinskii hysteresis model

The above model was applied to predict the rate-dependent hysteresis properties of a piezoceramic actuator. The reasonably good validity of the model was further demonstrated by comparing the model predictions with the measured data over a wide frequency range of inputs (1 to 500 Hz). Owing to the symmetric nature of the rate-dependent play operator, the modified model predictions were limited to symmetric hysteresis properties only (Al Janaideh et al. 2007a, 2007b). Alternatively, the classical generalised play operator, described in Brokate and Sprekels (1996) and Krasnoselskii and Pokrovskii (1989) could be utilised to realise asymmetric input-output relationships. These operators may be further refined to derive rate-dependent input-output relations using the methodology described in Al Janaideh et al. (2007a, 2007b).

The generalised play operator is a non-linear play operator, where an increase in input v causes the output w to increase along the curve γ_l, as shown in Figure 2. A decrease in input v, however, causes the output w to decrease along another curve γ_r, resulting in asymmetric hysteresis loops about the input or the output. The minor loops of the input v and the output w are bounded by the curves γ_l and γ_r, which are continuous functions with $\gamma_l \le \gamma_r$ (Brokate and Sprekels, 1996; Krasnoselskii and Pokrovskii, 1989). For a given input $v(t)$, Lipschitz-continuity of the generalised play operator can also be ensured provided that the curves γ_l and γ_r are Lipschitz continuous.

Figure 2 Input output relationship of a generalised play operator



Analytically, the generalised play operator for any input $v(t) \in C_m[0, t_E]$ is defined by (Brokate and Sprekels, 1996; Krasnoselskii and Pokrovskii, 1989)

$$F_\gamma[v](0) = f_\gamma\big(v(0), 0\big) = w(0)\ ;$$

$$F_\gamma[v](t) = f_\gamma\big(v(t), F_\gamma[v](t_i)\big);\ \text{for } t_i < t \le t_{i+1} \text{ and } 0 \le i \le N-1 \quad (5)$$

where, $f_\gamma(v, w) = \max\big(\gamma_l(v) - r, \min(\gamma_r(v) + r, w)\big)$.

The generalised Prandtl-Ishlinskii model is subsequently formulated using the generalised play operator $F_\gamma[v](t)$ to yield output $y_{P\gamma}(t)$ as

$$y_{P\gamma}(t) = qv(t) + \int_0^R p(r) F_\gamma[v](t)\,dr\,. \quad (6)$$

The dependence of the output on the rate of input could be characterised by introducing the dynamic threshold $\bar{r} = r(\dot{v})$ to the generalised play operator in a manner similar to that applied to the symmetric operators (Al Janaideh et al., 2007a, 2007b). The resulting modified generalised play hysteresis operator could describe the rate-dependent asymmetric hysteresis non-linearities, and can be expressed as

$$\begin{aligned} &F_{\bar{\gamma}}[v](0) = f_{\bar{\gamma}}\big(\gamma(v(0)), 0\big) = w(0)\,, \\ &F_{\bar{\gamma}}[v](t) = f_{\bar{\gamma}}\big(\gamma(v(t)), F_{\bar{\gamma}}[v](t_i)\big); \\ &\text{for } t_i < t \le t_{i+1} \text{ and } 0 \le i \le N-1 \\ &f_{\bar{\gamma}}(v, w) = \max\big(\gamma_l(v) - \bar{r}, \min(\gamma_r(v) + \bar{r}, w)\big) \end{aligned} \quad (7)$$

where the dynamic threshold $\bar{r}$ can be expressed as (Al Janaideh et al. 2007a, 2007b)

$$\bar{r} = \alpha \prod_{l=1}^{z} \ln\left(\beta_l + \lambda_l \left|\dot{v}(t)\right|^{\varepsilon_l}\right) \quad (8)$$

where λ_l and α are positive constants, $\beta_l \ge 1$, and $\varepsilon_l \ge 1$, while the order of the rate-dependent threshold is determined by constant z. The fundamental properties of this hysteresis operator can be effectively applied to characterise the rate-dependent asymmetric hysteresis effects of smart actuators. Moreover, the modified generalised operator can also characterise symmetric hysteresis loops.

Illustration example

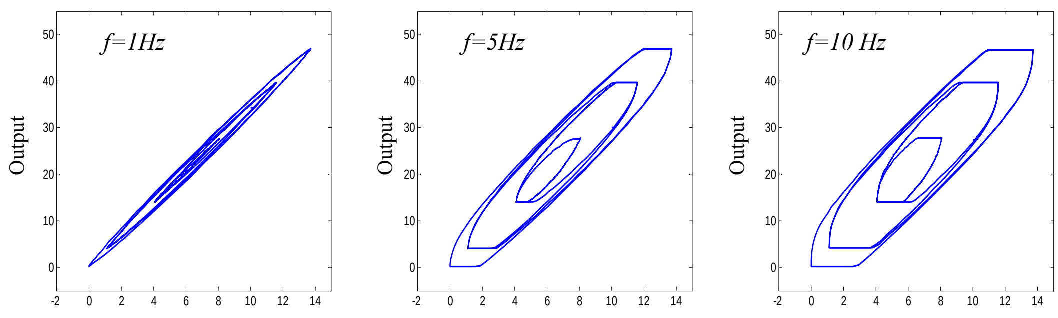
As an example, the response characteristics of the proposed symmetric and asymmetric generalised rate-dependent operators are illustrated for a complex harmonic input of the form: $v(t) = 7 + 4\sin(2\pi ft) + 3\cos(2.4\pi ft)$. This input allows for evaluating the major and minor loops in the operators' outputs. Based on the observed variations in hysteresis behaviour of a piezoceramic actuator over a wide range of input frequencies, a dynamic threshold function was formulated as (Al Janaideh et al., 2007a, 2007b)

$$\bar{r} = \alpha \ln\left(\beta_1 + \lambda_1 \left|\dot{v}(t)\right|^{\varepsilon_1}\right) \quad (9)$$

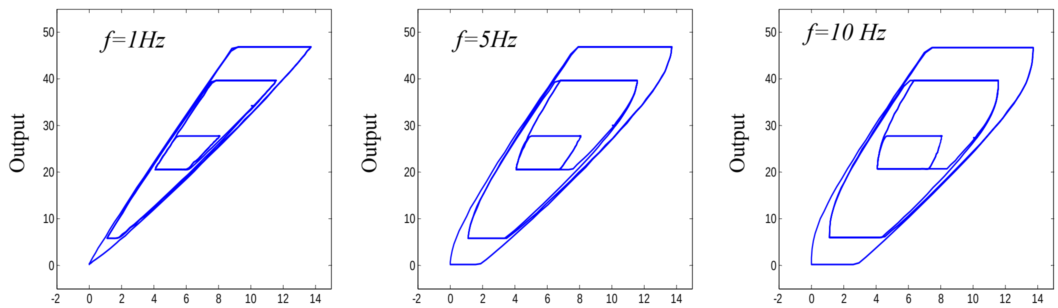
where $\alpha > 0$, $\beta_1 \ge 1$, $\lambda_1 > 0$ and $\varepsilon_1 \ge 1$ are constants.

The simulations are performed under different input fundamental frequencies (1, 5 and 10 Hz) to study the influence of rate of input, while the constants in the dynamic threshold model are chosen as: $\alpha = 2$, $\beta_1 = 1$, $\lambda_1 = 0.0083$ and $\varepsilon_1 = 2$. Two different envelop functions, $\gamma_l(v)$ and $\gamma_r(v)$, are chosen to obtain symmetric and asymmetric operator properties. For the symmetric case, identical values of the envelop functions are chosen, such that $\gamma_l(v) = \gamma_r(v) = 3.4v + 0.2$. Different functions were subsequently chosen for the asymmetric input-output relationship of the play operator, such that $\gamma_l(v)=3.4v+0.2$ and $\gamma_r(v)=5v+0.2$. Figures 3(a) and (b) illustrate the responses of the symmetric and asymmetry rate-dependent play operators, respectively, for the three selected fundamental frequencies. The relaxation slopes of the major and minor loops of the two rate-dependent play operators are determined by the envelope functions, $\gamma_l(v)$ and $\gamma_r(v)$.

Figure 3 Simulation results obtained for the generalised rate-dependent play operator under an input, v(t)=7+4sin(2πft)+ 3cos(2.4πft), (a) Symmetric – $\gamma_r=\gamma_l(v)=3.4v+0.2$ and (b) Asymmetric – $\gamma_r(v)=5v+0.2$ and $\gamma_l(v)=3.4v+0.2$ (see online version for colours)



(a)



(b)

The results clearly suggest that the proposed rate-dependent generalised play operator could effectively produce symmetric, $\gamma_l(v)=\gamma_r(v)$, as well asymmetric, $\gamma_l(v)\neq\gamma_r(v)$, hysteresis loops. Moreover, the outputs exhibit increasing hysteresis width of the operator with increase in the input frequency, which is attributed to the rate-dependent property of the operator.

The generalised rate-dependent Prandtl-Ishlinskii model is formulated upon integrating the above generalised play operator together with a density function, $p(\bar{r})$. A density function is expected to enhance prediction abilities of the generalised model for symmetric as well as asymmetric hysteresis non-linearities. The model output $y_{P\bar{\gamma}}$ is then obtained from equation (10):

$$y_{P\bar{\gamma}}(t)=qv(t)+\int_0^R p(\bar{r})F_{\bar{\gamma}}[v](t)d\bar{r}\,. \qquad (10)$$

The numerical implementation of the generalised Prandtl-Ishlinskii model is formulated using discrete inputs $v\in C[0,t_E]$ with a step size of h as

$$y_{P\bar{\gamma}}(t)=qv(t)+\sum_{j=1}^{N_R}p(\bar{r}_j)F_{\bar{\gamma}}[v](t) \qquad (11)$$

where j = 1,2,…, N_R, and N_R is the number of the generalised rate-dependent play operators that are used in the implementation. The time rate of the input is estimated from the discrete inputs v_k corresponding to interval k (k = 0,1,2,...., N ; $N=t_E/h$), such that:

$$\dot{v}_k=\left(v(t_k)-v(t_{k-1})\right)/\left(t_k-t_{k-1}\right). \qquad (12)$$

On the basis of observed hysteresis properties of a magnetostrictive actuator (Tan and Baras, 2004), higher-order polynomial envelope functions are defined as follows to enhance the asymmetric hysteresis predictions ability of the operator and the model

$$\gamma_r(v)=\sum_{n=1}^{s_r}a_n v^n+a_o$$

$$\gamma_l(v) = \sum_{n=1}^{s_l} b_n v^n + b_o \tag{13}$$

where n is a positive integer, and s_l and s_r are the orders of $\gamma_l(v)$ and $\gamma_r(v)$, respectively. The constants of the proposed $\gamma_l(v)$ and $\gamma_r(v)$ curves can be selected to describe different hysteresis properties. A density function of the following form is formulated upon integrating the dynamic threshold $\bar{r}$ of order ($z = 2$)

$$p(\bar{r}) = \rho e^{-\tau\bar{r}} \tag{14}$$

where ρ ($\rho > 0$) and τ are constants.

It should be mentioned that the proposed density function and the envelop functions, equation (13), of the generalised play operator are not unique; they depend upon the nature of hysteresis of particular material or actuator to be considered.

4 Simulation results

Reported experimental results of two different actuators are considered for examining the prediction abilities of the proposed generalised rate-dependent Prandtl-Ishlinskii model. These include a piezoceramic and a magnetostrictive actuator, which respectively exhibit rate-dependent symmetric and asymmetric hysteresis properties (Tan and Baras, 2004; Yu et al, 2002; Al Janaideh et al., 2007a, 2007b; Mrad and Hu, 2002). The measured data for both actuators revealed strong dependence of the output on the rate of input, while the major as well as minor hysteresis loops of the piezoceramics actuator were nearly symmetric (Al Janaideh et al., 2007a, 2007b).

The output of the Prandtl-Ishlinskii model directly relates to the properties of the play operator, dynamic threshold and the envelope function. The simulations were performed using the rate-dependent $F_{\bar{r}}[v](t)$ and the generalised rate-dependent $F_{\bar{\gamma}}[v](t)$ play operators, defined in equations (3) and (7), respectively, in order to illustrate their relative effects on the outputs under harmonic inputs at different frequencies. As an example, a dynamic threshold function of the form is chosen as:

$$\bar{r} = \eta_1 + \eta_2|\dot{v}| + \eta_3\sqrt{|\dot{v}|} \tag{15}$$

where η_1, η_2 and η_3 are positive constants.

An input signal of the form: $v(t) = 3 + 10\sin(2\pi ft) + 5\sin(3\pi ft)$ was considered to evaluate minor as well as major hysteresis loops, while four different fundamental frequencies were considered (f = 1, 2, 3 and 4 Hz). The simulation parameters were chosen as: $t_E = 6/f$, $h = 0.02/f$, $w(0) = 0$, $q = 0.457$. The parameters for the proposed threshold function, described in (15), were taken as: $\eta_1 = 1.4$, $\eta_2 = 0.0545$ and $\eta_3 = 1.060$. The effect of the density function was relaxed by selecting a constant density function, $p(\bar{r}) = 0.01$, for both rate-dependent Prandtl-Ishlinskii models. The envelop functions for the generalised rate-dependent play operator were chosen as: $\gamma_r = 3.4v - 1$ and $\gamma_l = 6v + 1.233$, in order to investigate the model prediction abilities for asymmetric hysteresis effects.

Figure 4 Simulation results attained from the Prandtl-Ishlinskii model employing the rate-dependent play operator, $F_{\bar{r}}[v](t)$, at different fundamental frequencies of the input (see online version for colours)

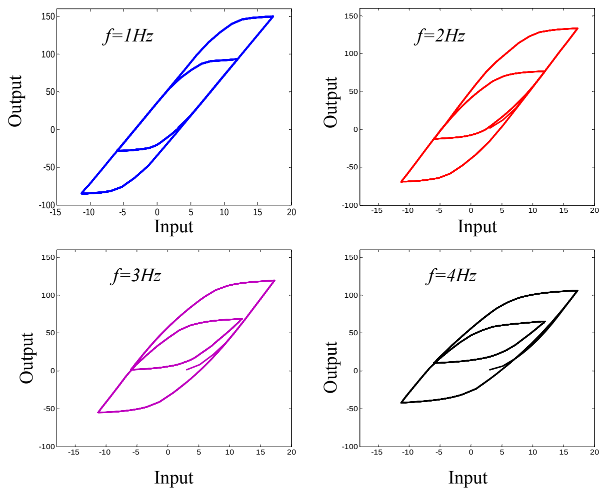
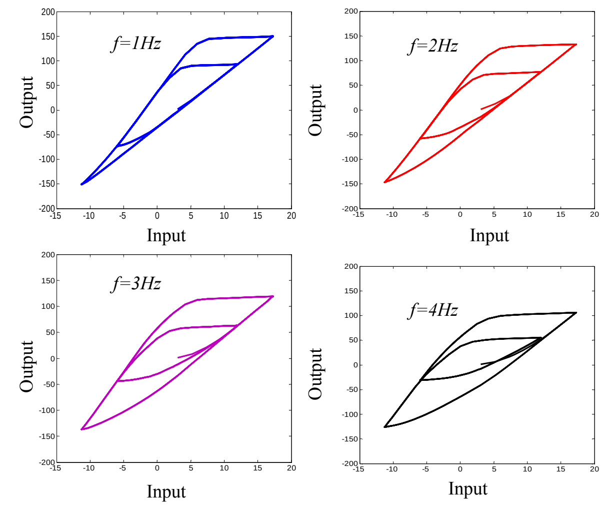


Figure 5 Simulation results of the Prandtl-Ishlinskii model employing generalised rate-dependent play operator, $F_{\bar{\gamma}}[v](t)$, at different fundamental frequencies of the input (see online version for colours)



Figures 4 and 5 illustrate the output-input simulation results of the two Prandtl-Ishlinskii models using rate-dependent $F_{\bar{r}}[v](t)$ and the generalised rate-dependent $F_{\bar{\gamma}}[v](t)$ play operators, respectively, in terms of the input-output properties. The results obtained from both models show an increase in the hysteresis and a decrease in amplitude of the output of the major as well as minor loops, as the fundamental frequency increases. The results further show that the model employing the generalised rate-dependent play operator yields asymmetric hysteresis loops, while employing the rate-dependent operator can provide only symmetric effects. It is thus ascertained that the generalised play operator not only relaxes the symmetry of the rate-dependent Prandtl-Ishlinskii model, but it could also yield the rate-dependent hysteresis effects.

5 Experimental verifications

The parameters of the generalised operator, dynamic threshold and density function, defined in equations (7), (8), (13) and (14), respectively, need to be defined on the basis of known characteristics of specific actuators. The generalised rate-dependent play operator is analysed using linear (s_r = s_l = 1) and non-linear (s_l = s_r = 3) envelop functions, as described in equation (13), respectively, in order to illustrate the influence of the order on the outputs of the rate-dependent Prandtl-Ishlinskii model under inputs at varying rates.

The experimental data obtained for a magnetostrictive actuator (Tan and Baras, 2004) and a piezoceramic actuator (Al Janaideh et al., 2007b) under harmonic inputs at various frequencies in the 10–100 and 1–200 Hz ranges, respectively, were considered for model parameter identification. The parameter identification, however, was limited to the generalised model alone in order to investigate its ability to predict both symmetric and asymmetric hysteresis effects of the respective actuators. The model parameters were identified through minimisation of an error sum-squared function given by

$$J = \sum_{i=1}^{n_1} \sum_{j_f=1}^{n_2} \sum_{a_k=1}^{n_3} C_{jf} \left(y_{P\overline{\gamma}} - y_m \right)^2 \tag{16}$$

subject to: $\beta_l \geq 1$ and $\lambda_l > 0$ (l=1,2) ; and $\rho > 0$

where $y_{P\overline{\gamma}}$ is the displacement response of the generalised Prandtl-Ishlinskii model corresponding to a particular excitation frequency, and y_m is the measured displacement under the same excitation frequency. The error function is constructed through summation of squared errors over a range of input frequencies and amplitudes, denoted by j_f (j = 1…n_2) and a_k (k = 1…n_3), respectively. The index i (i = 1…n_1) refers to the number of data points considered to compute the error function J for one complete hysteresis loop. For the magnetostrictive actuator, one level of input current amplitude of 0.8 A ($n_3 = 1$) with a bias of 0.1 A ($n_3 = 1$) could be considered, since the data was available only under this excitation. Four different excitation frequencies ($n_2 = 4$), namely, 10, 20, 50, and 100 Hz were considered, while a total of 60 data points ($n_1 = 60$) were available for each hysteresis loop. For the piezoceramic actuator, the input voltage amplitude was limited to 40V ($n_3 = 1$), while a total of four different frequencies of 1, 50, 100, and 200 Hz ($n_2 = 4$) were considered with a total of 50 data points ($n_1 = 50$) for each hysteresis loop. A weighting constant C_{jf} (j_f=1, 2, 3 and 4) was introduced corresponding to selected excitation frequencies (10, 20, 50 and 100 Hz for the magnetostrictive; and 1, 50, 100 and 200 Hz for the piezoceramic actuator) to emphasise the error minimisation at higher frequencies. The weighting constants were selected through several trial solutions of the minimisation problem and summarised in Table 1. The error minimisation problem was solved using the MATLAB optimisation toolbox. Solutions were attained for a number of starting values of the parameter vector, which converged to very similar parameter values for both actuators (Table 2) for the selected weighting constants.

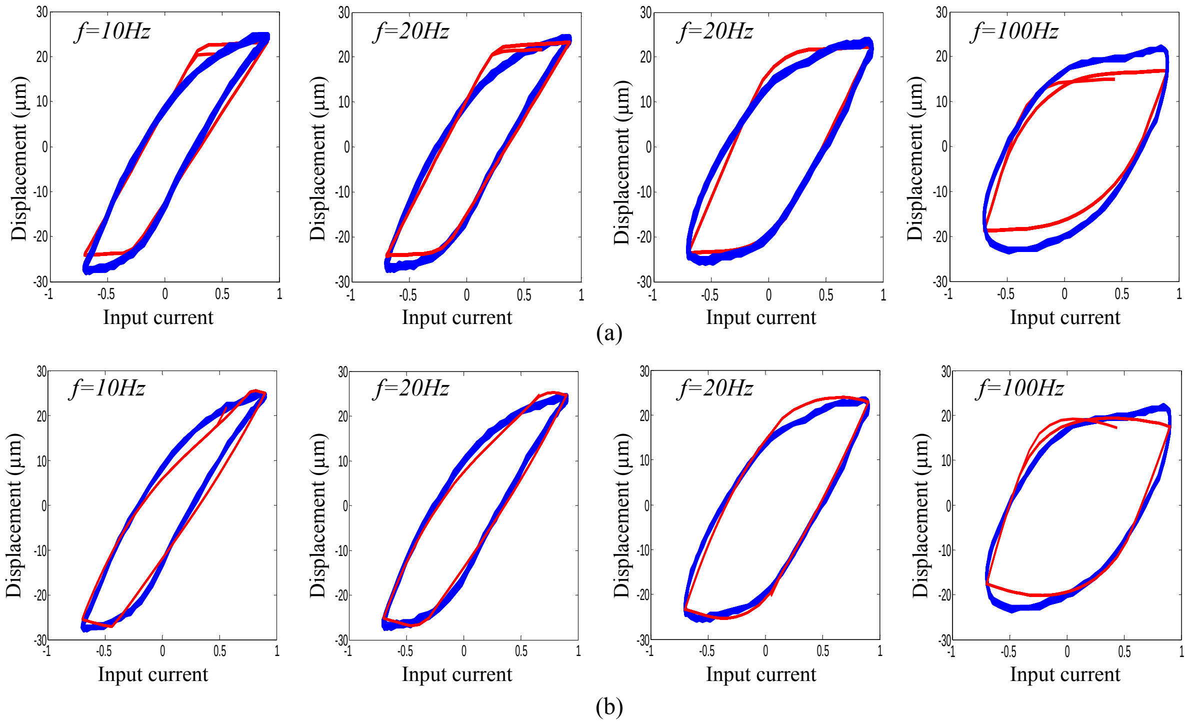
Table 1 Weighting constants C_{jf} applied in the minimisation function for identification of parameters based upon magnetostrictive and piezoceramic actuator data

Magnetostrictive actuator				*Piezoceramic actuator*		
f (Hz)	*m*	C_{jf}		*f (Hz)*	C_{jf}	
		$s_l = s_r = 1$	$s_l = s_r = 3$		$s_l = s_r = 1$	$s_l = s_r = 3$
10	1	13	11	1	22	14
20	2	15	29	50	27	12
50	3	24	12	100	28	22
100	4	33	14	200	44	32

Table 2 Identified parameters of the generalised rate-dependent Prandtl-Ishlinskii model using rate-dependent play operator of linear ($s_l = s_r = 1$) and non-linear ($s_l = s_r = 3$) envelop functions for the magnetostrictive and piezoceramic actuators with asymmetric and symmetric hysteresis properties

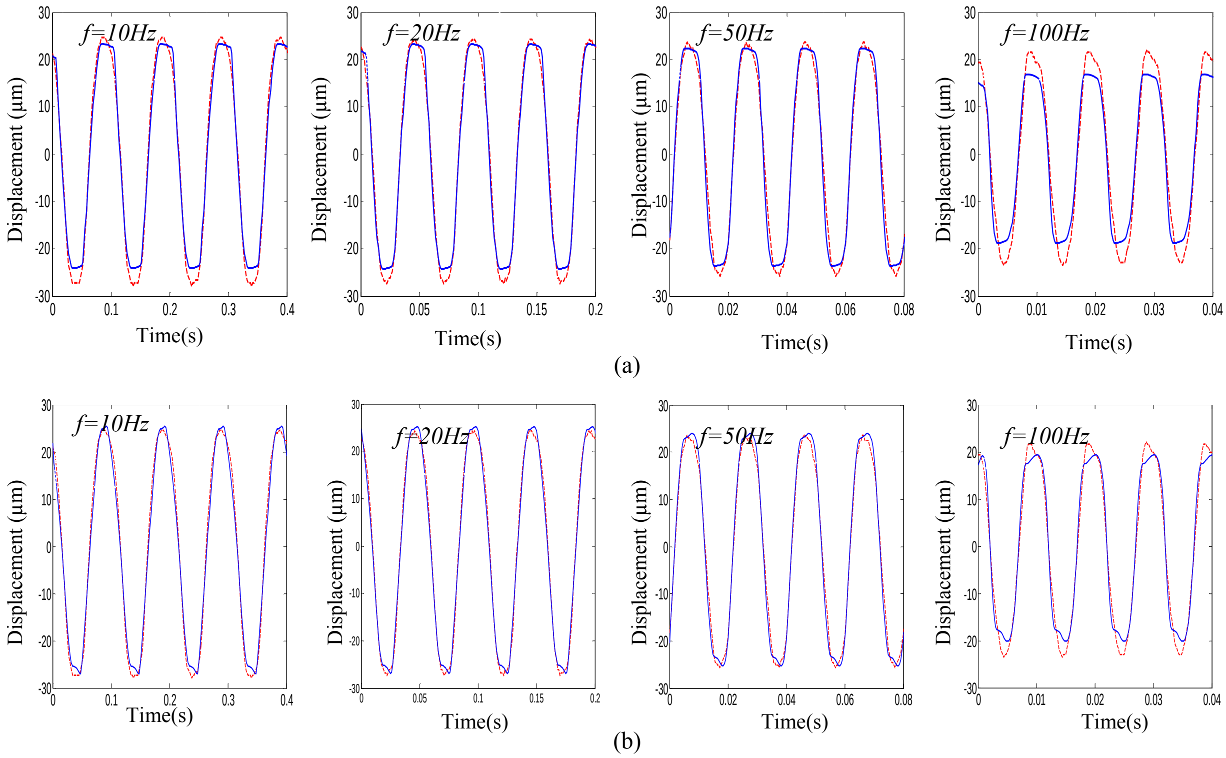
Parameter	*Linear rate-dependent play operator (s=1)*		*Non-linear rate-dependent play operator (s=3)*	
	Mgento.	*Piezo.*	*Mgento.*	*Piezo.*
α	3.721	2.795	4.848	1.792
β_1	1.199	2.745	1.090	2.264
β_2	1.199	1.021	1.199	1.101
λ_1	0.063	1.137×10^{-3}	1.7312×10^{-2}	1.642×10^{-6}
λ_2	0.002	1.085×10^{-4}	6.322×10^{-3}	1.658×10^{-5}
ρ	0.005	0.011	0.005	0.020
τ	0.092	0.044	0.092	0.038
a_o	19.908	0.681	–7.737	0.480
a_1	23.268	0.681	30.265	0.239
a_2	–	–	1.629	1.611×10^{-4}
a_3	–	–	2.861	5.609×10^{-5}
b_0	–5.4238	–0.1989	–3.338	–0.199
b_1	3.1901	0.4584	25.276	0.248
b_2	–	–	–9.789	-6.883×10^{-4}
b_3	–	–	9.098	3.785×10^{-5}
q	1.408	0.217	–7.510	0.218

Figure 6 Comparisons of displacement responses of the generalised rate-dependent model with the measured responses of a magnetostrictive actuator under different input frequencies, (a) Rate-dependent play operator with linear envelop function, $s_l = s_r = 1$ and (b) Rate-dependent play operator with non-linear envelop function, $s_l = s_r = 3$ (see online version for colours)



Notes: Measured Model

Figure 7 Comparisons of time histories of displacement responses of models with the measured data of a magnetostrictive actuator at different input frequencies, (a)Rate-dependent play operator with linear envelop function, $s_l = s_r = 1$ and (b) Rate-dependent play operator with non-linear envelop function, $s_l = s_r = 3$ (see online version for colours)



Notes: Measured ——— Model - - - - - - -

Table 3 Peak displacement error in μm and percent error between responses of the models based on linear (s = 1) and non-linear (s = 3) envelop functions of rate-dependent play operator and the measured responses of the magnetostrictive actuator at different excitation frequencies

Frequency(Hz)	*Peak error in μm (% error)*	
	$s_l = s_r = 1$	$s_l = s_r = 3$
10	4.50 (8.58)	3.75 (7.15)
20	5.13 (9.92)	3.09 (5.98)
50	6.01 (12.19)	4.36 (8.78)
100	5.97 (13.08)	5.00 (10.95)

Table 4 Peak displacement error in μm and percent error between responses of the models based on linear (sl = sr = 1) and non-linear (sl = sr = 3) envelop functions of rate-dependent play operator and the measured responses of the piezoceramic actuator at different excitation frequencies

Frequency(Hz)	*Peak error in μm (% error)*	
	$s_l = s_r = 1$	$s_l = s_r = 3$
10	5.87 (7.48)	4.99(6.36)
50	5.44(7.24)	5.49(7.30)
100	6.08(8.11)	3.81(5.09)
200	6.84(9.39)	4.87(6.68)

Figure 8 Time histories of errors between the model and measured displacement responses of the magnetostrictive actuator at different input frequencies, (a) Rate-dependent play operator with linear envelop function, $s_l = s_r = 1$ and (b) Rate-dependent play operator with non-linear envelop function, $s_l = s_r = 3$ (see online version for colours)

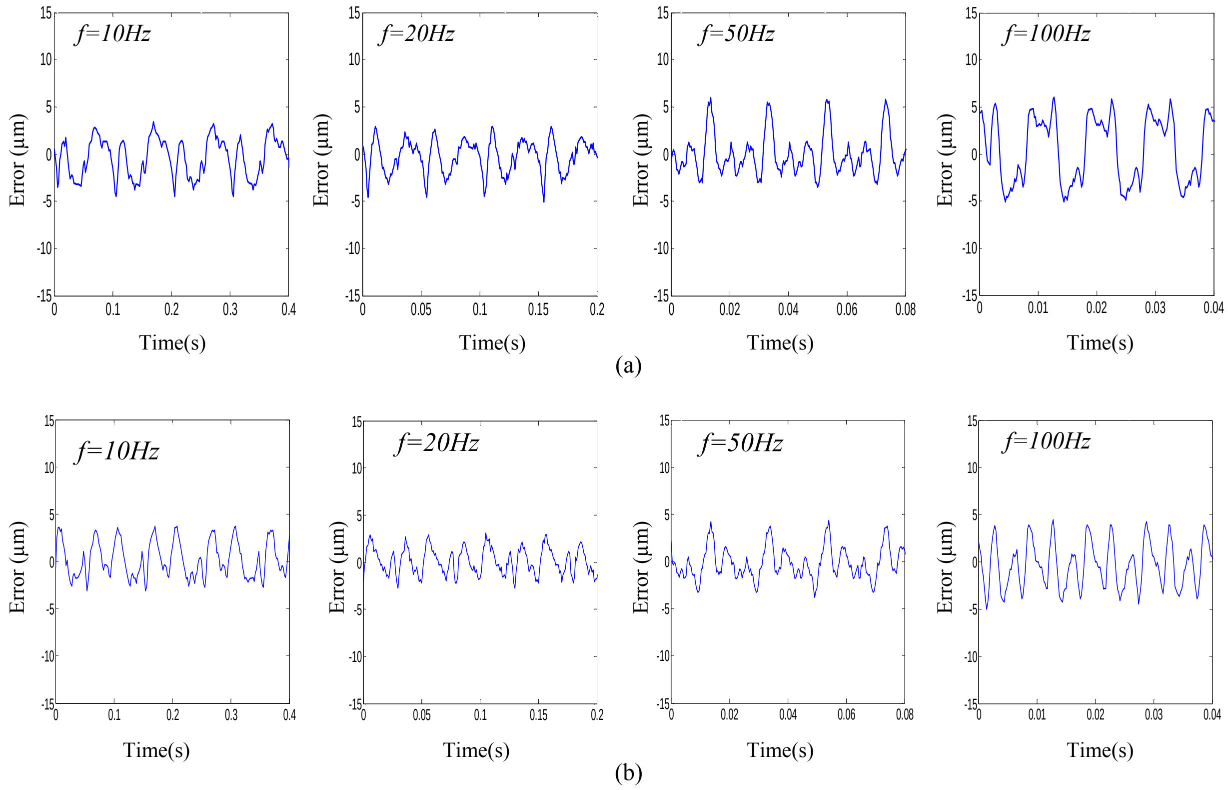
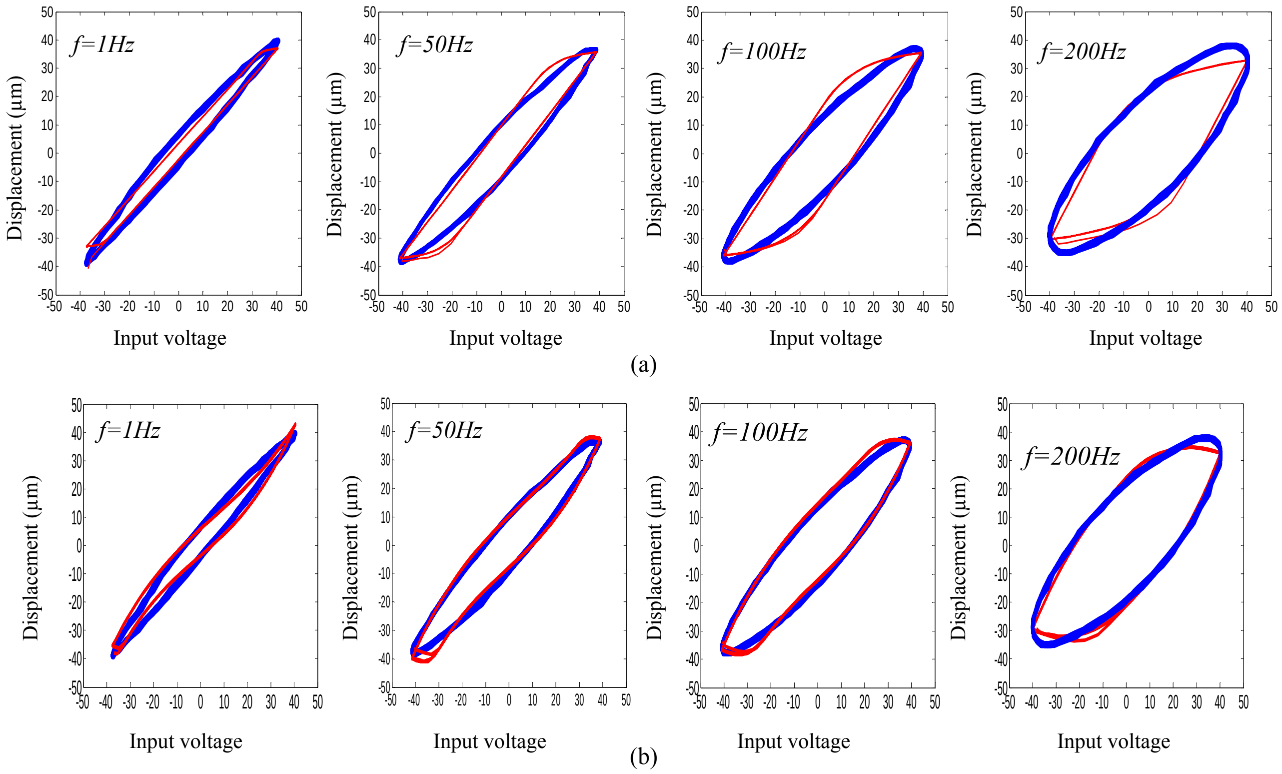
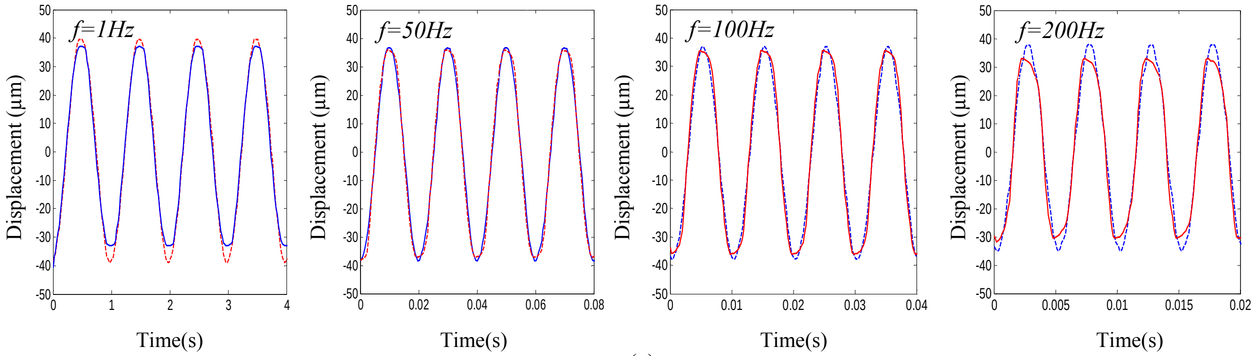


Figure 9 Comparisons of measured displacement responses of the generalised rate-dependent model with the measured responses of a piezoceramic actuator under different input frequencies, (a) Rate-dependent play operator with linear envelop function, $s_l = s_r = 1$ and (b) Rate-dependent play operator with non-linear envelop function, $s_l = s_r = 3$ (see online version for colours)

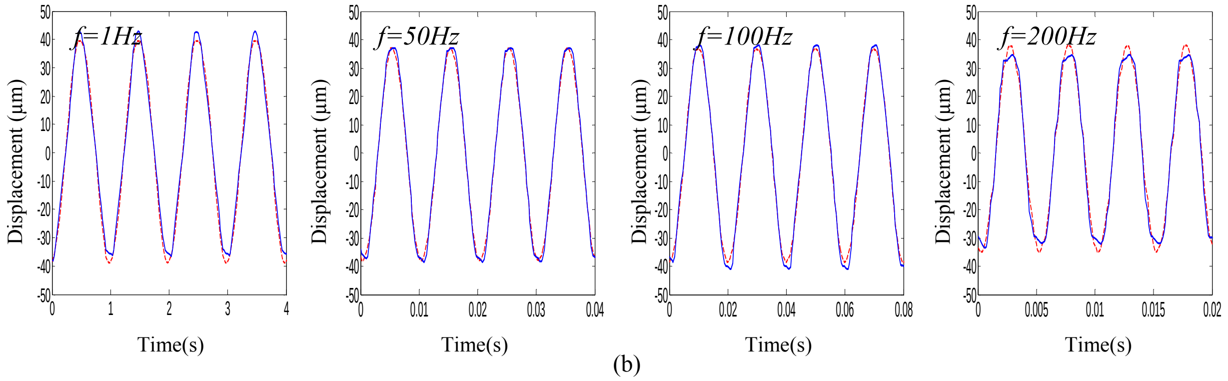


Note: Measured ▬ Model ▬

Figure 10 Time histories of displacement responses of the model and a magnetostrictive actuator at different input frequencies, (a) Rate-dependent play operator with linear envelop function, $s_1 = s_r = 1$ and (b) Rate-dependent play operator with non-linear envelop function, $s_1 = s_r = 3$ (see online version for colours)



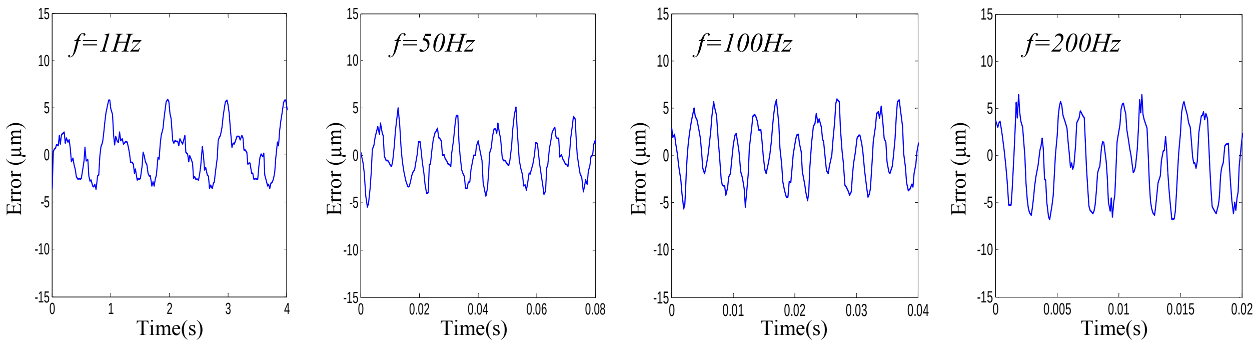
(a)



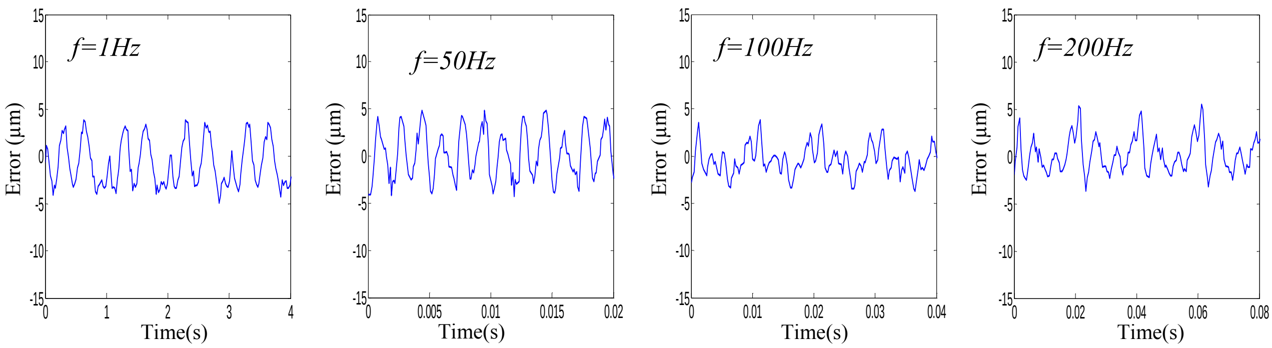
(b)

Note: Measured ______ Model - - - - - -

Figure 11 Time histories of errors between the model and measured displacement responses of the piezoceramic actuator at different input frequencies, (a) Rate-dependent play operator with linear envelop function, $s_1 = s_r = 1$ and (b) Rate-dependent play operator with non-linear envelop function, $s_1 = s_r = 3$ (see online version)



(a)



(b)

6 Conclusions

A generalised rate-dependent play operator together with a dynamic threshold function is proposed for formulating the Prandtl-Ishlinskii model capable of predicting symmetric and asymmetric rate-dependent hysteresis properties of smart actuators. By way of an example, it is shown that the generalised play operator permits for relaxation of the slopes of the major as well as minor hysteresis loops, and can thus characterise symmetric as well asymmetric input-output relations. The proposed operator also yields increase in the width of the hysteresis loop and decrease in the output magnitude as the rate of input is increased, which has been widely observed from the measured responses of different smart actuators. The effectiveness of the models is demonstrated by comparing the model responses with the measured symmetric and asymmetric hysteresis loops of the piezoceramic and magnetostrictive actuators, respectively, under different excitation frequencies. From the results, it can be concluded that the Prandtl-Ishlinskii model comprising the generalised rate-dependent play operator described by non-linear envelope function can provide better predictions of the symmetric as well as asymmetric hysteresis properties under different rates of inputs. Furthermore, the generalised model can also predict the rate-independent hysteresis characteristics reasonably well, as it is evident from the responses under low frequency inputs. The use of a higher order envelop functions of the rate-dependent play operator helps reduce the prediction error of the rate-dependent Prandtl-Ishlinskii model.

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