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RISK, INTEREST RATES AND THE FORWARD EXCHANGE *

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It is the purpose of this article to elaborate on the relationship between risk, the forward exchange, and the equilibrium interest rate that exists for any country engaging in foreign trade.[1] After analysis of several relevant equilibrium models, a framework for testing expectations on the future price of the British pound is developed, and estimates of various expectation parameters are derived.

I. Relationship of Foreign Exchange Prices and Interest Rates under Risk Neutrality

Let us assume that a particular country in question (henceforth, the domestic country) is small with respect to the rest of the world (henceforth, the foreign country). Let c_0 be the spot price of the foreign currency in terms of the domestic currency. Let $c_t{}^a$ be the anticipated spot price at time t. $c_t{}^a$ is a random variable if expectations are not held with certainty. Likewise, let $c_t{}^f$ be the forward price of the currency at time t. $c_t{}^f$ is the price traded in current markets and therefore is not a random variable. For interest rates, similar notation is used. Let $r_t{}^d$ be the present certain domestic stream of interest (interest rate) on a bond of period t where the interest is reinvested continually at the contracted rate. $r_0{}^d$ is therefore the present instantaneous domestic interest rate. We let r^f be the constant foreign interest rate for all durations.

Even in the simple case where investors are risk-neutral, i.e., equate only means of various investments, it can be shown that the forward price of the foreign currency is not an unbiased estimator

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1. The reader is also referred to the works of M. S. Feldstein, "Uncertainty and Forward Speculation," *Review of Economics and Statistics*, L (May 1968), 182–92, and Egon Sohmen, *Flexible Exchange Rates: Theory and Controversy* (Chicago: University of Chicago Press, 1961).

of the anticipated future price. Under circumstances of risk neutrality, investors must be indifferent between the yields from holding either domestic or foreign securities. Analytically, we can say that if E is the statistical expectation operator, then equating yields at the end of period t requires

(1) $$\exp(r_t^d t) = E(c_0 \cdot \exp(r^f t)/c_t^a) = \exp(r^f t) \cdot E(c_0/c_t^a).$$

r_t^d is therefore the "required" rate of return on domestic sure securities, given c_0, c_t^a, and r^f, for asset equilibrium to exist. Similarly, the forward price c_t^f must simply reflect the arbitrage condition

(2) $$\exp(r_t^d t) = \exp(r^f t) \cdot c_0/c_t^f.$$

If the random variable c_t^a has positive variance, then

(3) $$E(c_0/c_t^a) > c_0/E(c_t^a),$$

from which it follows from (1) and (2) that

(4) $$E(c_t^a) > c_t^f;$$

i.e., the expected value of the anticipated future spot price is greater than the forward price even though investors are risk-neutral.

It should be noted that, if we quoted the prices as domestic currency in terms of foreign currency, this seeming paradox would fail to materialize since $E(C_t^a/C_0) = E(C_t^a)/C_0$ for $C = 1/c$. However, since the natural numeraire is the unit of domestic currency, the preceding analysis is the proper formulation.

One can also derive the relationship between the equilibrium interest rate r_t^d and the expected value of the future spot price. Expanding $1/c_t^a$ around $1/E(c_t^a)$ by Taylor's series and taking expected values, we obtain the following approximation (let $c'_t = E(c_t^a)$):

(5) $$E(1/c_t^a) = (1 + \sigma_t^2/c'^2_t)/c'_t,$$

where σ_t^2 *is* the variance of c_t^a. Therefore, we have from (1) that

(6) $$r_t^d t = ln[(c_0/c'_t) \cdot (1 + \sigma_t^2/c'^2_t)] + r^f t,$$

or, assuming that σ_t^2/c'^2_t is small compared to 1,

(7) $$r_t^d = (ln(c_0/c'_t) + \sigma_t^2/c'^2_t)/t + r^f.$$

If the anticipated future spot price is held with certainty, then (7) reduces to

(7a) $$r_t^d = (ln(c_0/c_t^a))/t + r^f.$$

It can be seen that if the future expected price is equal to the spot price, then the domestic interest rate is higher than the foreign price by the coefficient of variation squared per unit time. For instance, if the standard deviation of the future spot price is 20 percent of the mean, then the domestic interest rate will be 0.04 higher than the foreign rate would have been if the expected future spot price were held with certainty.

II. An Application of the Model to Fixed Spot Rates

2.1. Theoretical Derivation

Let us formulate the solution of the interest rate relationships under the present regime of fixed international spot prices. In this case, an investor must anticipate both the magnitude of the future expected exchange price change and the time of its occurrence. Although the magnitude of the change can be certain, the time of its occurrence certainly cannot be since this implies an infinite rate of return on spot-forward arbitrage from the instant before to the instant after the parity change. In general, under a system of fixed spot prices (1) becomes

$$\exp(r_t^d t) = p(t)(c_0/c_1)\exp(r^f t) + (1-p(t))\exp(r^f t), \tag{8}$$

where c_1 is the expected (certain) devalued or revalued price and

$$p(t) = \int_0^t f(s)\,ds \tag{9}$$

is the cumulative distribution function of the density function $f(t)$, the time of the parity change. In other words, $p(t)$ is the probability that the exchange rate will have been changed by time t. Solving (8) for r_t^d, the required domestic interest rate, we have

$$r_t^d = ln\ (1+gp(t))/t + r^f, \tag{10}$$

where $g = (c_0 - c_1)/c_1$, the expected percentage change in the parity rate.

This required domestic interest rate is the return that must be offered investors if they are to be indifferent between holding foreign or domestic securities. In perfect markets, exogenous shifts of investors' expectations of future prices will *induce* changes in domestic interest rates to bring about this equilibrium. A fuller macroeconomic examination of this system would then specify the effect of these higher induced interest rates on spending, the price level, and hence the expected future spot price.

The forward price of the currency is easily derived to be

$$c_t^f = c_0/(1+gp(t)), \tag{11}$$

and the discount (premium) on a t period contract is

$$d(t) = c_0 gp(t)/(1+gp(t)). \tag{12}$$

This equation is very useful for the estimation of $p(t)$ since we know the discount or premium on the forward currency, and we can estimate the expected percentage parity change.

The nature of the speculators' distribution function $f(t)$ and the expected percentage parity change g are all that are needed to determine the equilibrium endogenous interest rate by (10). For instance, if the probability distribution is exponential, $f(t)=\theta e^{-\theta t}$, then

(13) $r_t^d - r^f = ln(1+g(1-e^{-\theta t}))/t,\ \theta > 0.$

In this case, $r_0^d - r^f = g\theta$, and $r_\infty^d = r^f$, with $\partial r_t^d/\partial_t < 0$, for all $t>0$. Note that the instantaneous interest rate differential between foreign and domestic interest rates is the expected capital gain divided by the mean expected time of devaluation. However, this is not always so. Applying de l'Hôpital's Rule to (10), we can see that in general

(14) $r_0^d - r^f = gf(0),$

which of course can be zero for many plausible probability distributions. One can see from (10), however, that $r_\infty^d = r^f$ for all probability distributions. This seemingly paradoxical result that the "consol" or perpetuity rate remains constant is due to our assumption that all interest from domestic bonds is instantaneously reinvested at the originally contracted domestic rate. If interest payments are reinvested at the foreign rate, the equilibrium consol rate can be shown to be $r^f(1+g)$. This definitional difference becomes negligible for finite term bonds.

In general, the sign of $\partial r_t^d/\partial t$ is indeterminate. As previously shown, if $f(0)=0$, then the interest rate differential is 0 for the instantaneous rate and consol rates. If the expected parity change is small, it can be shown that

(15) $\partial r_t^d/\partial t \gtreqless 0$ if and only if $\text{elas}(p(t)) \gtreqless 1$,

where elas $(p(t))$ is the elasticity of the cumulative distribution function with respect to time. Whenever $f(0)=0$, the elasticity of $p(t)$ at 0 is 1. Thus, for the exponential distribution discussed above, the slope of $\partial r_t^d/\partial t$ is 0 at 0 and negative for all $t>0$. For a uniform distribution from 0 to T, the elasticity of the cumulative distribution function is 1, and hence the interest rate is constant up to T. When $t>T$, the interest rate declines.

2.2. Application to 1969 British Exchange Crisis

Empirical work on currency crises is often complicated by massive official government forward exchange operations that distort the equilibrium values derived above from speculators' expectations.

Of course, if speculators' funds for necessary collateral are unlimited, such "twisting" of the forward rate would be impossible. However, lags and limited funds, among other imperfections, have been shown to be an important feature of short-term international capital markets.[2]

Despite these difficulties, a survey of the sterling crisis of May 1969 does reveal some interesting characteristics. In this crisis, the Bank of England did not support the forward pound, although it did give support to the spot exchange as required by IMF regulations. In the earlier crisis of November 1967, when the pound was in fact devalued, massive forward intervention by the Central Bank resulted in huge reserve losses for England after the devaluation, and the bank was determined not to allow the same situation to develop.[3] Figure I derives the implicit interest rate on one-, three-,

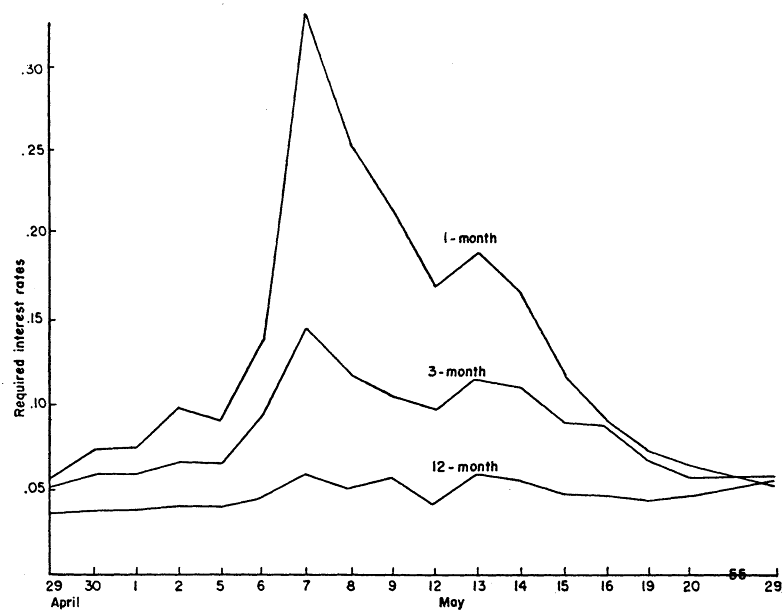

FIGURE I

and twelve-month contracts using (2), since the spot and forward prices are known. This interest rate did not in fact prevail in the English money market due to government restriction on capital

2. See L. H. Officer and T. D. Willett, "The Covered-Arbitrage Schedule: A Critical Survey of Recent Developments," *Journal of Money, Credit, and Banking,* II (May 1970), 247–57.
3. London *Financial Times,* var. ed., May 1969.

flows. However, sterling contracts were freely traded in Paris, and the prevailing interest rates there closely resembled those shown in Figure I.

One characteristic is immediately noticeable. Throughout most of the period the "equilibrium" interest rate was a declining function of contract length and therefore must satisfy conditions of (15). Note that at the height of the crisis (May 7), the interest rate gap widened to its maximum. One week forward sterling necessitated a 77 percent interest rate on May 7. The exponential distribution is therefore not an appropriate representation of the expectation function since the slope of the interest rate is 0 at $t=0$. The uniform distribution also cannot represent these expectations since it requires a nearly constant interest rate over the relevant interval as mentioned earlier. However, at the end of the crisis period, interest rates were nearly similar for all contracts, indicating that the uniform distribution would again become eligible.

In general one can say that the crisis probably did change the expectation structure or probability distribution. Once the height of the crisis passed, it seemed like the long contracts felt a strong lingering effect — nearly as strong as when the crisis peak occurred. In other words, there was a feeling that although this crisis was "weathered" and calm might reign for some short period of time, similar crises were likely to occur in the future before the long con-

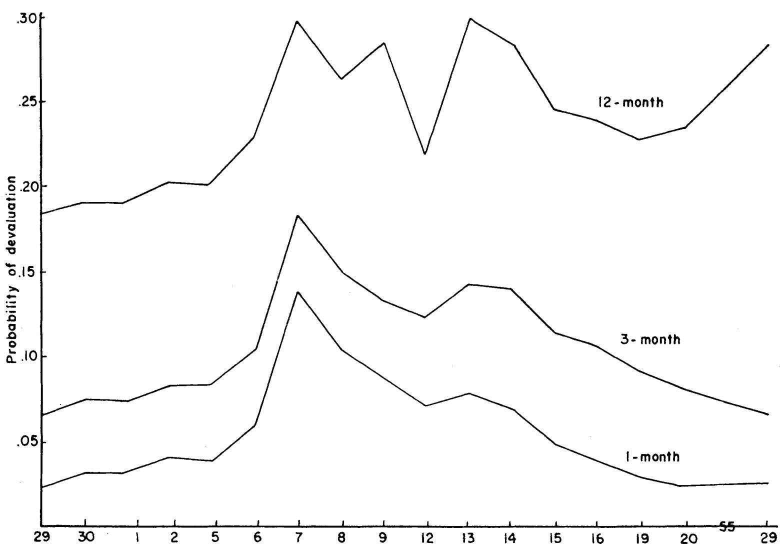

FIGURE II

tracts expired. It has indeed occurred that the British pound has experienced sharp acute crises regularly, followed by periods of relative calm in the foreign exchange markets.

Figure II graphs the cumulative probability function $p(t)$ derived by (12). Of course, knowledge of g is necessary, and I estimated that a reasonable value of the devalued pound was \$2.00 from the then spot price of \$2.40. This gives g a value of 0.20. In any case, different estimates of g would affect the level of $p(t)$ more than its shape relative to different contract lengths. In this figure, it is even more noticeable that the "fear" of devaluation did not diminish on the long contracts after the peak of the crisis passed. For the twelve-month contract, the probability remained above 23 percent for the entire period following the peak, while the one-month contract declined from 16 percent to below 2½ percent in less than ten trading days.

III. Conclusion

In the first section it was shown that, even for the special case of risk neutrality, the forward price of a currency does not bear a simple relationship to the expected future spot price. Fuller analysis would have to take into account not only the risk preferences (utility functions) of individuals, but the correlation of the price of foreign exchange with other assets in an economy.[4] Applications of this analysis in the present system of fixed international exchange rates may be important for the development of central bank policy. For instance, if the government believes that the implicit probability of devaluation of speculators calculated from (11) is too high, then official support will be not only desirable but also profitable (in an expected value sense) for the Central Bank. If the government actually believes that there is a greater probability than indicated by $p(t)$ that devaluation will occur (whether forced by speculators or not), support is not desirable insofar as it will often lead to windfall profits to speculators and a loss of reserves, as occurred in Britain in 1967.

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4. See H. G. Grubel, "Internationally Diversified Portfolios: Welfare Gains and Capital Flows," *American Economic Review*, LVIII (Dec. 1968), 1299–1314, and S. C. Tsiang, "The Theory of Forward Exchange and Effects of Government Intervention on the Forward Exchange Market," *International Monetary Fund Staff Papers*, April 1959, pp. 75–106.