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Research Article

Solution and Stability of a Mixed Type Additive, Quadratic, and Cubic Functional Equation

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We obtain the general solution and the generalized Hyers-Ulam-Rassias stability of the mixed type additive, quadratic, and cubic functional equation $f(x+2y) - f(x-2y) = 2(f(x+y) - f(x-y)) + 2f(3y) - 6f(2y) + 6f(y)$.

1. Introduction

The stability problem of functional equations originated from a question of Ulam [1] in 1940, concerning the stability of group homomorphisms. Let $(G_1, \cdot)$ be a group, and let $(G_2, *)$ be a metric group with the metric $d(\cdot,\cdot)$. Given $\epsilon > 0$, does there exist a $\delta > 0$, such that if a mapping $h : G_1 \to G_2$ satisfies the inequality $d(h(x \cdot y), h(x) * h(y)) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \epsilon$ for all $x \in G_1$? In other words, under what condition does there exist a homomorphism near an approximate homomorphism?

In 1941, Hyers [2] gave a first affirmative answer to the question of Ulam for Banach spaces. Let $f : E \to E'$ be a mapping between Banach spaces such that

$$\left\| f(x+y) - f(x) - f(y) \right\| \le \delta, \tag{1.1}$$

for all $x, y \in E$ and for some $\delta > 0$. Then there exists a unique additive mapping $T : E \to E'$ such that

$$\left\| f(x) - T(x) \right\| \le \delta, \tag{1.2}$$