

Day 6 Math 1 Unit 2

Arithmetic Sequences

Warm up: 10 minutes on
Project and/or braingenie.

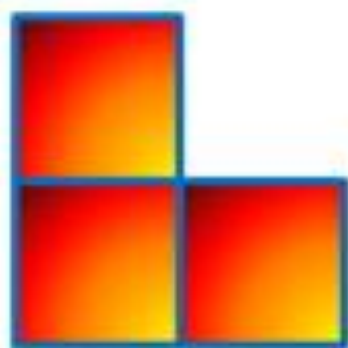


Diagram 1

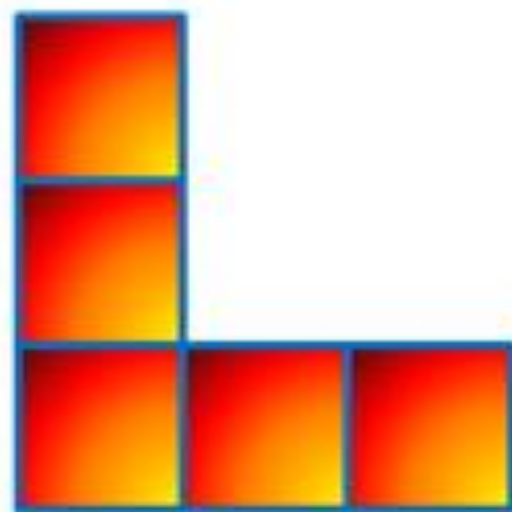


Diagram 2

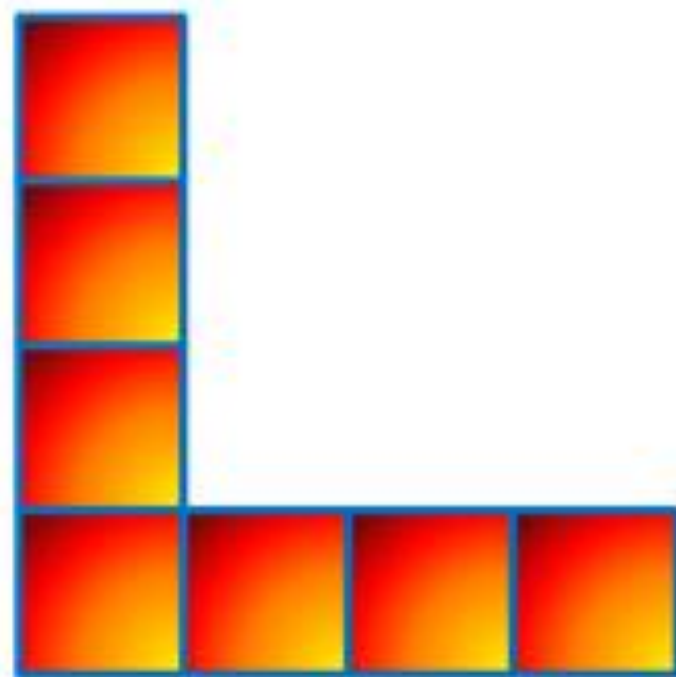
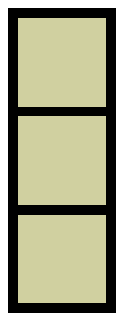


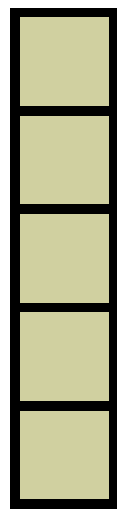
Diagram 3



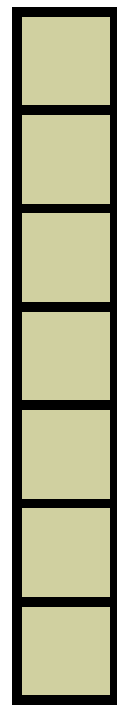
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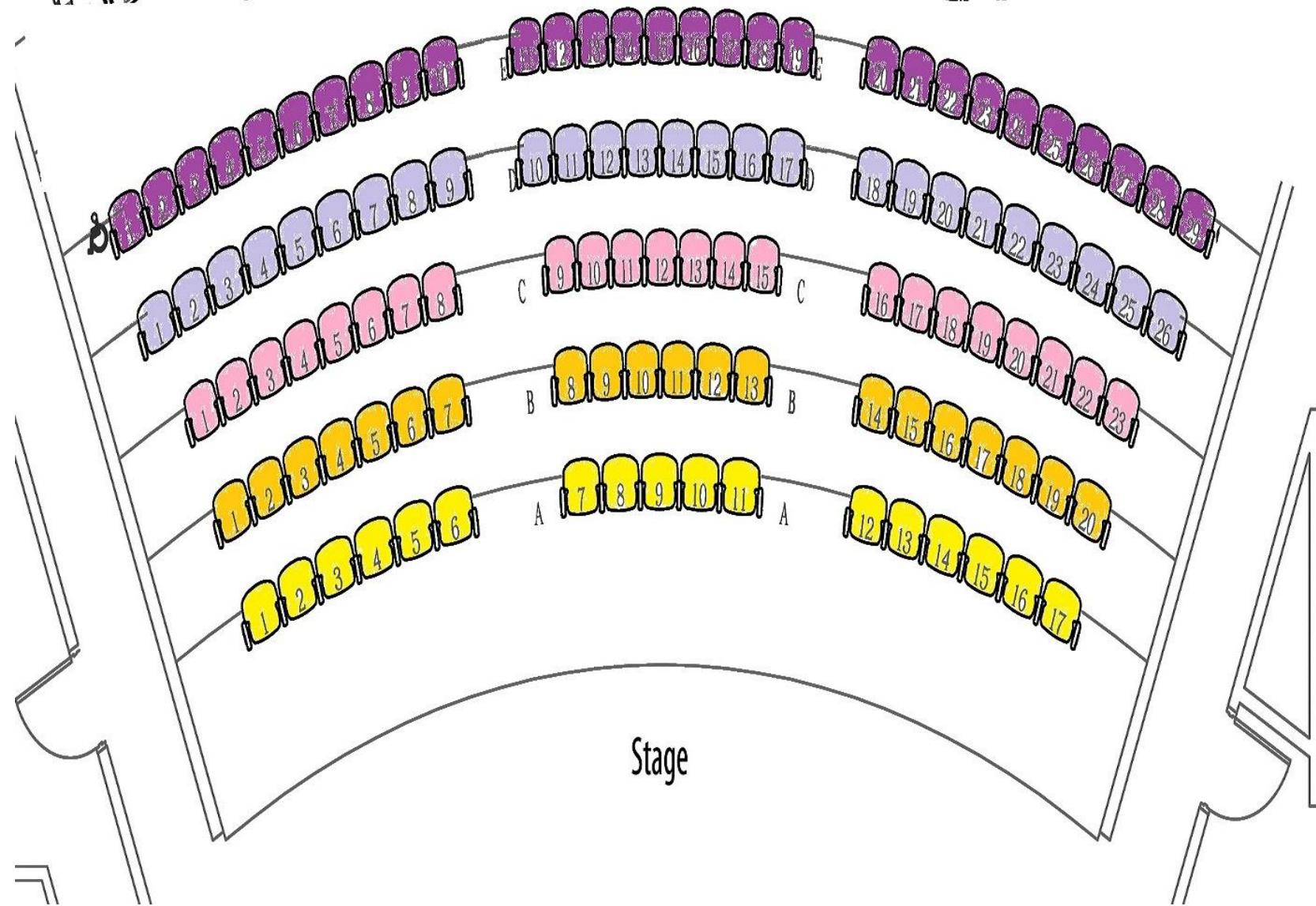
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Arithmetic Sequences

Definition

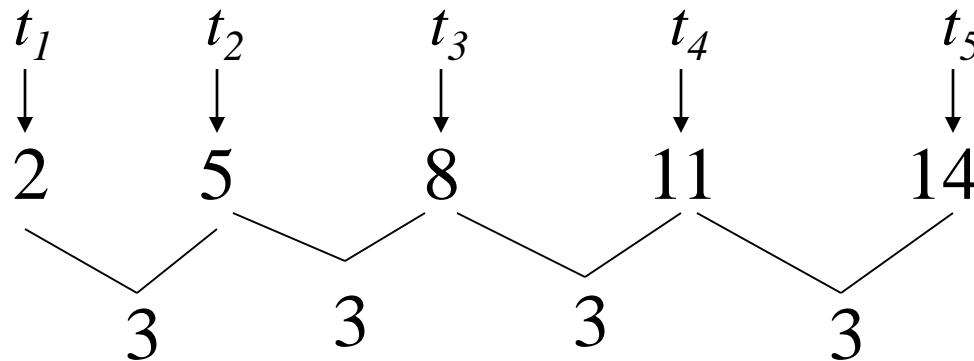
A sequence in which a constant (d) can be added to each term to get the next term is called an

Arithmetic Sequence.

The constant (d) is called the

Common Difference.

To find the common difference (d), subtract any term from one that follows it.



Examples:

Find the first term and the common difference of each arithmetic sequence.

1.) 4, 9, 14, 19, 24

First term (a): 4

Common difference (d): $a_2 - a_1 = 9 - 4 = 5$

2.) 34, 27, 20, 13, 6, -1, -8,

First term (a): 34

Common difference (d): -7

BE CAREFUL: ALWAYS CHECK TO MAKE SURE THE DIFFERENCE IS THE SAME BETWEEN EACH TERM !

Now you try!

Find the first term and the common difference of each of these arithmetic sequences.

a) $1, -4, -9, -14, \dots$

b) $11, 22, 36, 49, \dots$

Answers with solutions

a) 1, -4, -9, -14,

$a = 1$ and

$$d = a_2 - a_1 = -4 - 1 = -5$$

b) 11, 23, 35, 47,

NO Common Difference so NOT
arithmetic!

How do we write Arithmetic Sequences?

- List of numbers

Ex: $\{1, 6, 11, 16, 21, 26, 31, 36, \dots\}$

- Recursive formula

Ex: $a_1 = -4$
 $a_n = a_{n-1} + 5$

- Explicit formula

Ex: $a_n = a_1 + (n - 1)d$

Recursive Formula

Tells you what to do:

$$b_1 = 3$$

$$b_{n+1} = b_n + 5$$

First term =

Next term = what we have now + 5

$$a_1 = 25$$

$$a_{n+1} = a_n - 3$$

First = 25

Next = now + 5

Recursive formulas are often written as “Now and Next”

- Now-next

$$\text{FIRST} = \#$$

$$\text{NEXT} = \text{NOW} + d$$

- Formal definition:

$$a_1 = \#$$

$$a_n = a_{n-1} + d$$

Practice

Write the now/next formula:
(recursive formula)

Given this recursive formula, write
the first five terms of the
sequence.

Practice – Recursive Formula

What is the 10th term of this sequence?

5, 11, 17, 23, ...

Explicit Formula

- What happens when we want the 100th term of a sequence?

The first term of an arithmetic sequence is (a). We add (d) to get the next term.

3, 7, 11, 15, We know $a = 3$ and $d = 4$

$$a_1 = 3$$

$$a_2 = a_1 + d = 3 + 4 = 7$$

$$a_3 = a_1 + d + d = a_1 + 2d = 3 + 2(4) = 11$$

$$a_4 = a_1 + d + d + d = a_1 + 3d = 3 + 3(4) = 15$$

$$a_{10} = a_1 + d + \dots + d + d = a_1 + \underline{\quad}d = 3 + \underline{\quad}(4) = 15$$

The nth term of an arithmetic sequence is given by:

$$a_n = a_1 + (n - 1) d$$

The last # in the
sequence/or the #
you are looking for

First
term

The position
the term is in

The common
difference

Examples: Find the 14th term of the arithmetic sequence
4, 7, 10, 13,

$$t_n = a + (n - 1) d$$

$$t_{14} = 4 + (14 - 1) 3$$

$$= 4 + (13) 3$$

$$= 4 + 39$$

$$= 43$$

You are
looking for
the term!

**The 14th term in this sequence
is the number 43!**

Now you try!

Find the 10th and 25th term given the following information. Make sure to derive the general formula first and then list what you have been provided.

a) 1, 7, 13, 19

b) The first term is 3 and the common difference is -21

c) The second term is 8 and the common difference is 3

Answers with solutions

a) 1, 7, 13, 19
 $a = 1$ and $d = a_2 - a_1 = 7 - 1 = 6$

$$t_n = a + (n-1)d = 1 + (n-1)6 = 1 + 6n - 6 \quad \text{So } t_n = \mathbf{6n-5}$$

$$t_{10} = 6(10) - 5 = 55$$

$$t_{25} = 6(25) - 5 = 145$$

b) The first term is 3 and the common difference is -21

$$a = \mathbf{3} \quad \text{and} \quad d = \mathbf{-21}$$

$$t_n = a + (n-1)d = 3 + (n-1)(-21) = 3 - 21n + 21 \quad \text{So } t_n = \mathbf{24-21n}$$

$$t_{10} = 24 - 21(10) = -186 \quad t_{25} = 24 - 21(25) = -501$$

c) The second term is 8 and the common difference is 3

$$a = 8 - 3 = \mathbf{5} \quad \text{and} \quad d = \mathbf{3}$$

$$t_n = a + (n-1)d = 5 + (n-1)3 = 5 + 3n - 3 \quad \text{So } t_n = \mathbf{3n+2}$$

$$t_{10} = 3(10) + 2 = 32 \quad t_{25} = 3(25) + 2 = 77$$

<https://www.youtube.com/watch?v=JtsyP0tnVRY>

https://www.youtube.com/watch?v=_cooC3yG_p0

Examples: Find the 14th term of the arithmetic sequence with first term of 5 and the common difference is -6.

$$a = 5 \text{ and } d = -6$$

$$t_n = a + (n - 1) d$$

$$t_{14} = 5 + (14 - 1) - 6$$

$$= 5 + (13) * -6$$

$$= 5 + -78$$

$$= -73$$

You are looking for *the term!* List which variables from the general term are provided!

The 14th term in this sequence is the number -73!

Examples: In the arithmetic sequence 4, 7, 10, 13, ..., which term has a value of 301?

$$t_n = a + (n - 1)d$$

$$301 = 4 + (n - 1)3$$

$$301 = 4 + 3n - 3$$

$$301 = 1 + 3n$$

$$300 = 3n$$

$$100 = n$$


You are
looking
for n !

**The 100th term in this
sequence is 301!**

Examples:

In an arithmetic sequence, term 10 is 33 and term 22 is -3. What are the first four terms of the sequence?

$t_{10} = 33$
 $t_{22} = -3$



Use what you know!

$$t_n = a + (n - 1)d$$

For term 10: $33 = a + 9d$

$$t_n = a + (n - 1)d$$

For term 22: $-3 = a + 21d$



HMMM! Two equations you can solve!

SOLVE:

$$33 = a + 9d$$
$$-3 = a + 21d$$

By elimination

$$-36 = 12d$$
$$-3 = d$$

SOLVE:

$$33 = a + 9d$$
$$33 = a + 9(-3)$$
$$33 = a - 27$$
$$60 = a$$

The sequence is 60, 57, 54, 51,