Dhirubhai Ambani
Institute of Information and Communication Technology

1 Mathematical Aptitude and Reasoning
All candidates must attempt Part-1

1. If the arithmetic mean of a and b is

$$\frac{a^n + b^n}{a + b}$$

then

   (a) $n = 0$
   (b) $n = 1$
   (c) $n = 2$
   (d) $n = -1$
   (e) none of these

2. What is the value of the following limit?

$$\lim_{x \longrightarrow 4} \ \lfloor x - 4 \rfloor + \mid x + 4 \mid .$$

where $\lfloor x \rfloor$ is the greatest integer function.

   (a) 0
   (b) 4
   (c) 2
   (d) 8
   (e) none of these

3. Which of the following statement is true for the function $f(x) = \lfloor x + 9 \rfloor$?

   (a) The function $f(x)$ is continuous at $x = -9$ but not differentiable at $x = -9$
   (b) The function $f(x)$ is continuous at $x = -9$ but differentiable at $x = -9$
   (c) The function $f(x)$ is continuous and differentiable at $x = -9$
   (d) The function $f(x)$ is neither continuous nor differentiable at $x = -9$
   (e) none of these

4. The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is

(a) convergent
(b) divergent for $n = -2$
(c) divergent
(d) divergent for $n = 2$
(e) none of these

5. $\frac{dy}{dx} + y = 0$ has solution

(a) $y = e^{-x} + c$
(b) $x = e^{y} + c$
(c) $y = e^{x} + c$
(d) $x = e^{-y} + c$
(e) none of the above

6. The propositional logic formula $p \Rightarrow q$ is equivalent to:

(a) $p \wedge q$
(b) $\overline{p} \vee \overline{q}$
(c) $\overline{p} \vee q$
(d) $\overline{p} \wedge \overline{q}$
(e) none of the above

7. Consider the **First Order Logic** formulae $\mathcal{F}_1 = \exists x \forall y(\ldots)$ and $\mathcal{F}_2 = \forall y \exists x(\ldots)$. Which of the following is true?

(a) $\mathcal{F}_1 \Rightarrow \mathcal{F}_2$
(b) $\mathcal{F}_1 \Leftrightarrow \mathcal{F}_2$
(c) $\mathcal{F}_2 \Rightarrow \mathcal{F}_1$
(d) It depends on the propositional logic form represented in the bracket as $(\ldots)$
(e) none of the above

8. Suppose two unbiased (six-sided) dice are rolled. The outcome represented as the total of the values shown by the two dice is a random variable and can take values in the range $\{2, \ldots, 12\}$. Which of these outcomes has highest probability of occurrence?

(a) 9
(b) 12
(c) 3
(d) 7
(e) none of the above

9. Rank of the matrix A given below is

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$$

   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) none of the above

10. The Eigen values of the matrix A given below are

$$A = \begin{pmatrix} 2 & 8 & 16 \\ 0 & 3 & 9 \\ 0 & 0 & 4 \end{pmatrix}$$

   (a) 2,2,2
   (b) 2,3,4
   (c) 0,1,2
   (d) 3,4,0
   (e) none of the above

11. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ is

   (a) convergent
   (b) absolutely divergent
   (c) divergent
   (d) divergent for $n = 2$
   (e) none of these

12. The differential equation $(\frac{dy}{dx})^2 + y^2 = 0$ has

   (a) only one real solution
   (b) no real solution
   (c) two real solutions
   (d) three complex solutions
   (e) none of these

13. If set $A = \{2, 4, \ldots, 2n\}$ then power set of A has

   (a) 2^{n-1} elements
   (b) 2^n elements
   (c) 2^{n+1} elements

(d) 2^{n-5}

(e) none of these

14. A function f is said to be one-to-one and onto if

    (a) Whenever $f(x) = f(y) \Rightarrow x = y$
    (b) Whenever $f(x) = f(y) \Rightarrow x \neq y$
    (c) Whenever $f(x) \neq f(y) \Rightarrow x \neq y$
    (d) Whenever f is injective
    (e) none of these

15. A function f is said to be one-to-one if

    (a) Image set is super set of codomain
    (b) Image set is sub set of codomain
    (c) Image set is dense
    (d) Image set is equal to codomain
    (e) none of these

2 Mathematics, Physics, Computer Science and IT, Electronics and Communication, Bioinformatics and Biomedical Engineering

All candidates must attempt atleast one section

A. Mathematics and Physics

1. Give an example of a real-valued function which is discontinuous everywhere on $\mathbb{R}$. Justify your answer.

2. How many groups are there of order 4? Justify your answer by writing their Cayley Tables.

3. For a prime number p define a $p \times p$ matrix V_p as

$$V_p = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 2^2 & \dots & 2^{p-1} \\ 1 & 3 & 3^2 & \dots & 3^{p-1} \\ 1 & 4 & 4^2 & \dots & 4^{p-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & (p-1) & (p-1)^2 & \dots & (p-1)^{p-1} \\ 1 & p & p^2 & \dots & p^{p-1} \end{bmatrix}.$$

   Find the rank of V_p.

4. Show that the complex valued function $f(z) = |z|^2$ is differentiable only at $z = 0$. Will it be analytic at origin? Justify your answer.

5. Solve the following differential equation.

$$\frac{d^2y}{dx^2} + 9y = \sin 2x + \cos(\sqrt{3}x).$$

6. A free particle of mass m is given an initial velocity v_0. If the dissipative force acting on the particle is given by $-\gamma v^2$ (v is the speed of the particle and γ is the dissipation constant), calculate and sketch v as a function of time. Compare your result with the one obtained when the dissipative force varies linearly with the velocity ($-\gamma v$).

7. The electric field of an electromagnetic wave propagating through vacuum is given by

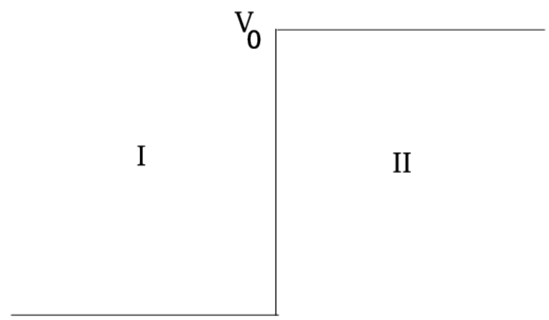
$$\vec{E}(\vec{r}, t) = E_0 \cos(4\pi x - 3\pi y - \omega t)$$

   (a) Calculate the wave vector $\vec{k}$.

(b) If this field is acting on a point charge q moving with velocity $v = v_0\hat{x}$ at $t = 0$, calculate the instantaneous Lorentz force acting on the point charge.

8. A particle with mass m moves in a potential given by

$$V(x) = \begin{cases} V_0, & x > 0, \\ 0, & otherwise \end{cases}$$



(a) What is the general solution for the wave functions in regions **I & II**, if the energy of the particle $E < V_0$.

(b) Calculate and make a plot of the reflection coefficient for $0 < E < 2V_0$.

9. Consider a Young's 2-slit experiment in which the separation between the two slits is a.

(a) Calculate and sketch the interference pattern obtained on a screen kept far away from the slits when they are illuminated by monochromatic light.

(b) Explain the effect on the interference pattern when the slit separation is increased by means of a representative sketch.

(c) What would happen to the interference fringes in (a) when a thin film of refractive index μ and thickness t is placed in the path of one of the interfering beams?

B. Computer Science and Information Technology

1. Consider a stack $S[1, \ldots, n]$ of n elements stored in an array implementation with the array running from index 1 to index n (top of the stack). Write a pseudocode for reversing the elements t of the stack such that $i \leq t \leq j$. The other elements of the stack must retain their positions at **the end** of the process. You are allowed to use **two** extra stacks S_1 and S_2 each the same size as S. What is the time complexity of the procedure as a function of n, i, j? Would a similar problem take **more or less** time if we were given the input in a queue, and allowed to use two extra queues?

2. Assume we have a connected simple undirected graph G with a specific adjacency list representation. Suppose running BFS and DFS on this graph starting from the same source vertex, results in the *same tree* (BFS Tree and DFS Tree respectively). Additionally, the *order* in which the various vertices of the graph are visited by the two algorithms is the *same*. What can you conclude about the graph we started with and also the vertex from which BFS and DFS were initiated?

3. Consider two random permutations of the integers $\{1, \ldots, n\}$ and you need to determine which of the two appears first in the lexicographic ordering of all the permutations. Will you look for the first discrepancy starting from the left end, starting from the right end or any arbitrary position where the two permutations differ? Explain your decision.

4. Consider a DFA (deterministic finite automaton) $\mathcal{A}$ with 8 states. Let the language accepted by it be represented by $L(\mathcal{A})$. The automaton $\mathcal{A}$ accepts a particular word of length 12 characters. Given this information, determine the number of words in $L(\mathcal{A})$.

5. Draw a DFA (deterministic finite state automaton) which accepts the language over the alphabet $\Sigma = \{a, b\}$ in which each word contains an odd number of a's and an odd number of b's.

6. Using counting semaphores, solve the readers-writers problem with following attributes: If both Readers and Writers are present, they must alternate with each having exclusive access to the resource.

7. Explain the basic attributes of a memory subsystem that supports demand paging.

8. Briefly outline the relative merits and demerits of ALOHA and Slotted ALOHA protocols. Give example where the use of one protocol is preferable over the others.

9. A group of 10 class-C address blocks are required to serve an organization. Using a concrete example, describe a CIDR (classless inter-domain routing) scheme for efficient routing.

C. Electronics and Communication

1. Medium $1(z \geq 0)$ has a dielectric constant of 2 and a conductivity of 40 $\mu S/m$. Medium $2(z \leq 0)$ has a dielectric constant of 5 and a conductivity of 50 nS/m. If $\vec{J_2}$ has a magnitude of 2 A/m^2, and $\theta_2 = 60$ with the normal to the interface, compute $\vec{J_1}$ and θ_1. What is the surface charge density at the interface? $\left(\epsilon_0 = \frac{10^9}{36\pi}\mathrm{F/m}\right)$

2. (a) Using the concept of displacement current, explain how the differential form of Ampere's law in static magnetic field is modified for time-varying electromagnetic field.

   (b) A lossless transmission line is 80 cm long and operates at a frequency of 600 MHz. The line parameters are L = 0.25 H/m and C = 100 pF/m. Find the characteristic impedance, the phase constant, the velocity on the line, and the input impedance for load .

3. Consider the first-order difference equation $y[n] - \frac{1}{2}y[n-1] = x[n]$. Assuming the condition of *initial rest* (i.e., if $x[n] = 0$ for $n < n_0$, then system output $y[n] = 0$ for $n < n_0$), find the impulse response of the system.

4. Using zero locations of FIR filters, justify clearly as to why we can't design FIR filters in analog domain.

5. Let $s(t)$ be a general angle modulated signal given by $s(t) = A_c \cos[\theta_i(t)] = A_c \cos[\omega_c t + \phi(t)]$. It is given that when the modulating signal is $m(t) = \cos(\omega_m t)$, $s(t)$ has the instantaneous frequency given by $f_i(t) = f_c + 2\pi k f_m^2 \cos(\omega_m t)$, where k is a suitable constant.

   (a) Find the expression for $\theta_i(t)$ and $s(t)$.

   (b) Now, if the modulating signal is $m(t)$, what could be the general expression for $\theta_i(t)$ and $s(t)$.

6. Suppose that X takes on one of the values $0, 1, 2$ and there exists a constant c such that $P(X = i) = cP(X = i - 1), i = 1, 2$. What would be the value of c such that $E(X) = 10/7$?

7. A random variable X has the following probability function

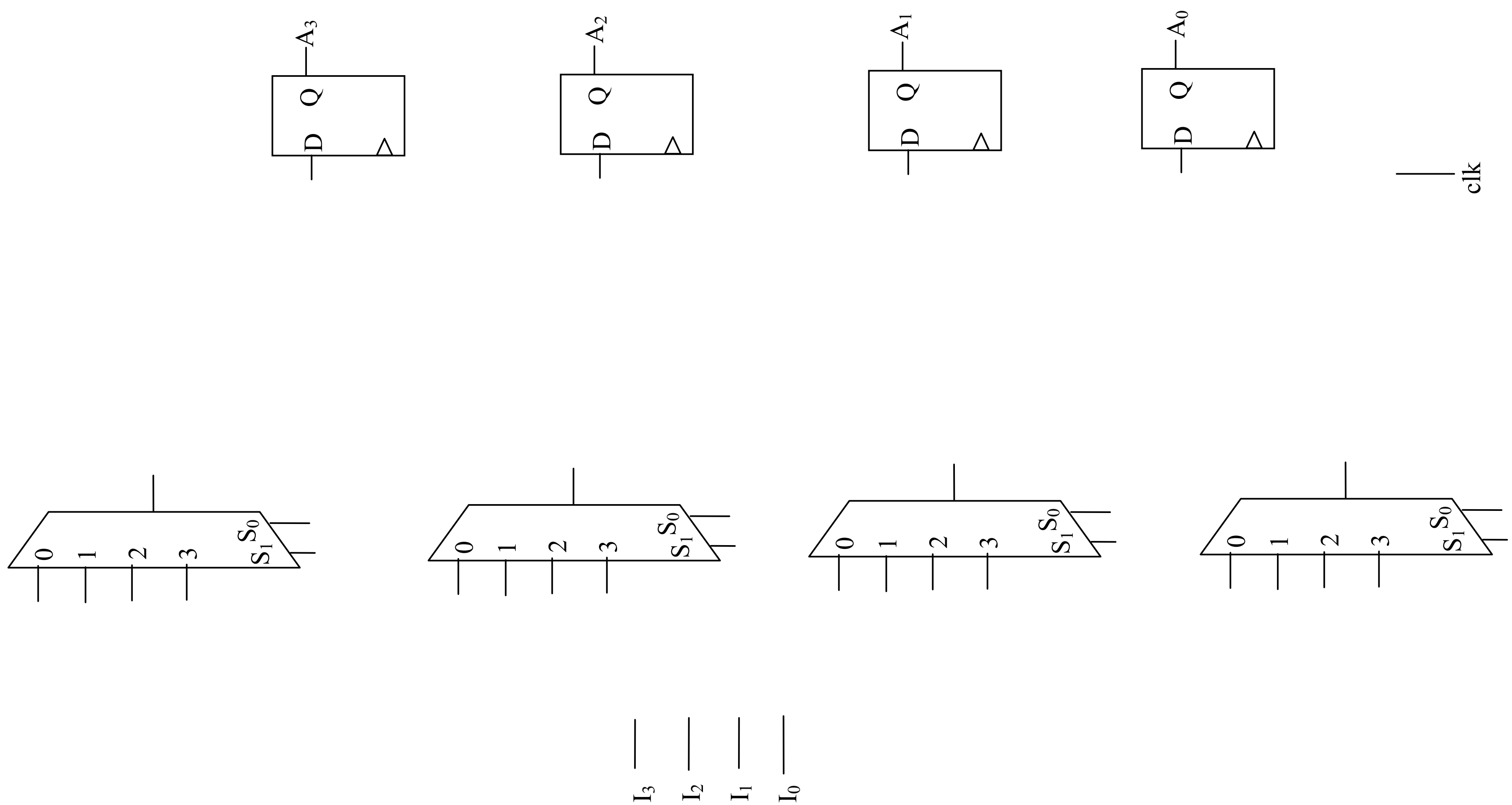
x	0	1	2	3	4	4	6	7
P(X=x)	0	k	2 k	2 k	3 k	k^2	$2k^2$	$k^2 + 7k$

Find i) The value of k ii) E(X)

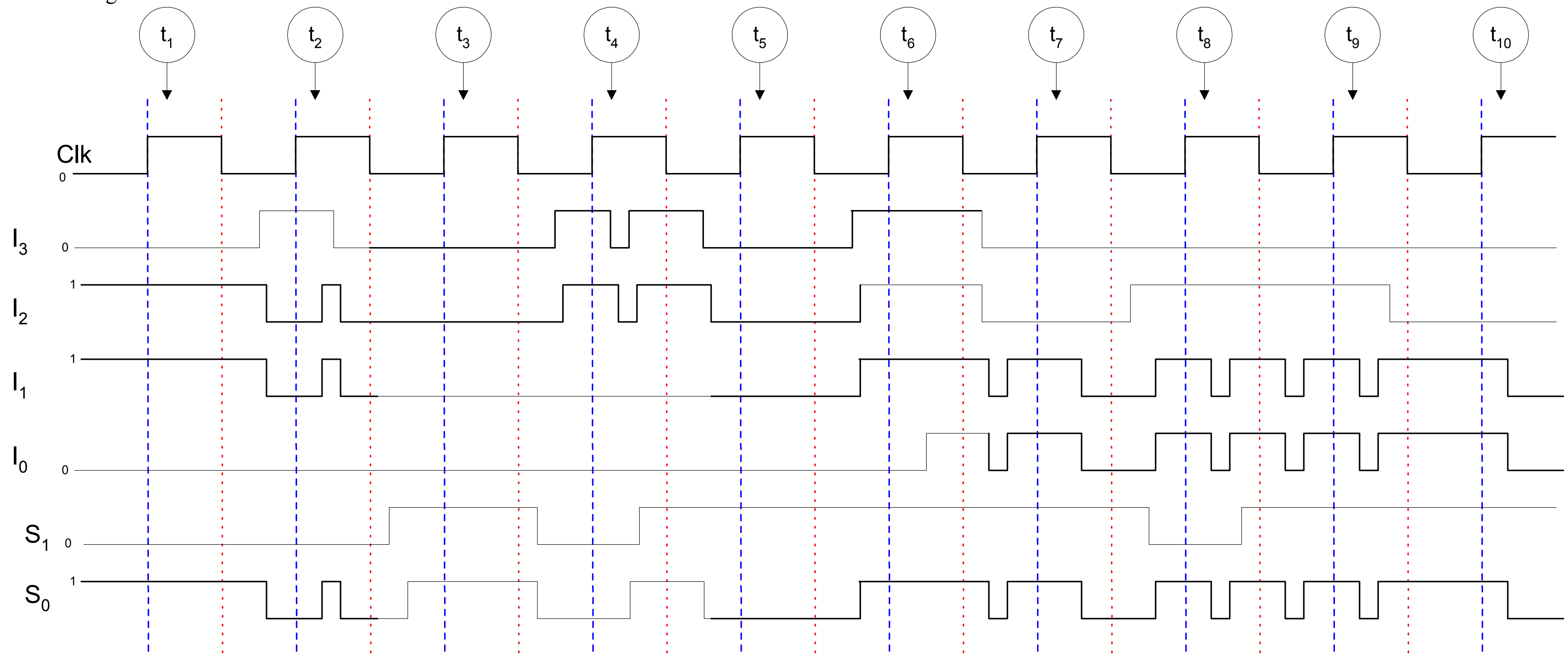
8. Answer the following questions related to a shift-register:

8.1 Complete the following schematic. This schematic should implement a shift-register with the functionality as given in the table below. This shift-register has four FFs (outputs of these FFs are A_3 to A_0), the external inputs are I_3, to I_0, and two select lines S_1 and S_0 (for the four *4-to-1 MUX*). For multi-bit variables A, I, and S, assume that subscript-0 is the LSB bit.

$\{S_1,S_0\}$	Function
00	Load back the present output of the FFs (@ clk-edge)
01	Load external inputs (@ clk-edge)
10	Load back the inverted version of the present output of the FFs (@ clk-edge)
11	Shift (and rotate) left (@ clk-edge)



8.2 For the above shift-register schematic, the various input signal waveforms are shown below. Based on those waveforms, fill out the appropriate binary (0 or 1) values in the following table.



	A_3	A_2	A_1	A_0
At time t_1				
At time t_2				
At time t_3				
At time t_4				
At time t_5				
At time t_6				
At time t_7				
At time t_8				
At time t_9				
At time t_{10}				

9. For the circuit given in Fig.1, find the branch currents I_1, I_2, and I_3, at node A.

Answer: I_1 =______________ I_2 = ________________ I_3 = _____________

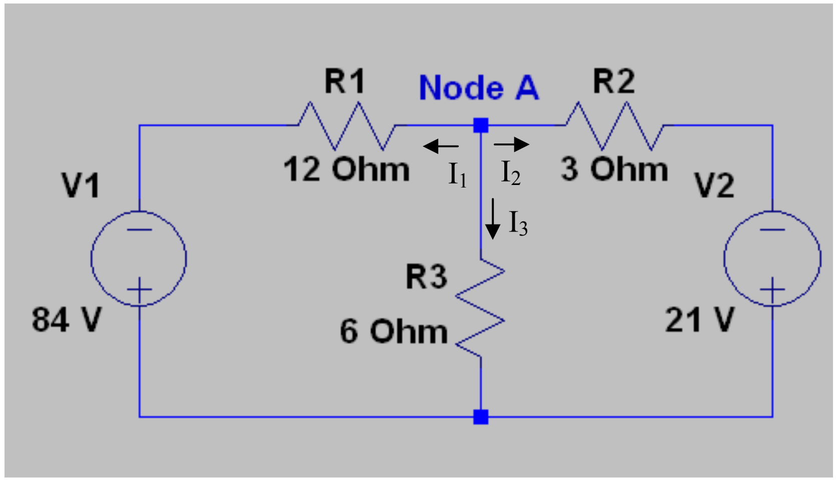


Fig.1

10. In Fig. 2, a circuit based on an *NPN* BJT is given. As shown, V_{in}= 10 V, and V_{CC} = 10 V. Assume, $V_{BE\text{-}sat}$ = 0.7 V, $V_{CE\text{-}sat}$ = 0.1 V, and h_{FE} = 30. Now answer the following questions:

10.1 Find the values of base current I_B and collector current I_C.

Answer: I_B = _______

I_C = _______

10.2 Find $I_{B\text{-sat}}$. Is the BJT operating in Saturation?

Answer: $I_{B\text{-sat}}$ = _______

Saturation? (YES / NO)

10.3 Find the maximum value of R_B for which the transistor will be in saturation?

Answer: R_B = ______

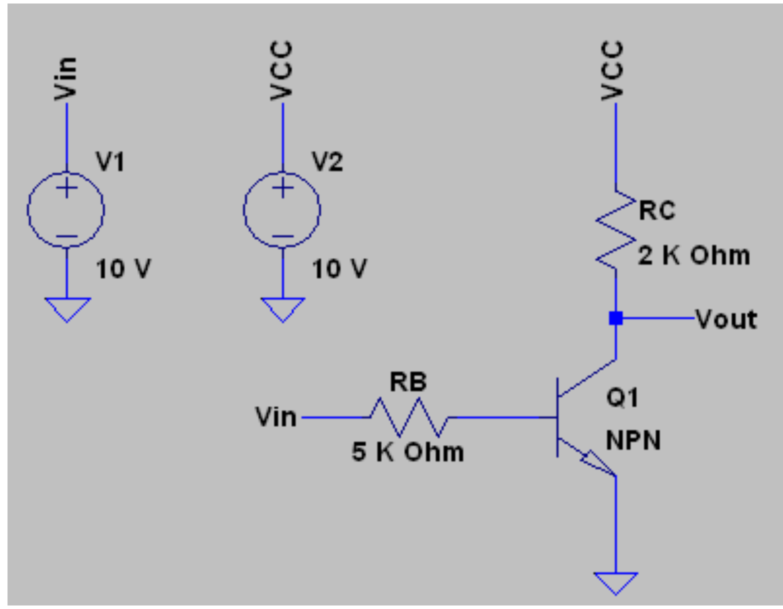


Fig. 2.

11. Draw the schematic diagram for a **4-to-16 decoder with Enable** using four **2-to-4 decoder with Enable**, one **2-to-4 decoder without Enable,** and some gates. The **4-to-16 decoder with Enable** has inputs [A_3 A_2 A_1 A_0] and enable E_n, and outputs [D_0 to D_{15}]. Use the symbol (shown in Fig. 3) for the **2-to-4 decoder with Enable** and **2-to-4 decoder without Enable**. For multi-bit variables A, D, and S, assume that subscript-0 is the LSB bit. Clearly label all your inputs and outputs in the entire schematic.

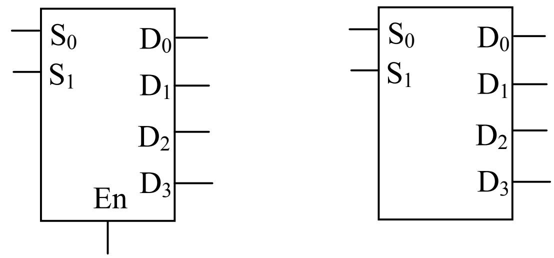


Fig. 3

D. Bioinformatics and Biomedical Engineering

1. Suppose you downloaded the genome of a species from NCBI. It is given below:

```
LOCUS       DQ823385                 675 bp    mRNA    linear   VRL 08-APR-2009
DEFINITION  Influenza A virus (A/swine/Hangzhou/1/2006(H9N2)) nonstructural
            protein 1 (NS1) mRNA, complete cds.
ACCESSION   DQ823385
VERSION     DQ823385.1  GI:110809858
KEYWORDS    .
SOURCE      Influenza A virus (A/swine/Hangzhou/1/2006(H9N2))
  ORGANISM  Influenza A virus (A/swine/Hangzhou/1/2006(H9N2))
            Viruses; ssRNA negative-strand viruses; Orthomyxoviridae;
            Influenzavirus A.
REFERENCE   1  (bases 1 to 675)
  AUTHORS   Wang,F.-K., Yuan,X.-F., Wang,Y.-C., Zhang,C., Xu,L.-H. and
            Liu,S.-D.
  TITLE     Cloning, prokaryotic expression and antigenicity analysis of NS1
            gene of H9N2 swine influenza virus
  JOURNAL   Unpublished
REFERENCE   2  (bases 1 to 675)
  AUTHORS   Wang,F.-K., Yuan,X.-F., Wang,Y.-C., Zhang,C., Xu,L.-H. and
            Liu,S.-D.
  TITLE     Direct Submission
  JOURNAL   Submitted (26-JUN-2006) Veterinary Medicine, Shandong Agricultural
            University, Daizong Road No.61, Tai'an, Shandong 271018, China
FEATURES             Location/Qualifiers
     source          1..675
                     /organism="Influenza A virus
                     (A/swine/Hangzhou/1/2006(H9N2))"
                     /mol_type="mRNA"
                     /strain="A/swine/Hangzhou/1/2006"
                     /serotype="H9N2"
                     /db_xref="taxon:395639"
     gene            1..675
                     /gene="NS1"
     CDS             1..675
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                     /codon_start=1
                     /product="nonstructural protein 1"
                     /protein_id="ABG91296.1"
                     /db_xref="GI:110809859"
                     /translation="MDSNTVSSFQVDCFLWHVRKRFADQELGDAPFLDRLRRDQKSLR
                     GRSSTLGLDIRTATREGKHIVERILEEESDEALKTTIASVPSPRYLTDMTLEEMSRDW
                     LMLIPKQKVTGSLCIRMDQAIVDKNIILKANFSVIFNRLEALILLRAFTEEGAIVGEI
                     SPLPSLPGHTDKDVKNAIGVLIGGFEWNDNTVRVSEALQRFAWRSSDEDGRPPLSPLE
                    HHHHHH"
```

```
ORIGIN
        1 atggattcca atactgtgtc aagcttccag gtagactgct ttctttggca tgtccgcaaa
       61 cggtttgcag accaagaact gggtgatgcc ccatttctag accggcttcg ccgggatcag
      121 aagtccctga gaggaagaag cagcactctt ggtctggaca tcagaaccgc cactcgtgaa
      181 ggaaagcata tcgtggagcg gattctggag gaagagtcag atgaagcact taaaacgact
      241 attgcttcag tgccatctcc acgctatcta actgacatga ctcttgaaga aatgtcaaga
      301 gattggttaa tgctcattcc caaacagaaa gtgacagggt ccctttgcat tagaatggac
      361 caagcaatag tagacaaaaa catcatattg aaagcaaatt tcagtgtgat tttcaatcga
      421 ctggaagctc taatactact tagagctttt actgaagaag gagcaatagt gggcgaaatc
      481 tcaccattac cttctcttcc aggacatact gataaggatg tcaaaaatgc aattggggtc
      541 ctcatcggag gatttgaatg gaatgataac acagttcgag tctctgaagc tctacagaga
      601 ttcgcttgga gaagcagcga tgaggatggg agacctccac tctctccact cgagcaccac
      661 caccaccacc actga
//
```

Explain as much as you can (explain each term) from the above data about this species.

2. Find the ORF (Open Reading Frame) from the following $mRNA$ sequence:

$$5' \text{ UUGGAGUAAAAUAUCAUGAAUCAA } 3'$$

3. The edit distance $d_e(\mathbf{s}, \mathbf{t})$ between two strings $\mathbf{s}$ and $\mathbf{t}$ of lengths m and n can be computed by the following recurrence relation

$$\begin{aligned} d[i,o] &= i; 0 \leq i \leq m \\ d[0,j] &= j; 1 \leq j \leq n \\ d[i,j] &= \min\{d[i-1,j]+1, d[i,j-1]+1, d[i-1,j-1]+t(i,j)\}; 1 \leq j \leq n, 1 \leq i \leq m \end{aligned}$$

where $t(i,j) = 1,$ if $\mathbf{s}_i \neq \mathbf{s}_j$ and has a value 0 if they are equal.
Explain the reasoning why we can do that ? Using above arguments and the table business compute the edit distance between *saturday* and *sunday*.

4. State and prove Fourier Slice Theorem for 2-dimension.

5. Find the Radon transform for the function

$$f(x,y) = \begin{cases} 1 & \text{if } (x,y) = (0,0) \\ 0 & \text{if } (x,y) \neq (0,0). \end{cases}$$