each of the coexisting networks can only precode for their own channel. Evidently, in the absence of a cognitive relay the two systems can not access the same resources for transmission simultaneously.

B. Conventional ZF Relay Precoding

Since the interference between the channels within each of the systems is removed by the precoders of each BS, the aim of the relay is to cancel the cross-interference between the two systems, denoted by the red arrows in Fig. 2. A precoder designed according to the ZF criterion at the relay aims to zero the mean square error (MSE)

$$\varepsilon = E\left\{\left\|\mathbf{R}\cdot\mathbf{b}+\mathbf{Q}\cdot\mathbf{x}_r-\mathbf{b}\right\|^2\right\} \tag{9}$$

Note that, since a common scaling factor is determined after the precdoing optimization, the scaling factor *f* that appears in (7) is ignored in (9). It can easily be proven that the ZF solution to the minimization of ε is

$$\hat{\mathbf{x}}_r = \mathbf{Q}^{-1}\cdot\left(\mathbf{I}-\mathbf{R}\right)\cdot\mathbf{b} \tag{10}$$

and exists if $\mathbf{Q}$ is positive semi-definite. The relay then calculates the global scaling factor f and transmits $\mathbf{x}_r = f\cdot\hat{\mathbf{x}}_r$.

C. Scaling of Transmitted Power

Since the precoding matrices tend to increase the power of the transmitted signal a scaling factor f is required to normalize the power at the output of the antennas. For the interference elimination by the relay to be possible it is required that f is common for the three simultaneous transmissions (PBS, SBS, Relay). If the f is different for each of the transmissions then (10) does not zero the MSE of (9). Consequently, the relay that has complete information of all transmissions periodically needs to calculate f and forward it to the BSs. For practicality reasons this needs to be a data independent scaling factor that ensures average transmit power normalization and is updated only when the channel coefficients change. Based on the power of the precoded symbols on (3),(10) and assuming, without loss of generality, a restriction of the total transmit power to unity f can be calculated as

$$f = tr\left[\left(\mathbf{I}-\mathbf{R}\right)^H\cdot\left(\mathbf{Q}^{-1}\right)^H\cdot\mathbf{Q}^{-1}\cdot\left(\mathbf{I}-\mathbf{R}\right)+\left(\mathbf{H}_p\cdot\mathbf{H}_p^*\right)^{-1}+\left(\mathbf{H}_s\cdot\mathbf{H}_s^*\right)^{-1}\right]^{-\frac{1}{2}} \tag{11}$$

III. CONSTRUCTIVE INTERFERENCE AND PROPOSED PRECODING

A. Constructive Interference

The main idea behind this work is the fact that in some cases and for some symbol combinations, the interference between the transmitted symbols can add to the desired symbol energy, leading to a higher received power. By allowing this power at the receiver the instantaneous signal to noise ratio (SNR) is increased and performance is improved. This is done by exploiting 'green' signal energy that already exists in the communication system and therefore the performance benefits are yielded without the need to increase the transmitted per symbol power. On the other hand, by completely removing interference as done conventionally, this important source of green energy is lost. The above generic concept is thoroughly explained in [10] for the conventional downlink. For this reason only a brief illustration of its key elements is included in this paper.

The generic interference caused to antenna u from the transmission of antenna k is expressed as

$$I_{k,u} = b_k\cdot\rho_{k,u} \tag{12}$$

where b_k is the interfering symbol and $\rho_{k,u}$ is the correlation element in matrix $\mathbf{R}$ shown above. This interference is constructive to the desired symbol energy when it moves the received symbol away from the decision thresholds in the modulation constellation. The example of quadrature phase shift keying (QPSK), assumed in the following simulations, is shown in Fig. 3, where possible received symbols are shown in the QPSK constellation. Constructive interference is denoted by green and destructive interference is denoted by red colour. The mathematical criterion for constructive interference is shown in [11] for QPSK as

$$D_R = \mathrm{Re}\left(b_k\cdot\rho_{k,u}\right)\cdot\mathrm{Re}\left(b_u\right)>0 \;\&\; D_I = \mathrm{Im}\left(b_k\cdot\rho_{k,u}\right)\cdot\mathrm{Im}\left(b_u\right)>0 \tag{13}$$

where Re(x) and Im(x) denote the real and imaginary parts of the complex number x. It can be seen in Fig.3 that when interference is constructive the constellation is effectively spread and detection is more tolerant to noise. By completely removing all interference by means of ZF precoding this benefit is lost. The following sections propose two adaptive precoding techniques that exploit the constructive interference energy.

B. Selective Relay Precoding (SRP)

This technique comprises of an adaptation of [11], proposed for the downlink, to the cognitive radio scenario investigated here. From the above analysis and (13) it is clear that some of the correlation elements instantaneously benefit the received symbols' energy. Moreover, using the knowledge of data and channels at the relay transmitter the interference can be predicted and characterized. It is therefore intuitive to design a precoder that retains the correlation elements in $\mathbf{R}$ that yield constructive interference and eliminates the elements that result in destructive interference. To achieve this, the

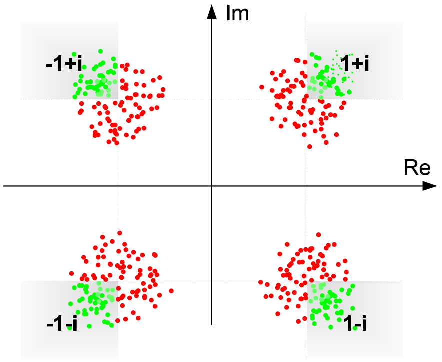

Fig. 3: The constructive (green) - destructive (red) interference sectors in the QPSK constellation