Appendix A

Abitur Exams from Central City

Abitur exam 1991
Basic Mathematics

Suggestion 1

Problem 1

Given is the function

$$f:\ x \longrightarrow f(x) = \text{arc}\ \cos(2x^2 - 1);\ x \in D_f.$$

Its graph is referred to as G_f. D_f is the domain for the function f.

1. 1. a) Determine the maximum possible domain for f.

b) Examine G_f for symmetry.

1. 2. a) Determine whether f is constant at $x=0$.

b) Calculate $f'(x)$ and determine the area in which f descends strictly monotonously.

c) Determine whether the limit $\lim_{h->0} f'(0+h)$ exists for $h>0$.

d) Compare the result of (c) with $\lim_{h->0} f'(0+h)$. Determine if the function f is differentiable at $x=0$.

1. 3. Show that in D_f the following are valid:

$$f(x) = 2 \cdot \text{arc}\cos x + c_1 \quad \text{for } x > 0$$

and

$$f(x) = -2 \cdot \text{arc}\cos x + c_2 \quad \text{for } x < 0$$

Give evidence to support this and determine the constants C_1 and C_2.

1. 4. Find the equation for the tangent that intersects the curve at the point with the x-coordinate 0.5.

1. 5. Sketch the graph G_f based on the results of these problems. Set one unit = 2cm.

Problem 2

In a Cartesian coordinate system, the lines

$$l_1: \quad \xi = \begin{pmatrix} 0 \\ 2 \\ 4.5 \end{pmatrix} + \sigma \begin{pmatrix} 3 \\ 1 \\ -1.5 \end{pmatrix} \quad \text{with } \sigma \in \mathbf{R}$$

and

$$l_2: \quad \xi = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} + \mu \cdot \begin{pmatrix} 3 \\ -1 \\ -0.5 \end{pmatrix} \quad \text{with } \mu \in \mathbf{R}$$

are given.

2. 1. a) Show that l_1 and l_2 are skewed.

b) Determine the intersection point D of the line l_2 through the x_1-x_2 plane.

2. 2. The plane P_1 is parallel to l_1 and contains l_2 .

a) Find an equation for the normal to P_1 . (Possible result: $2x_1 + 3x_2 + 6x_3 - 18 + 0$)

b) Find the coordinates of the points at which plane P_1 intersects the x and y-axes.

c) Create a sketch in which you show the positions of the lines l_1 and l_2 and the plane P_1 .

2. 3. The plane P_2 is perpendicular to P_1 and contains the line l_1.

a) Find the equation for the normal to P_2.

b) What is the equation for the line of intersection s between P_1 and P_2 ?

2. 4. What is the distance between P_1 and l_1 ?

Work time: 3 hours

Materials allowed: mathematical formula collection and calculator

Abitur exam 1993 Advanced Mathematics Examiner:

Suggestion I

Problem 1:

Given: the family of functions $f(x) = \frac{sx}{e^{tx^2}}$ with $s, t \in \mathbf{R}$

a) Show that $f''(x) = -2stx\,(3 - 2tx^2)\,e^{-tx^2}$, and that all graphs in the group of curves are point symmetric to the origin.

Determine, by using l'Hôpital's rule, the horizontal asymptotes in terms of *s* and *t*.

b) Choose *s* and *t* so that the inflectional tangent at $x = 1$ has a slope of $-\frac{8}{\sqrt{e^3}}$.

Choose for parts (c) through (e): $s = 4$ and $t = 3/2$.

c) Determine the points of intersection with the y- and x-axes as well as the maximum, minimum and inflection points. Draw the graph in the area $-5 \quad x \quad 5$. (Origin in the center; 2cm = 1 unit)

d) Determine the equation of the inflectional tangent in the first quadrant. Where does this line intersect the x-axis?

e) Determine the area under the curve.

f) Given $s = 4$ and t is variable.

Show that the tangents of all inflection points in the first quadrant have the same slope:

$$-8 \cdot e^{-3/2}$$

On what curve do all inflection points lie? (Find the equation!) (1st quadrant!)

Determine whether all other inflection points lie on this line as well.

Draw this curve in the coordinate system of part (c).

Determine the equation of the inflectional tangent in terms of t.

For what value t^* does the inflectional tangent intersect the x-axis at the point S* (4.5 / 0) ?

Draw the corresponding graph in the coordinate system from part (c).

[to check, $t^* = 1/6$]

g) Another line goes through the point S*(4.5 / 0), intersecting the same graph ($t = t^*$).

Show that the x value of the corresponding point of intersection x_I fulfills the equation:

$$2x_I^3 - 9x_I^2 + 27 = 0 \quad .$$

Determine the point of intersection using the knowledge that the x value of the inflection point from part (f) also fulfills this equation.

Draw both tangents in the coordinate system from part (c) .

Do these tangents cross at a right angle?

Problem 2: Given the local vectors

$$\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}; \quad \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}; \quad \mathbf{c_t} = \begin{pmatrix} -5t \\ 6t+7 \\ -t-1 \end{pmatrix} \text{ with } t \in \mathbf{R} \text{ and } \mathbf{q} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}.$$

a) Can $\mathbf{q}$ be expressed using 2 of the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c_t}$?

If so, give their corresponding linear combination.

What follows from this for the coplanar relationship between 3 of the given vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c_t}$, and $\mathbf{q}$?

Express $\mathbf{q}$ using all 3 vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c_t}$.

Are all vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c_t}$ linearly independent? If so, for what values of t ?

b) Find the triple scalar products $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{q}$ and $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c_t}$.

What is the relationship between the vectors?

How can the results be interpreted geometrically?

c) Determine the area of the geometric figure formed by the points 0 (= origin), A, and B together with

I) the point Q

II) the group of points C_t (in terms of t).

d) Determine an equation for a line that is normal to the plane P created by the points O, A, and B. [to check: $x+y+z = 0$]

Give a parameter equation for the line l, on which all points C_t lie.

What is the relationship between l and P ? (Give indicative sizes!)

e) Determine the equation of the group of spheres S_t, whose center points all lie on the line $l': x = \begin{pmatrix} 0 \\ -5 \\ 1 \end{pmatrix} + t \begin{pmatrix} 5 \\ -6 \\ 1 \end{pmatrix}$

and for which the plane P is the tangential plane.

Which sphere S* of this group touches the plane P in T(3 / -1 / ?) ?

What is the distance between the two intersection points of the line l´ with the sphere S* ?

f) Given F_t is the point of intersection of the line AB and a perpindicular line containing C_t. Determine the local vector $\mathbf{f_t}$ in terms of t. For what value t' do the points A, B, C_t, make an isosceles triangle with the base AB ?
In this case, where does the intersection point F´ lie?

Allowed: nonprogrammable calculator
mathematic formula collection Pohlmann or Hornschuh

Subject: chemistry Examiner:

Suggestion 1

The acid-base properties of organic compounds can be applied to the preservation of foodstuffs. The acidity of acids used for preservation and the pH value of the foodstuff decide which substance is best suited to a given foodstuff.

Explain the principal and the causes of acidity based on Brønsted-Lowry's definition.

What effects influence the acidity of a substance?

Derive the acid constants pK_a for the example of propanoic acid.

Table 1:

a) Acidity of propanoic and hydroxypropanoic acids

Acids	pK_a
propanoic acid (propionic acid)	4.87
2-hydroxypropanoic acid (lactic acid)	3.08
3-hydroxypropanoic acid	4.51

b) Acidity of benzoic and hydrozybenzoic acids

Acids	pK_{a1}	pK_{a2}
benzoic acid	4.19	—
m-hydroxybenzoic acid	4.06	9.92
p-hydroxybenzoic acid (PHB)	4.48	9.32
o-hydroxybenzoic acid (salicylic acid)	2.97	13.40

What is the relationship between the pK_a value and “acid strength”?

Explain, using the example of p-hydroxybenzoic acid, that two pK_a values are given for aromatic hydroxy acids, whereas only one is given for the aliphatic acids.

Sodium benzoate is used as a preservative for ketchup, mayonnaise, etc. But of course, only undissociated acids have an anti-microbial effect. What does the benzoic acid molecule / benzoate ion ratio depend on? Use reaction equations in answering the question.

Compare the pK_a values of the given propanoic acids and explain the differences.

As is shown by the pK_a values, m-hydroxybenzoic acid is a stronger acid than benzoic acid, but p-hydroxybenzoic acid is weaker. How do you explain this apparent contradiction?

Salicylic acid is significantly stronger than benzoic acid. Find an explanation for this, paying special attention to the structure of salicylic acid, and taking into account that the 2nd pK_a value is unusually high.