$A_{11}^{-1} = \dfrac{1}{4*1-3*1} * \begin{pmatrix} 4 & -3 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ -1 & 1 \end{pmatrix}$

$A_{22}^{-1} = \dfrac{1}{2*2-5*1} \begin{pmatrix} 2 & -1 \\ -5 & 2 \end{pmatrix} = -1* \begin{pmatrix} 2 & -1 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 5 & -2 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 4 & -3 & 23 & -10 \\ -1 & 1 & -7 & 3 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 5 & -2 \end{pmatrix}$

$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 4 & -3 & 23 & -10 \\ -1 & 1 & -7 & 3 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 5 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 1* \begin{pmatrix} 4 \\ -1 \\ 0 \\ 0 \end{pmatrix} + 1* \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + 1* \begin{pmatrix} 23 \\ -7 \\ -2 \\ 5 \end{pmatrix} + 1* \begin{pmatrix} -10 \\ 3 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 4-3+23-10 \\ -1+1-7+3 \\ 0+0-2+1 \\ 0+0+5-2 \end{pmatrix} = \begin{pmatrix} 14 \\ -4 \\ -1 \\ 3 \end{pmatrix}$

Solution :

$A = \begin{pmatrix} A_{11} & I_2 \\ 0 & A_{22} \end{pmatrix}$

avec $A_{11} = \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix}$ $A_{22} = \begin{pmatrix} 2 & 1 \\ 5 & 2 \end{pmatrix}$

$\begin{pmatrix} 1 & 3 & 1 & 0 \\ 1 & 4 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 5 & 2 \end{pmatrix}$

≈ $\text{ligne}_2 \leftarrow \text{ligne}_2 - \text{ligne}_1$

$\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 5 & 2 \end{pmatrix}$

≈ $\text{ligne}_3 \leftarrow \text{ligne}_3 / 2$

$\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 5 & 2 \end{pmatrix}$

≈ $\text{ligne}_4 \leftarrow \text{ligne}_4 - 5*\text{ligne}_3$

$\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & -1/2 \end{pmatrix}$

≈ $\text{ligne}_3 \leftarrow \text{ligne}_3 + \text{ligne}_4$

$\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1/2 \end{pmatrix}$

$v_1 - 2v_2 = 7* \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} - 2* \begin{pmatrix} 5 \\ -4 \\ -7 \end{pmatrix} = \begin{pmatrix} h=-3 \\ 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 5 & -3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

≈ ligne$_1$ ← ligne$_1$ – 5*ligne$_2$

$$\begin{pmatrix} \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \\ \text{[illegible]} & \text{[illegible]} & -\text{[illegible]} & \text{[illegible]} \\ \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \end{pmatrix}$$

On a donc $x_{\text{[illegible]}}\begin{pmatrix} \text{[illegible]} \\ -\text{[illegible]} \\ -\text{[illegible]} \end{pmatrix} + x_{\text{[illegible]}}\begin{pmatrix} \text{[illegible]} \\ -\text{[illegible]} \\ -\text{[illegible]} \end{pmatrix} + x_{\text{[illegible]}}\begin{pmatrix} h \\ \text{[illegible]} \\ \text{[illegible]} \end{pmatrix} = \begin{pmatrix} \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \end{pmatrix}$, ce qui correspond à la matrice $M = \begin{pmatrix} \text{[illegible]} & \text{[illegible]} & h & \text{[illegible]} \\ -\text{[illegible]} & -\text{[illegible]} & \text{[illegible]} & \text{[illegible]} \\ -\text{[illegible]} & -\text{[illegible]} & \text{[illegible]} & \text{[illegible]} \end{pmatrix}$

$$X = \begin{pmatrix} x_{\text{[illegible]}} \\ x_{\text{[illegible]}} \\ x_{\text{[illegible]}} \\ x_{\text{[illegible]}} \\ x_{\text{[illegible]}} \\ x_{\text{[illegible]}} \end{pmatrix} = \begin{pmatrix} \text{[illegible]} \\ x_{\text{[illegible]}} \\ x_{\text{[illegible]}} \\ -\text{[illegible]} \\ x_{\text{[illegible]}} \\ \text{[illegible]} \end{pmatrix} = \begin{pmatrix} \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \\ -\text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \end{pmatrix} + x_{\text{[illegible]}}*\begin{pmatrix} -\text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \end{pmatrix} + x_{\text{[illegible]}}*\begin{pmatrix} \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \end{pmatrix} + x_{\text{[illegible]}}*\begin{pmatrix} -\text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \\ -\text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \end{pmatrix}$$

$$A = \begin{pmatrix} \text{[illegible]} & -\text{[illegible]} & \text{[illegible]} \\ \text{[illegible]} & -\text{[illegible]} & \text{[illegible]} \\ -\text{[illegible]} & \text{[illegible]} & h^{\text{[illegible]}}-\text{[illegible]} \end{pmatrix} \qquad B = \begin{pmatrix} \text{[illegible]} \\ -\text{[illegible]} \\ h-\text{[illegible]} \end{pmatrix}$$

Ces deux matrices donnent la matrice augmentée suivante : $M = \begin{pmatrix} \text{[illegible]} & -\text{[illegible]} & \text{[illegible]} & \text{[illegible]} \\ \text{[illegible]} & -\text{[illegible]} & \text{[illegible]} & -\text{[illegible]} \\ -\text{[illegible]} & \text{[illegible]} & h^{\text{[illegible]}}-\text{[illegible]} & h-\text{[illegible]} \end{pmatrix}$

Procédons à la réduction des lignes :

$$\begin{pmatrix} \text{[illegible]} & -\text{[illegible]} & \text{[illegible]} & \text{[illegible]} \\ \text{[illegible]} & -\text{[illegible]} & \text{[illegible]} & -\text{[illegible]} \\ -\text{[illegible]} & \text{[illegible]} & h^{\text{[illegible]}}-\text{[illegible]} & h-\text{[illegible]} \end{pmatrix}$$

≈ L2 ← L2 – 3*L1

$$\begin{pmatrix} \text{[illegible]} & -\text{[illegible]} & \text{[illegible]} & \text{[illegible]} \\ \text{[illegible]} & \text{[illegible]} & -\text{[illegible]} & -\text{[illegible]} \\ -\text{[illegible]} & \text{[illegible]} & h^{\text{[illegible]}}-\text{[illegible]} & h-\text{[illegible]} \end{pmatrix}$$