Geometric Computer Vision

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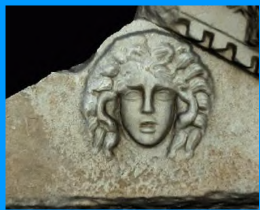
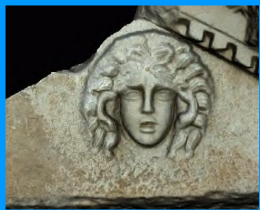
Fall 2009

http://www.inf.ethz.ch/personal/pomarc/courses/gcv/



Geometric Computer Vision course schedule
(tentative)

	Lecture	Exercise
Sept 16	Introduction	-
Sept 23	Geometry & Camera model	Camera calibration
Sept 30	Single View Metrology (Changchang Wu)	Measuring in images
Oct. 7	Feature Tracking/Matching	Correspondence computation
Oct. 14	Epipolar Geometry	F-matrix computation
Oct. 21	Shape-from-Silhouettes	Visual-hull computation
Oct. 28	Multi-view stereo matching	Project proposals
Nov. 4	Structure from motion and visual SLAM	Papers
Nov. 11	Multi-view geometry and self-calibration	Papers
Nov. 18	Shape-from-X	Papers
Nov. 25	Structured light and active range sensing	Papers
Dec. 2	3D modeling, registration and range/depth fusion (Christopher Zach?)	Papers
Dec. 9	Appearance modeling and image-based rendering	Papers
Dec. 16	Final project presentations	Final project presentations

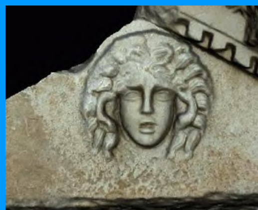


Projective Geometry and Camera model

Class 2

points, lines, planes
conics and quadrics
transformations
camera model

Read tutorial chapter 2 and 3.1
http://www.cs.unc.edu/~marc/tutorial/



Homogeneous coordinates

Homogeneous representation of 2D points and lines

$$ax + by + c = 0 \qquad\qquad (a,b,c)^{\mathsf{T}}(x,y,1) = 0$$

The point x lies on the line l if and only if

$$\mathrm{l}^{\mathsf{T}}\mathrm{x} = 0$$

Note that scale is unimportant for incidence relation

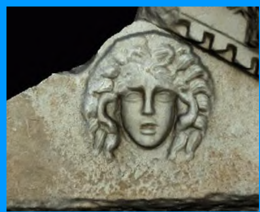
$$(a,b,c)^{\mathsf{T}} \sim k(a,b,c)^{\mathsf{T}}, \forall k \neq 0 \qquad (x,y,1)^{\mathsf{T}} \sim k(x,y,1)^{\mathsf{T}}, \forall k \neq 0$$

equivalence class of vectors, any vector is representative

Set of all equivalence classes in $\mathbf{R}^3 - (0,0,0)^{\mathsf{T}}$ forms $\mathbf{P}^2$

Homogeneous coordinates $(x_1, x_2, x_3)^{\mathsf{T}}$ but only 2DOF

Inhomogeneous coordinates $(x, y)^{\mathsf{T}}$



Points from lines and vice-versa

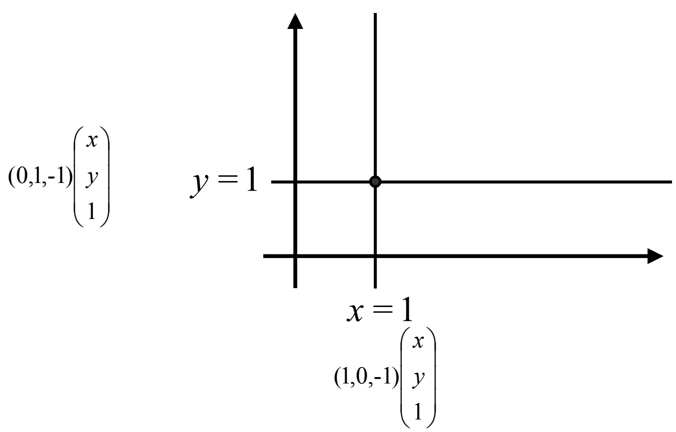
Intersections of lines

The intersection of two lines l and l' is $\mathrm{x} = \mathrm{l} \times \mathrm{l}'$

Line joining two points

The line through two points x and x' is $\mathrm{l} = \mathrm{x} \times \mathrm{x}'$

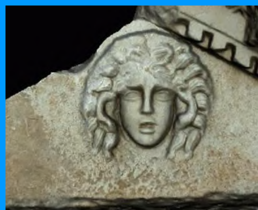
Example



Note:

$$\mathrm{x} \times \mathrm{x}' = [\mathrm{x}]_\times \mathrm{x}'$$

with $[\mathrm{x}]_\times = \begin{bmatrix} 0 & z & -y \\ -z & 0 & x \\ y & -x & 0 \end{bmatrix}$

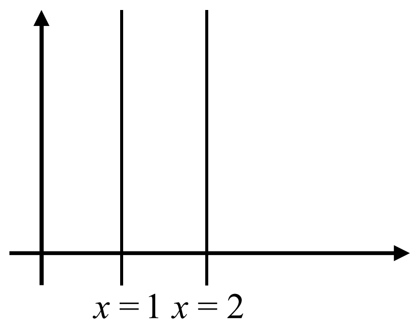


Ideal points and the line at infinity

Intersections of parallel lines

$$\mathbf{l} = (a, b, c)^{\mathsf{T}} \text{ and } \mathbf{l}' = (a, b, c')^{\mathsf{T}} \qquad \mathbf{l} \times \mathbf{l}' = (b, -a, 0)^{\mathsf{T}}$$

Example



$(b, \text{-}a)$ tangent vector

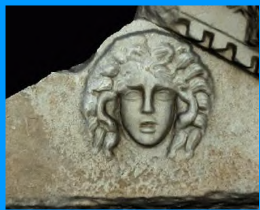
(a, b) normal direction

Ideal points $(x_1, x_2, 0)^{\mathsf{T}}$

Line at infinity $\mathbf{l}_\infty = (0, 0, 1)^{\mathsf{T}}$

$$\mathbf{P}^2 = \mathbf{R}^2 \cup \mathbf{l}_\infty$$

Note that in **P**^{2} there is no distinction between ideal points and others



3D points and planes

Homogeneous representation of 3D points and planes

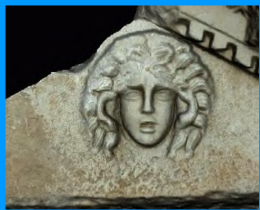
$$\pi_1 X_1 + \pi_2 X_2 + \pi_3 X_3 + \pi_4 X_4 = 0$$

The point X lies on the plane π if and only if

$$\pi^{\mathsf{T}} \mathrm{X} = 0$$

The plane π goes through the point X if and only if

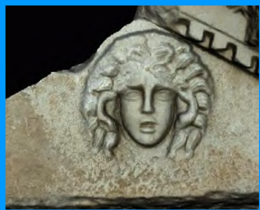
$$\pi^{\mathsf{T}} \mathrm{X} = 0$$



Planes from points

Solve π from $X_1^T\pi = 0$, $X_2^T\pi = 0$ and $X_3^T\pi = 0$

$$\begin{bmatrix} X_1^T \\ X_2^T \\ X_3^T \end{bmatrix}\pi = 0 \quad \text{(solve } \pi \text{ as right nullspace of } \begin{bmatrix} X_1^T \\ X_2^T \\ X_3^T \end{bmatrix}\text{)}$$



Points from planes

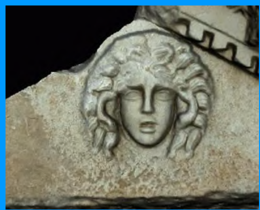
Solve X from $\pi_1^\mathsf{T}\mathrm{X}=0$, $\pi_2^\mathsf{T}\mathrm{X}=0$ and $\pi_3^\mathsf{T}\mathrm{X}=0$

$$\begin{bmatrix}\pi_1^\mathsf{T}\\ \pi_2^\mathsf{T}\\ \pi_3^\mathsf{T}\end{bmatrix}\mathrm{X}=0 \qquad \text{(solve } \mathrm{X} \text{ as right nullspace of } \begin{bmatrix}\pi_1^\mathsf{T}\\ \pi_2^\mathsf{T}\\ \pi_3^\mathsf{T}\end{bmatrix}\text{)}$$

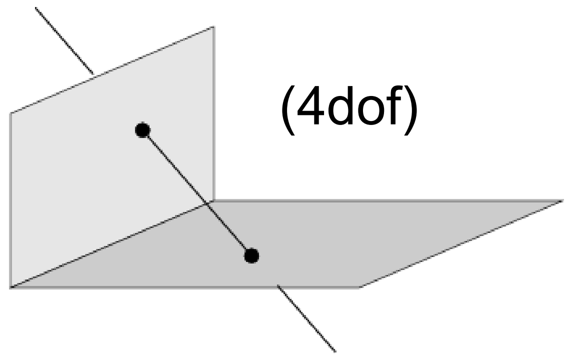
Representing a plane by its span

$$\mathrm{X}=\mathbf{M}\,\mathrm{x} \qquad \mathbf{M}=\left[\mathrm{X}_1\mathrm{X}_2\mathrm{X}_3\right]$$

$$\pi^\mathsf{T}\,\mathbf{M}=0$$



Lines



(4dof)

Representing a line by its span

$$W = \begin{bmatrix} A^{\mathsf{T}} \\ B^{\mathsf{T}} \end{bmatrix} \qquad \lambda A + \mu B$$

Dual representation

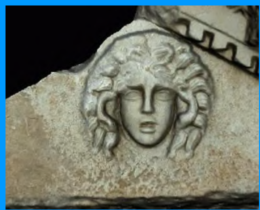
$$W^* = \begin{bmatrix} P^{\mathsf{T}} \\ Q^{\mathsf{T}} \end{bmatrix} \qquad \lambda P + \mu Q$$

$$W^* W^{\mathsf{T}} = W W^{*\mathsf{T}} = 0_{2\times 2}$$

Example: X-axis

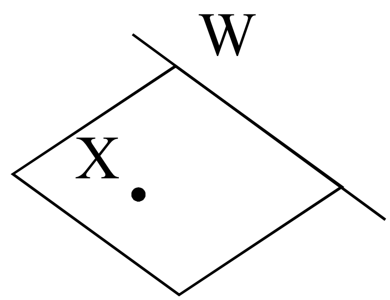
$$W = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad W^* = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

(Alternative: Plücker representation, details see e.g. H&Z)

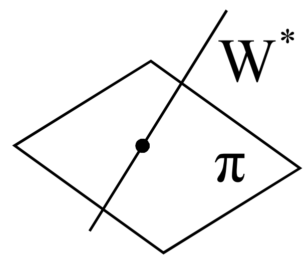


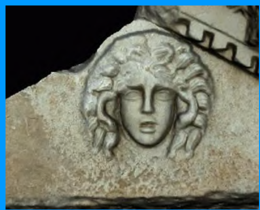
Points, lines and planes

$$\mathbf{M} = \begin{bmatrix} W \\ X^{\mathsf{T}} \end{bmatrix} \qquad \mathbf{M}\,\pi = 0$$



$$\mathbf{M} = \begin{bmatrix} W^{*} \\ \pi^{\mathsf{T}} \end{bmatrix} \qquad \mathbf{M}\,X = 0$$





Plücker coordinates

Elegant representation for 3D lines

$$l_{ij} = A_i B_j \quad B_i A_j$$ (with A and B points)

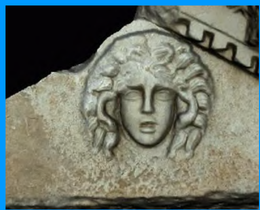
$$\mathsf{L} = [l_{12}, l_{13}, l_{14}, l_{23}, l_{42}, l_{34}]^{\mathsf{T}} \in \mathbf{P}^5$$

$$(\mathsf{L} \mid \hat{\mathsf{L}}) = l_{12}\hat{l}_{34} + l_{13}\hat{l}_{42} + l_{14}\hat{l}_{23} + l_{23}\hat{l}_{14} + l_{42}\hat{l}_{13} + l_{34}\hat{l}_{12}$$

$$(\mathsf{L} \mid \mathsf{L}) = 0$$ (Plücker internal constraint)

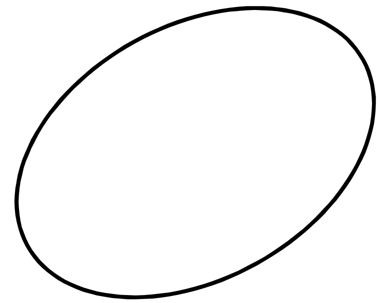
$$(\mathsf{L} \mid \hat{\mathsf{L}}) = \det[\mathrm{A}, \mathrm{B}, \hat{\mathrm{A}}, \hat{\mathrm{B}}] = 0$$ (two lines intersect)

(for more details see e.g. H&Z)



Conics

Curve described by 2nd-degree equation in the plane



$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

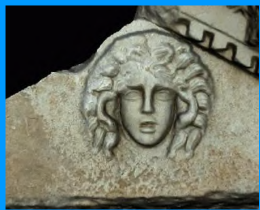
or *homogenized* $x \mapsto x_1/x_3,\ y \mapsto x_2/x_3$

$$ax_1^2 + bx_1x_2 + cx_2^2 + dx_1x_3 + ex_2x_3 + fx_3^2 = 0$$

or in matrix form

$$\mathrm{x}^{\mathsf{T}}\mathbf{C}\,\mathrm{x} = 0 \quad \text{with} \quad \mathbf{C} = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

5DOF: $\{a : b : c : d : e : f\}$



Five points define a conic

For each point the conic passes through

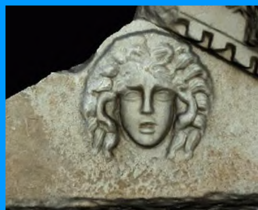
$$ax_i^2 + bx_i y_i + cy_i^2 + dx_i + ey_i + f = 0$$

or

$$\left(x_i^2, x_i y_i, y_i^2, x_i, y_i, 1\right)\mathbf{c} = 0 \qquad\qquad \mathbf{c} = \left(a, b, c, d, e, f\right)^{\mathsf{T}}$$

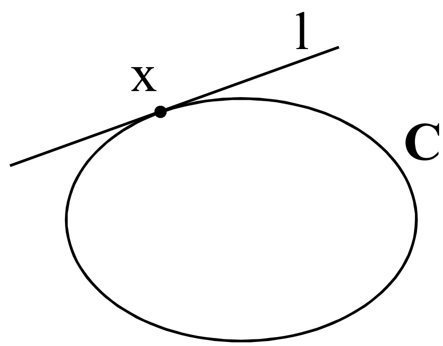
stacking constraints yields

$$\begin{bmatrix} x_1^2 & x_1 y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2 y_2 & y_2^2 & x_2 & y_2 & 1 \\ x_3^2 & x_3 y_3 & y_3^2 & x_3 & y_3 & 1 \\ x_4^2 & x_4 y_4 & y_4^2 & x_4 & y_4 & 1 \\ x_5^2 & x_5 y_5 & y_5^2 & x_5 & y_5 & 1 \end{bmatrix} \mathbf{c} = 0$$



Tangent lines to conics

The line l tangent to $\mathbf{C}$ at point x on $\mathbf{C}$ is given by $l=\mathbf{C}x$

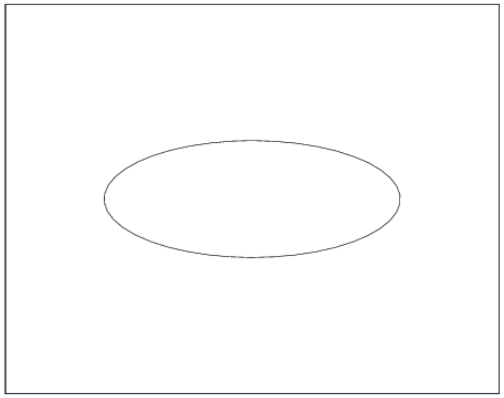


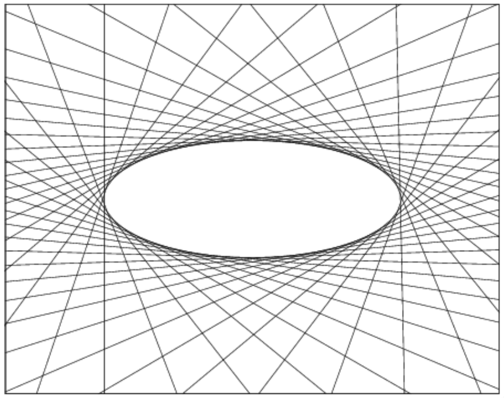
Dual conics

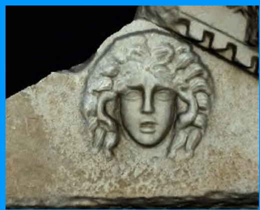
A line tangent to the conic $\mathbf{C}$ satisfies $\mathbf{l}^{\mathsf{T}}\,\mathbf{C}^{*}\,\mathbf{l}=0$

In general ($\mathbf{C}$ full rank): $\mathbf{C}^{*}=\mathbf{C}^{-1}$

Dual conics = line conics = conic envelopes



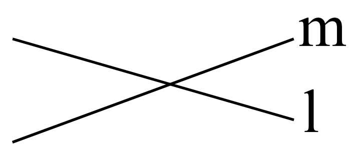




Degenerate conics

A conic is degenerate if matrix $\mathbf{C}$ is not of full rank

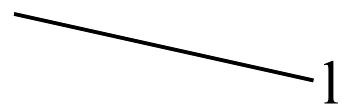
e.g. two lines (rank 2)



$$\mathbf{C} = \mathrm{l}\mathrm{m}^{\mathsf{T}} + \mathrm{m}\mathrm{l}^{\mathsf{T}}$$

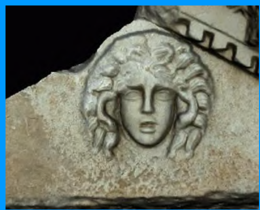
e.g. repeated line (rank 1)

$$\mathbf{C} = \mathrm{l}\mathrm{l}^{\mathsf{T}}$$



Degenerate line conics: 2 points (rank 2), double point (rank1)

Note that for degenerate conics $\left(\mathbf{C}^{*}\right)^{*} \neq \mathbf{C}$



Quadrics and dual quadrics

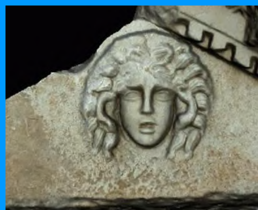
$\mathrm{X}^{\mathsf{T}}\mathrm{QX}=0$ (Q : 4x4 symmetric matrix)

$$\mathrm{Q}=\begin{bmatrix} \bullet & \bullet & \bullet & \bullet \\ \circ & \bullet & \bullet & \bullet \\ \circ & \circ & \bullet & \bullet \\ \circ & \circ & \circ & \bullet \end{bmatrix}$$

- 9 d.o.f.
- in general 9 points define quadric
- det Q=0 ↔ degenerate quadric
- tangent plane $\pi=\mathrm{QX}$

$\pi^{\mathsf{T}}\mathrm{Q}^{*}\pi=0$

- relation to quadric $\mathrm{Q}^{*}=\mathrm{Q}^{-1}$ (non-degenerate)



2D projective transformations

<u>Definition:</u>

A *projectivity* is an invertible mapping h from P^2 to itself such that three points x_1,x_2,x_3 lie on the same line if and only if $h(x_1),h(x_2),h(x_3)$ do.

<u>Theorem:</u>

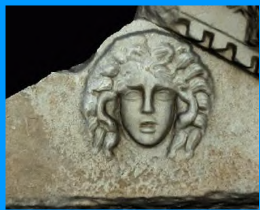
A mapping $h:P^2 \rightarrow P^2$ is a projectivity if and only if there exist a non-singular 3x3 matrix $\mathbf{H}$ such that for any point in P^2 reprented by a vector x it is true that $h(x)=\mathbf{H}x$

<u>Definition:</u> Projective transformation

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \text{or} \quad x' = \mathbf{H}\,x$$

8DOF

projectivity=collineation=projective transformation=homography



Transformation of 2D points, lines and conics

For a point transformation

$$\mathbf{x}' = \mathbf{H}\,\mathbf{x}$$

Transformation for lines

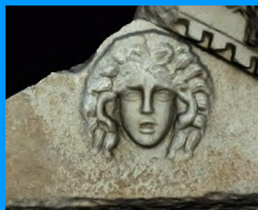
$$\mathbf{l}' = \mathbf{H}^{-\mathsf{T}}\,\mathbf{l}$$

Transformation for conics

$$\mathbf{C}' = \mathbf{H}^{-\mathsf{T}}\mathbf{C}\mathbf{H}^{-1}$$

Transformation for dual conics

$$\mathbf{C}'^{*} = \mathbf{H}\mathbf{C}^{*}\mathbf{H}^{\mathsf{T}}$$

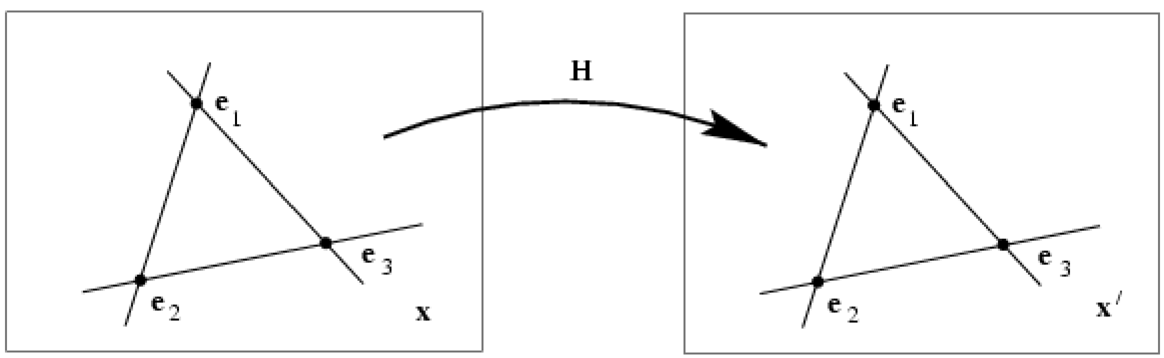


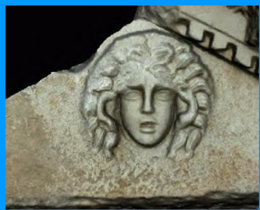
Fixed points and lines

$\mathbf{H}\,\mathrm{e} = \lambda\,\mathrm{e}$ (eigenvectors $\mathbf{H}$ =fixed points)

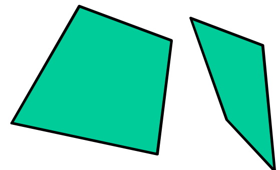
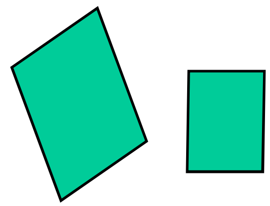
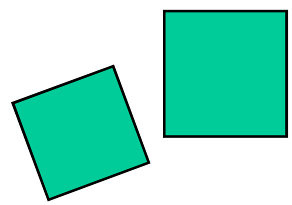
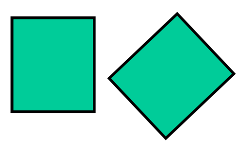
($\lambda_1 = \lambda_2 \Rightarrow$ pointwise fixed line)

$\mathbf{H}^{T}\,\mathrm{l} = \lambda\,\mathrm{l}$ (eigenvectors $\mathbf{H}^{-T}$ =fixed lines)

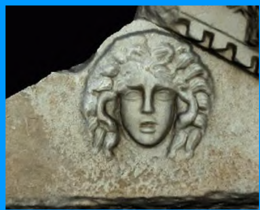




Hierarchy of 2D transformations

		transformed squares	invariants
Projective 8dof	$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$		Concurrency, collinearity, order of contact (intersection, tangency, inflection, etc.), cross ratio
Affine 6dof	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Parallellism, ratio of areas, ratio of lengths on parallel lines (e.g midpoints), linear combinations of vectors (centroids). **The line at infinity l_∞**
Similarity 4dof	$\begin{bmatrix} sr_{11} & sr_{12} & t_x \\ sr_{21} & sr_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Ratios of lengths, angles. **The circular points I,J**
Euclidean 3dof	$\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		lengths, areas.

ETH



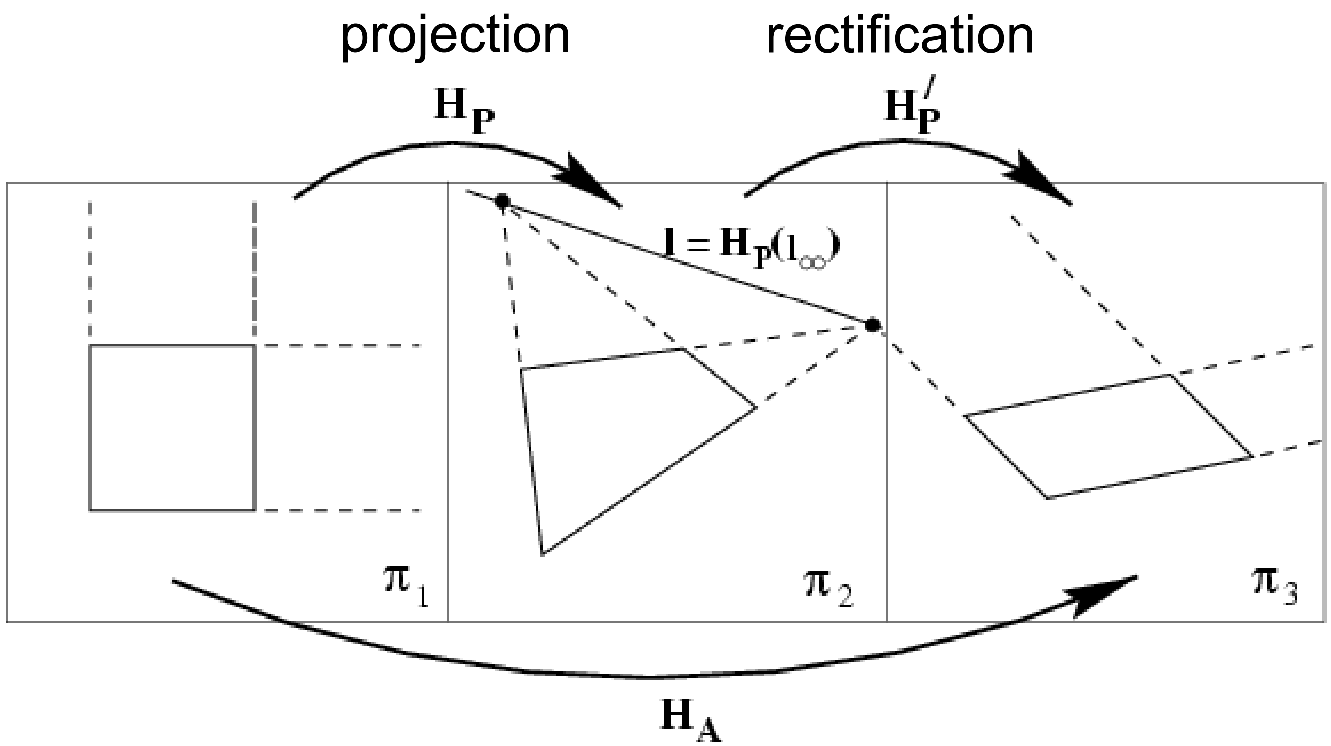
The line at infinity

$$l'_\infty = \mathbf{H}_A^{\mathsf{T}} l_\infty = \begin{bmatrix} \mathbf{A}^{\mathsf{T}} & 0 \\ \mathbf{A}\,t & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = l_\infty$$

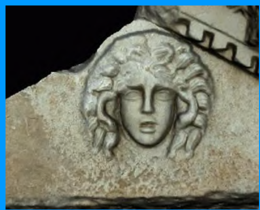
The line at infinity l_∞ is a fixed line under a projective transformation H if and only if H is an affinity

Note: not fixed pointwise

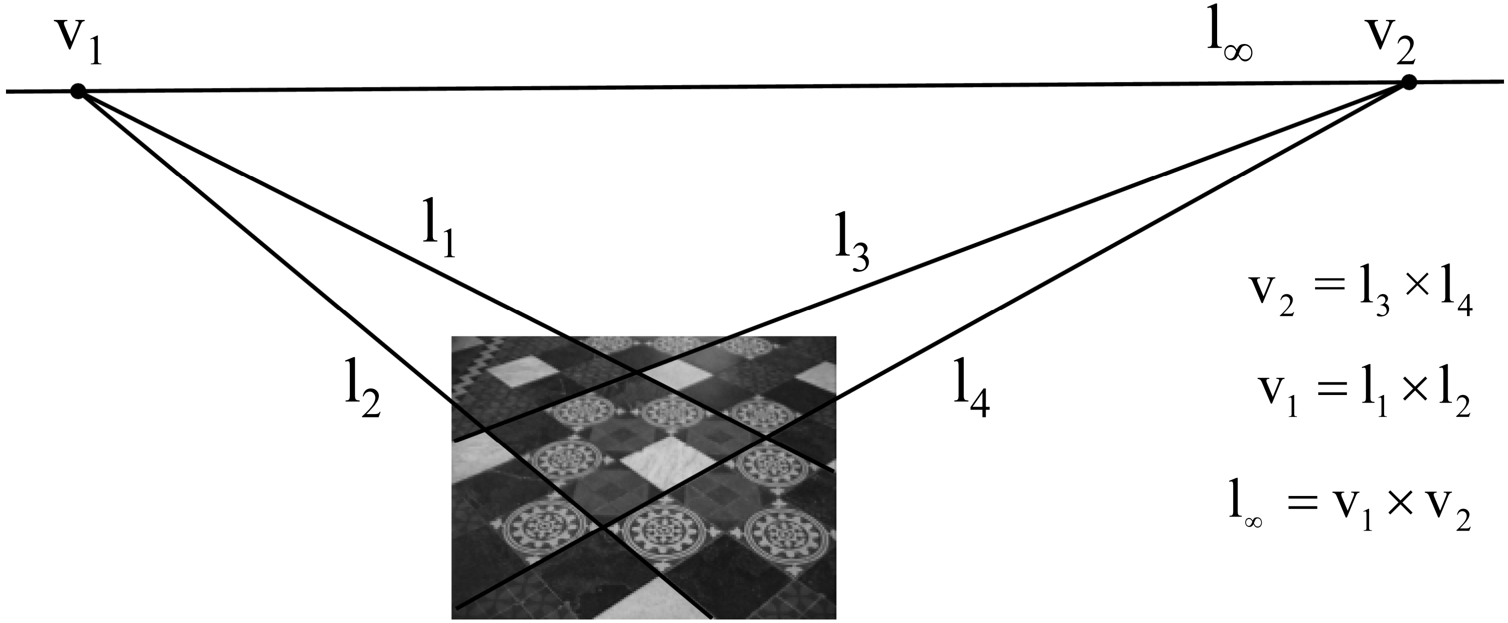
Affine properties from images



$$\mathbf{H}_{PA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix} \mathbf{H}_A \qquad \mathbf{l}_\infty = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}^\mathsf{T}, l_3 \neq 0$$



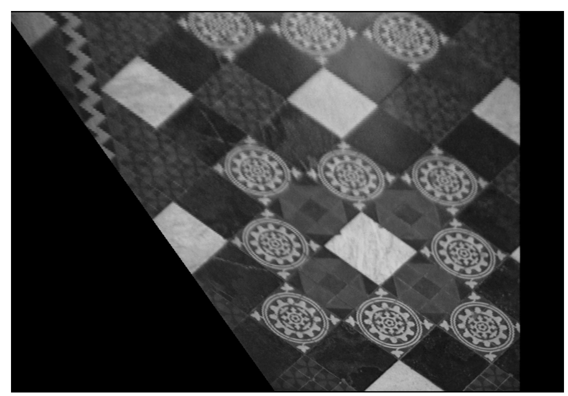
Affine rectification

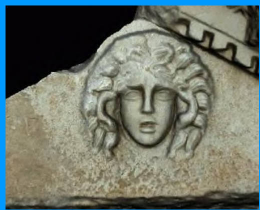


$$v_2 = l_3 \times l_4$$

$$v_1 = l_1 \times l_2$$

$$l_\infty = v_1 \times v_2$$





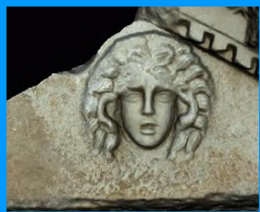
The circular points

$$\mathrm{I} = \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} \qquad \mathrm{J} = \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

$$\mathrm{I}' = \mathbf{H}_S\,\mathrm{I} = \begin{bmatrix} s\cos\theta & s\sin\theta & t_x \\ -s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} = se^{i\theta} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} = \mathrm{I}$$

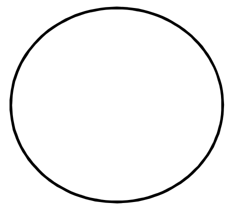
The circular points I, J are fixed points under the projective transformation $\mathbf{H}$ iff $\mathbf{H}$ is a similarity

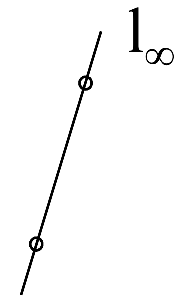
ETH



The circular points

"circular points"





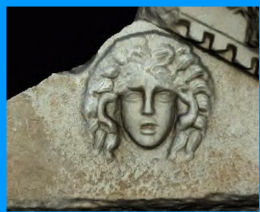
$$x_1^2 + x_2^2 + dx_1x_3 + ex_2x_3 + fx_3^2 = 0$$
$$x_3 = 0$$

$$x_1^2 + x_2^2 = 0$$

$$\mathrm{I} = (1, i, 0)^\mathsf{T}$$
$$\mathrm{J} = (1, -i, 0)^\mathsf{T}$$

Algebraically, encodes orthogonal directions

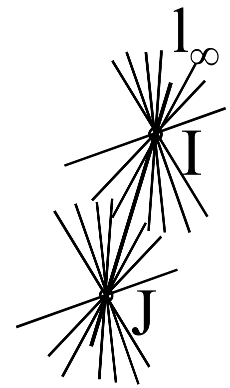
$$\mathrm{I} = (1,0,0)^\mathsf{T} + i(0,1,0)^\mathsf{T}$$



Conic dual to the circular points

$$\mathbf{C}^*_\infty = \mathrm{IJ}^\mathsf{T} + \mathrm{JI}^\mathsf{T}$$

$$\mathbf{C}^*_\infty = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



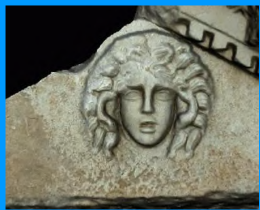
$$\mathbf{C}^*_\infty = \mathbf{H}_S \mathbf{C}^*_\infty \mathbf{H}_S^\mathsf{T}$$

The dual conic $\mathbf{C}^*_\infty$ is fixed conic under the projective transformation $\mathbf{H}$ iff $\mathbf{H}$ is a similarity

Note: $\mathbf{C}^*_\infty$ has 4DOF

l_∞ is the nullvector

ETH



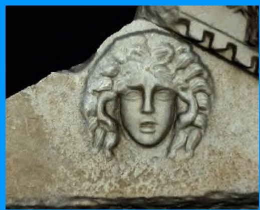
Angles

Euclidean: $\mathrm{l} = \left(l_1, l_2, l_3\right)^{\mathsf{T}} \qquad \mathrm{m} = \left(m_1, m_2, m_3\right)^{\mathsf{T}}$

$$\cos\theta = \frac{l_1 m_1 + l_2 m_2}{\sqrt{\left(l_1^2 + l_2^2\right)\left(m_1^2 + m_2^2\right)}}$$

Projective: $$\cos\theta = \frac{\mathrm{l}^{\mathsf{T}}\, \mathbf{C}_\infty^*\, \mathrm{m}}{\sqrt{\left(\mathrm{l}^{\mathsf{T}}\, \mathbf{C}_\infty^*\, \mathrm{l}\right)\left(\mathrm{m}^{\mathsf{T}}\, \mathbf{C}_\infty^*\, \mathrm{m}\right)}}$$

$\mathrm{l}^{\mathsf{T}}\, \mathbf{C}_\infty^*\, \mathrm{m} = 0$ (orthogonal)



Transformation of 3D points, planes and quadrics

For a point transformation (cfr. 2D equivalent)

$$\mathrm{X}' = \mathbf{H}\,\mathrm{X} \qquad (\mathrm{x}' = \mathbf{H}\,\mathrm{x})$$

Transformation for lines

$$\pi' = \mathbf{H}^{-\mathsf{T}}\,\pi \qquad (\mathrm{l}' = \mathbf{H}^{-\mathsf{T}}\,\mathrm{l})$$

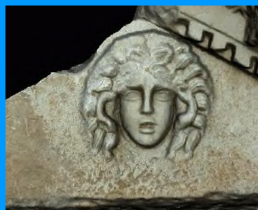
Transformation for conics

$$\mathrm{Q}' = \mathrm{H}^{-\mathsf{T}}\mathrm{Q}\mathrm{H}^{-1} \qquad (\mathbf{C}' = \mathbf{H}^{-\mathsf{T}}\mathbf{C}\mathbf{H}^{-1})$$

Transformation for dual conics

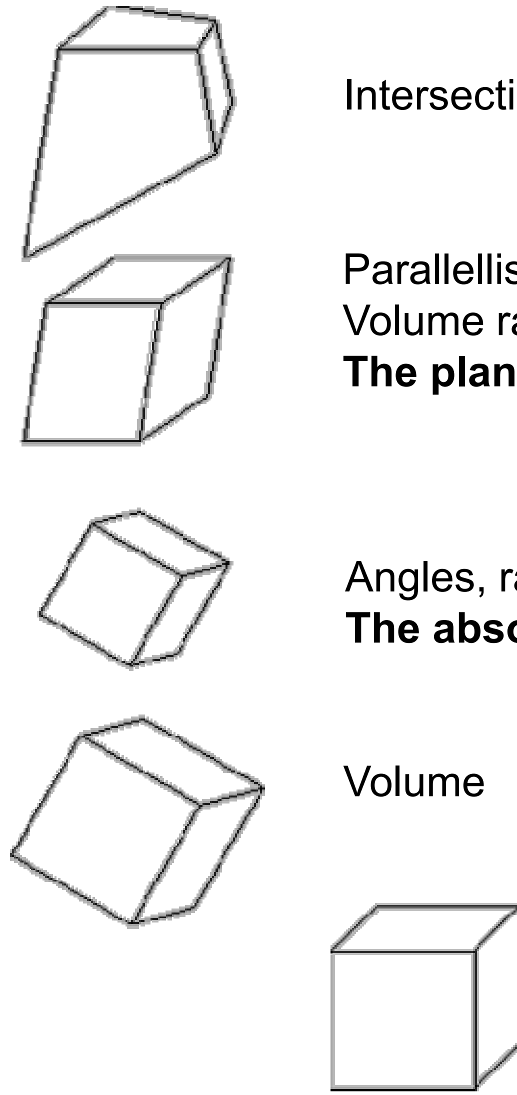
$$\mathrm{Q}'^{*} = \mathrm{H}\mathrm{Q}^{*}\mathrm{H}^{\mathsf{T}} \qquad (\mathbf{C}'^{*} = \mathbf{H}\mathbf{C}^{*}\mathbf{H}^{\mathsf{T}})$$

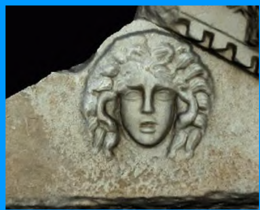
ETH



Hierarchy of 3D transformations

Projective 15dof	$\begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$	Intersection and tangency
Affine 12dof	$\begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$	Parallellism of planes, Volume ratios, centroids, **The plane at infinity π_∞**
Similarity 7dof	$\begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix}$	Angles, ratios of length **The absolute conic Ω_∞**
Euclidean 6dof	$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$	Volume



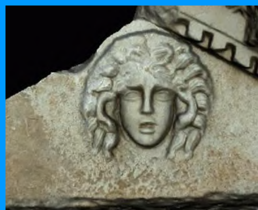


The plane at infinity

$$\pi'_\infty = \mathbf{H}_A{}^{\mathsf{T}}\pi_\infty = \begin{bmatrix} \mathbf{A}^{\mathsf{T}} & 0 \\ -\mathbf{A}\,\mathrm{t} & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \pi_\infty$$

The plane at infinity π_∞ is a fixed plane under a projective transformation H iff H is an affinity

1. canonical position $\pi_\infty = (0,0,0,1)^{\mathsf{T}}$
2. contains directions $\mathrm{D} = (X_1, X_2, X_3, 0)^{\mathsf{T}}$
3. two planes are parallel $\Leftrightarrow$ line of intersection in π_∞
4. line // line (or plane) $\Leftrightarrow$ point of intersection in π_∞



The absolute conic

The absolute conic Ω_∞ is a (point) conic on π_∞.

In a metric frame:

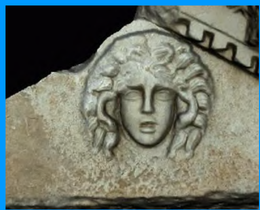
$$\left.\begin{array}{c} X_1^2 + X_2^2 + X_3^2 \\ X_4 \end{array}\right\} = 0$$

or conic for directions: $(X_1, X_2, X_3)\mathrm{I}(X_1, X_2, X_3)^{\mathsf{T}}$

(with no real points)

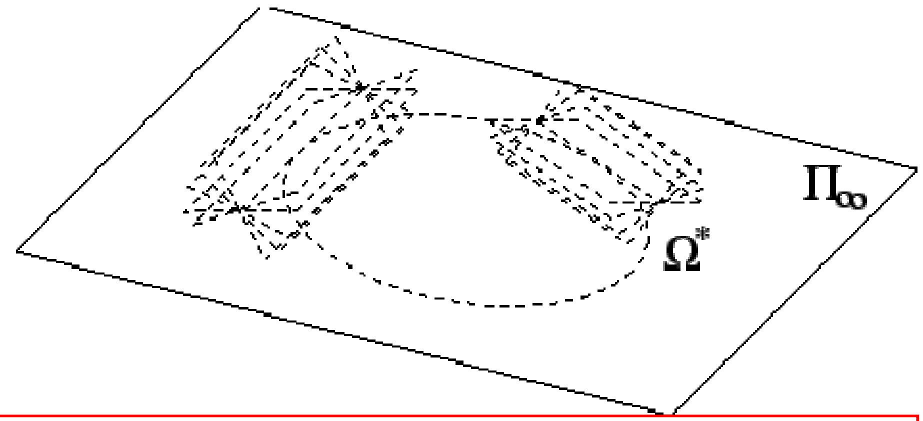
The absolute conic Ω_∞ is a fixed conic under the projective transformation $\mathbf{H}$ iff $\mathbf{H}$ is a similarity

1. Ω_∞ is only fixed as a set
2. Circle intersect Ω_∞ in two circular points
3. Spheres intersect π_∞ in Ω_∞



The absolute dual quadric

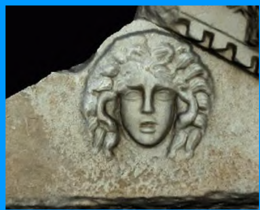
$$\Omega^*_\infty = \begin{bmatrix} I & 0 \\ 0^\mathsf{T} & 0 \end{bmatrix}$$



The absolute dual quadric Ω^*_∞ is a fixed conic under the projective transformation $\mathbf{H}$ iff $\mathbf{H}$ is a similarity

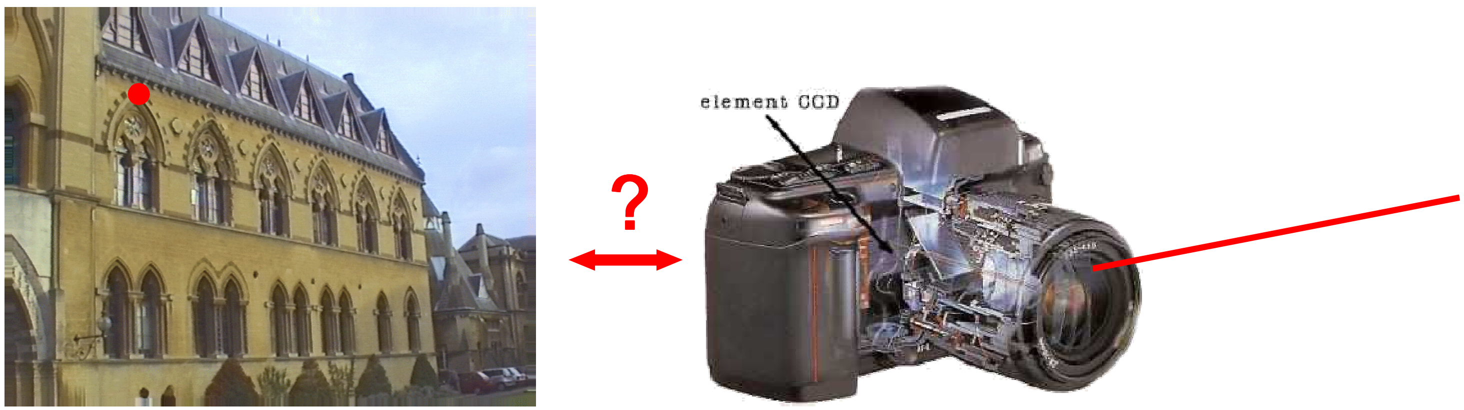
1. 8 dof
2. plane at infinity π_∞ is the nullvector of Ω_∞
3. Angles:

$$\cos\theta = \frac{\pi_1^\mathsf{T} \Omega^*_\infty \pi_2}{\sqrt{\left(\pi_1^\mathsf{T} \Omega^*_\infty \pi_1\right)\left(\pi_2^\mathsf{T} \Omega^*_\infty \pi_2\right)}}$$

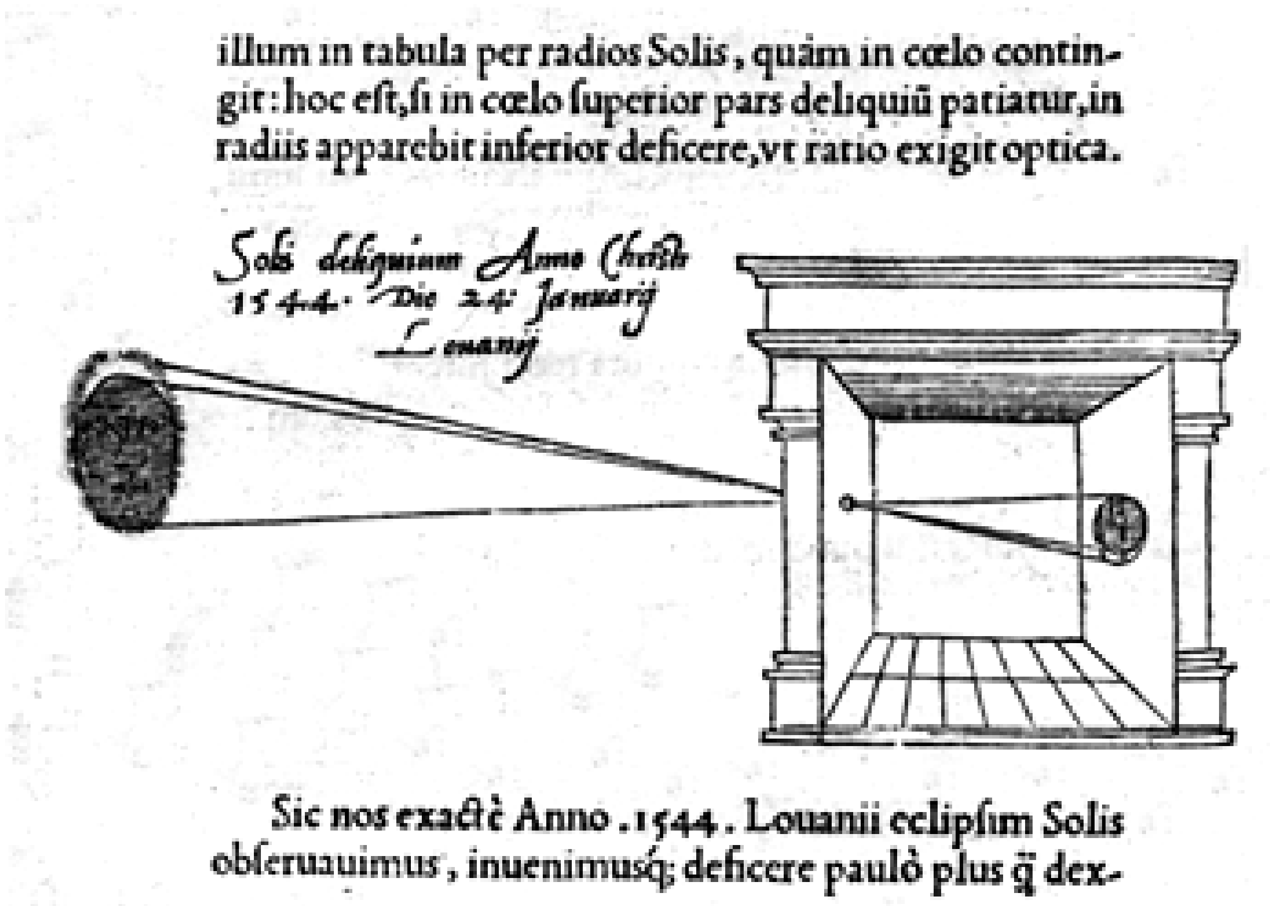


Camera model

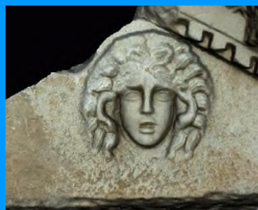
Relation between pixels and rays in space



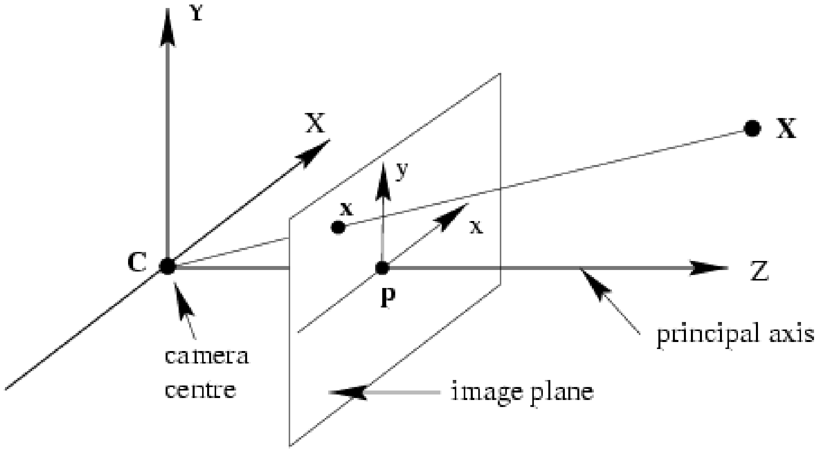
Pinhole camera

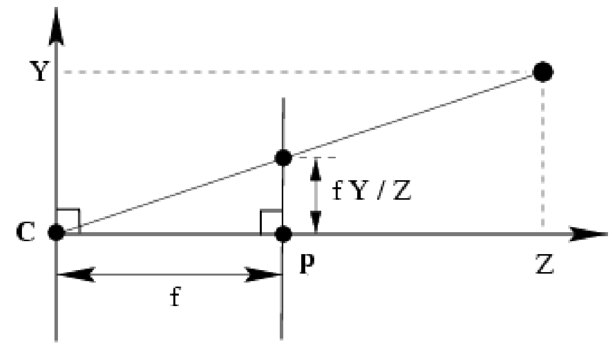

illum in tabula per radios Solis, quàm in cœlo contingit: hoc est, si in cœlo superior pars deliquiū patiatur, in radiis apparebit inferior deficere, vt ratio exigit optica.

Sic nos exactè Anno .1544. Louanii eclipsim Solis obseruauimus, inuenimusq; deficere paulò plus q̄ dex-



Pinhole camera model



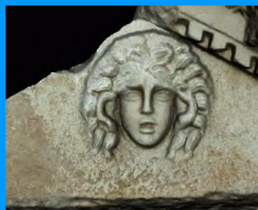


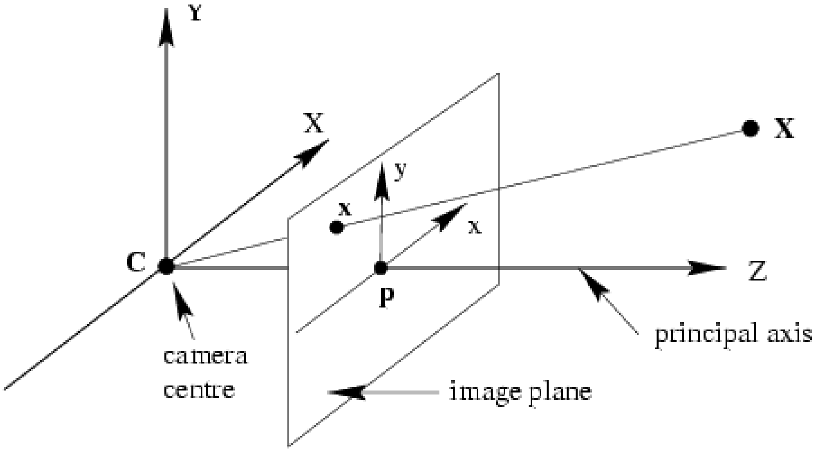
$$(X, Y, Z)^T \mapsto (fX / Z, fY / Z)^T$$

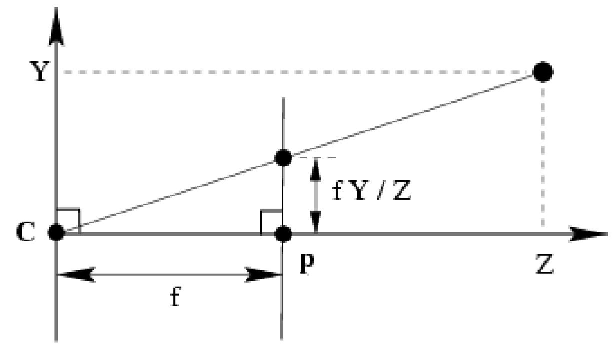
$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

linear projection in homogeneous coordinates!

Pinhole camera model





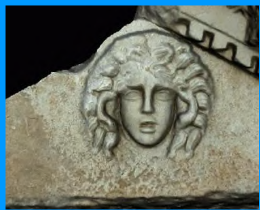


$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

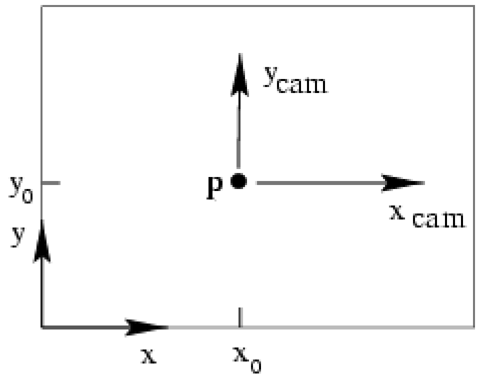
$$\mathrm{x} = \mathrm{PX}$$

$$\begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathrm{P} = \mathrm{diag}(f, f, 1)\left[\mathrm{I} \mid 0\right]$$



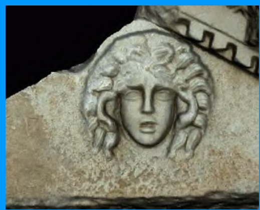
Principal point offset



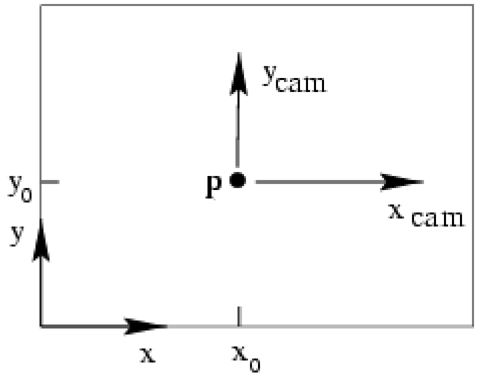
$$(X,Y,Z)^T \mapsto (fX/Z + p_x, fY/Z + p_y)^T$$

$(p_x, p_y)^T$ principal point

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX + Zp_x \\ fY + Zp_x \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



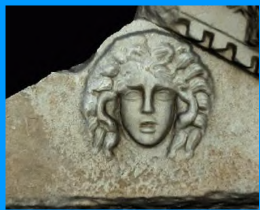
Principal point offset



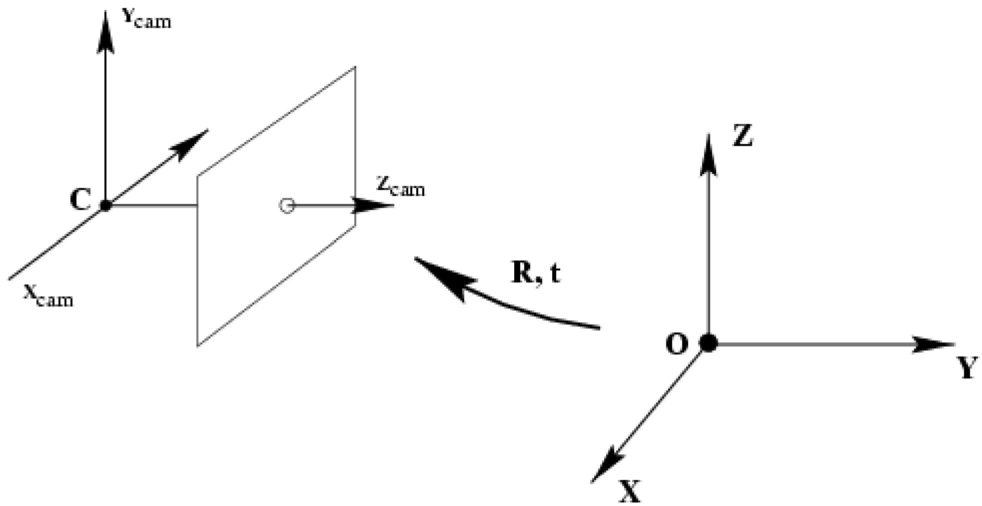
$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_x \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathrm{x} = \mathrm{K}[\mathrm{I} \mid 0]\mathrm{X}_{cam}$$

$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$ calibration matrix



Camera rotation and translation



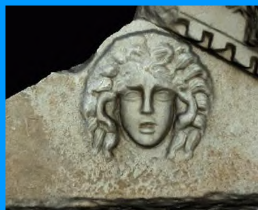
$$\tilde{X}_{cam} = R\left(\tilde{X} - \tilde{C}\right)$$

$$X_{cam} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} X$$

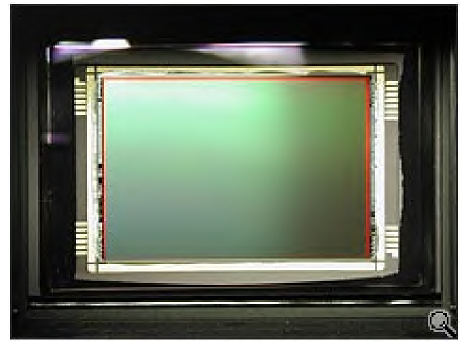
$$\mathbf{x} = \mathbf{K}[I \mid 0] X_{cam}$$

$$x = KR\left[I \mid -\tilde{C}\right] X$$

$$x = PX \qquad P = K[R \mid t] \qquad t = -R\tilde{C}$$

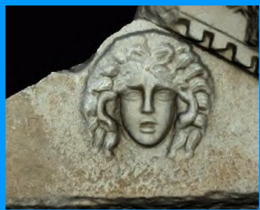


CCD camera



$$K = \begin{bmatrix} \alpha_x & & p_x \\ & \alpha_y & p_y \\ & & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$



General projective camera

$$K = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_x & p_y \\ & & 1 \end{bmatrix}$$

$$P = \underbrace{KR}_{\text{non-singular}}\left[I \mid \quad \tilde{C}\right]$$

11 dof (5+3+3)

$$P = K\left[R \mid t\right]$$

intrinsic camera parameters
extrinsic camera parameters

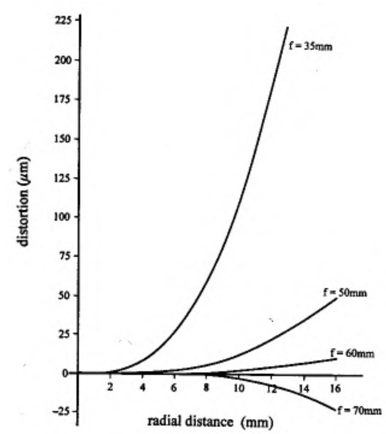
Radial distortion

- Due to spherical lenses (cheap)
- Model:

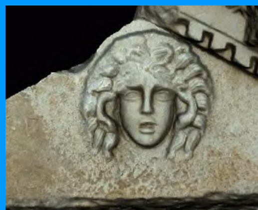
$$\begin{bmatrix}x\\y\\1\end{bmatrix} \sim \begin{bmatrix} f_x & s & c_x\\0&f_y&c_y\\0&0&1\end{bmatrix} R\left(\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}\begin{bmatrix}\mathbf{R}^\top & -\mathbf{R}^\top \mathtt{t}\\0_3^\top & 1\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}\right)$$

$$R \qquad (x,y) = (1+K_1(x^2+y^2)+K_2(x^4+y^4)+\ldots)\begin{bmatrix}x\\y\end{bmatrix}$$

straight lines are not straight anymore

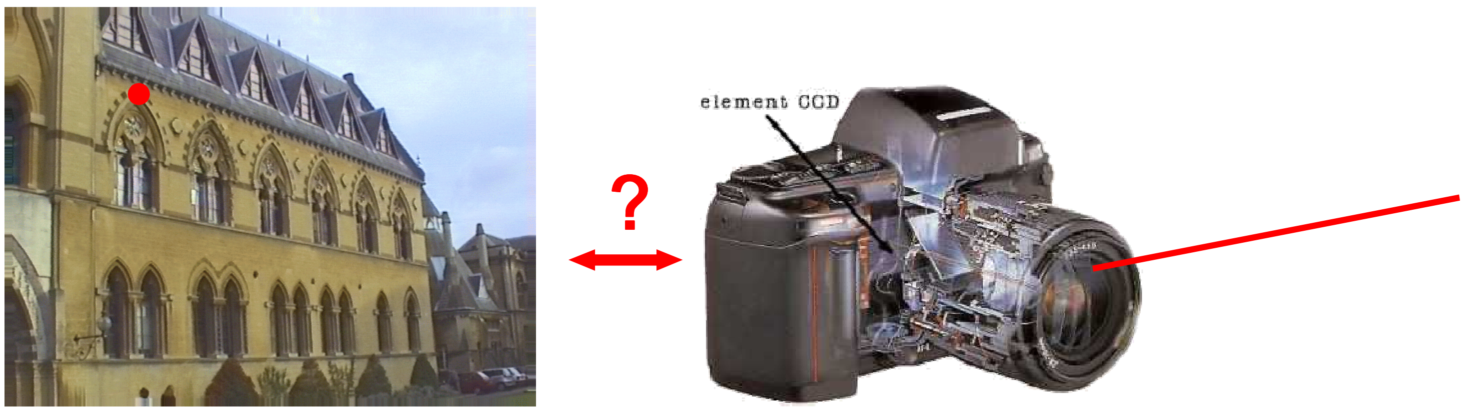


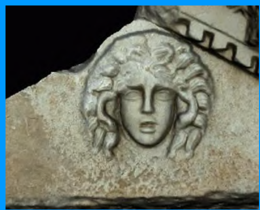
http://foto.hut.fi/opetus/260/luennot/11/atkinson_6-11_radial_distortion_zoom_lenses.jpg



Camera model

Relation between pixels and rays in space

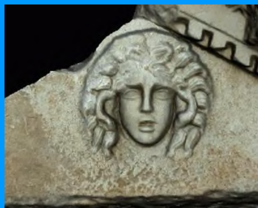




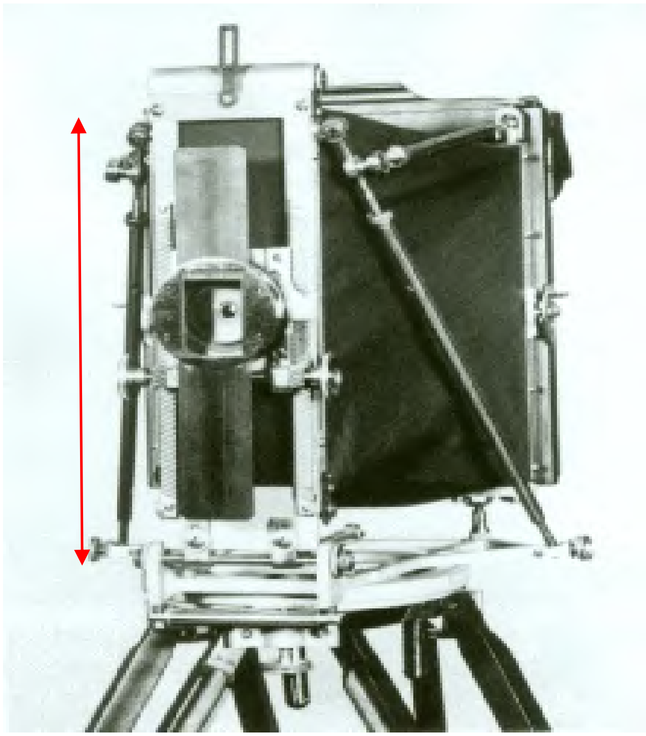
Projector model

Relation between pixels and rays in space
(dual of camera)

(main geometric difference is vertical principal point offset
to reduce keystone effect)



Meydenbauer camera



vertical lens shift
to allow direct
ortho-photographs

Fig. 5: The principle of »Plane-Table Photogrammetry«
(after an instructional poster of Meydenbauer's institute)

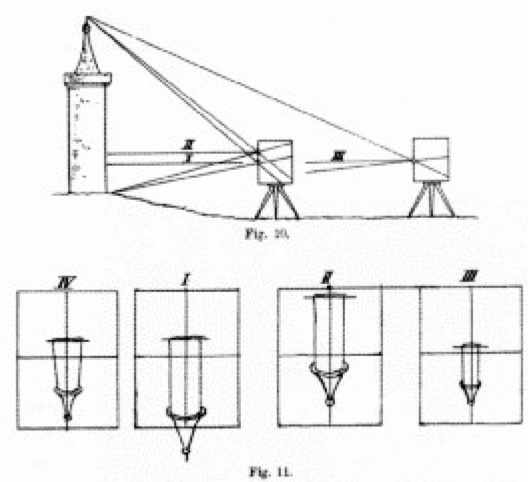
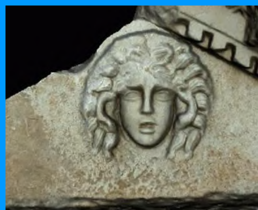
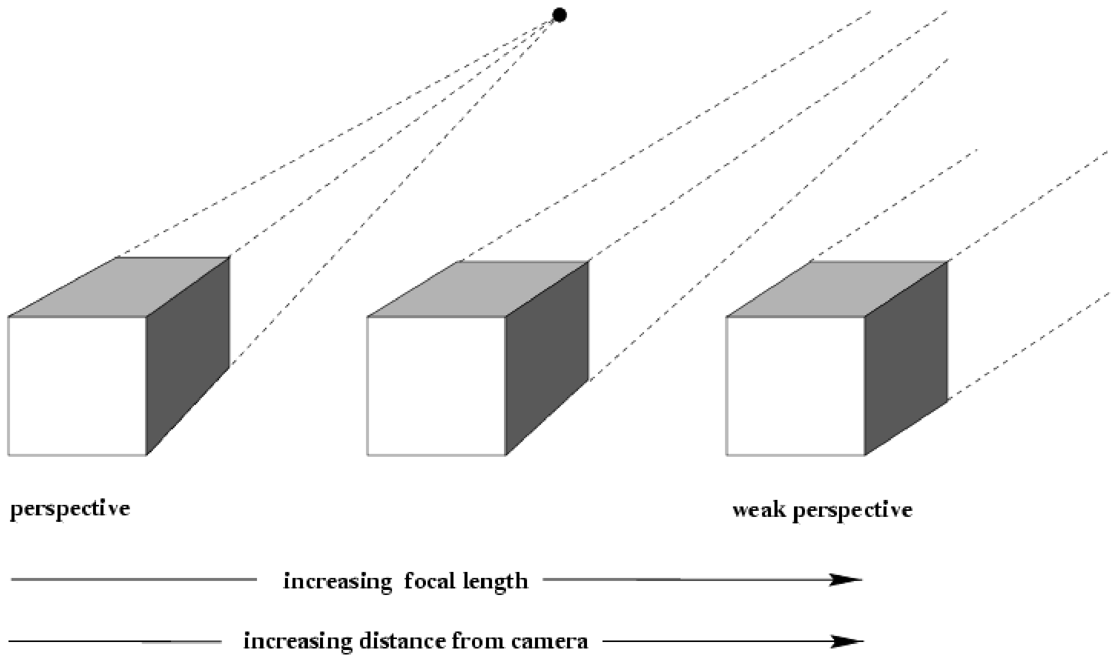
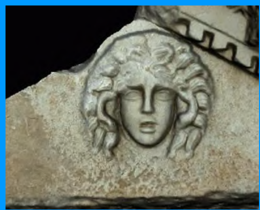


Fig. 6: The effect of a vertical shift of the camera lens;
the position II makes the best use of the image format
(after Meydenbauer's textbook from 1912)



Affine cameras





Action of projective camera on points and lines

projection of point

$$x = PX$$

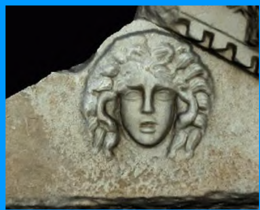
forward projection of line

$$X(\mu) = P(A + \mu B) = PA + \mu PB = a + \mu b$$

back-projection of line

$$\Pi = P^T l$$

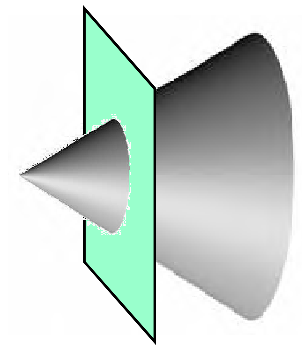
$$\Pi^T X = l^T PX \qquad \left(l^T x = 0;\ x = PX\right)$$



Action of projective camera on conics and quadrics

back-projection to cone

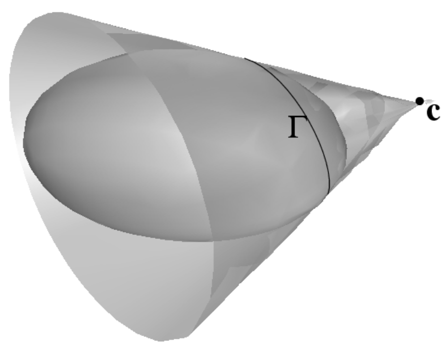
$$Q_{co} = P^T C P \qquad x^T C x = X^T P^T C P X = 0$$



$$(x = PX)$$

projection of quadric

$$C^* = P Q^* P^T \qquad \Pi^T Q^* \Pi = l^T P Q^* P^T l = 0$$



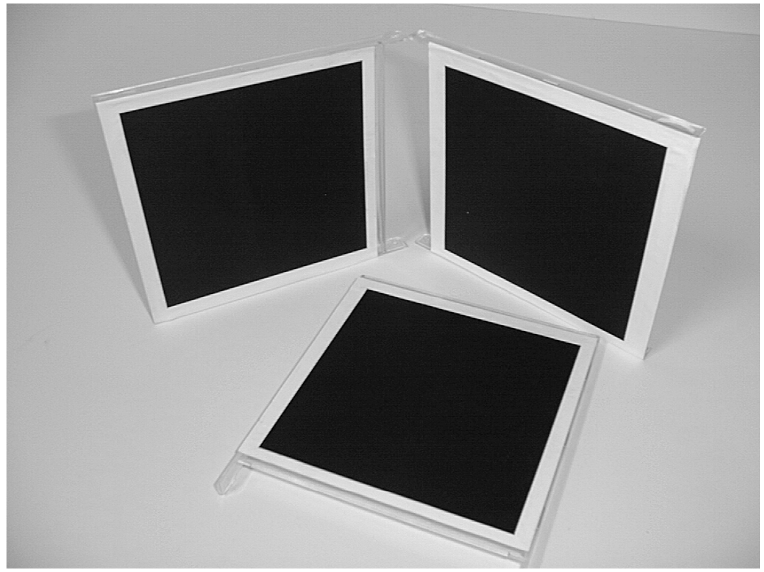
$$(\Pi = P^T l)$$

Image of absolute conic

$$
\begin{aligned}
\omega^* &= \mathbf{P}\Omega^*\mathbf{P}^\top \\
&= \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathtt{t} \end{bmatrix}\begin{bmatrix} \mathbf{I} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{R}^\top \\ \mathtt{t}^\top \end{bmatrix}\mathbf{K}^\top \\
&= \mathbf{K}\mathbf{K}^\top
\end{aligned}
$$

$$
\omega = \mathbf{K}^{-1}\mathbf{K}^{-\top}
$$

A simple calibration device



(i) compute H for each square (corners @ (0,0),(1,0),(0,1),(1,1))

(ii) compute the imaged circular points $H(1,\pm i,0)^T$

(iii) fit a conic to 6 circular points

(iv) compute K from ω through cholesky factorization

(≈ Zhang's calibration method)

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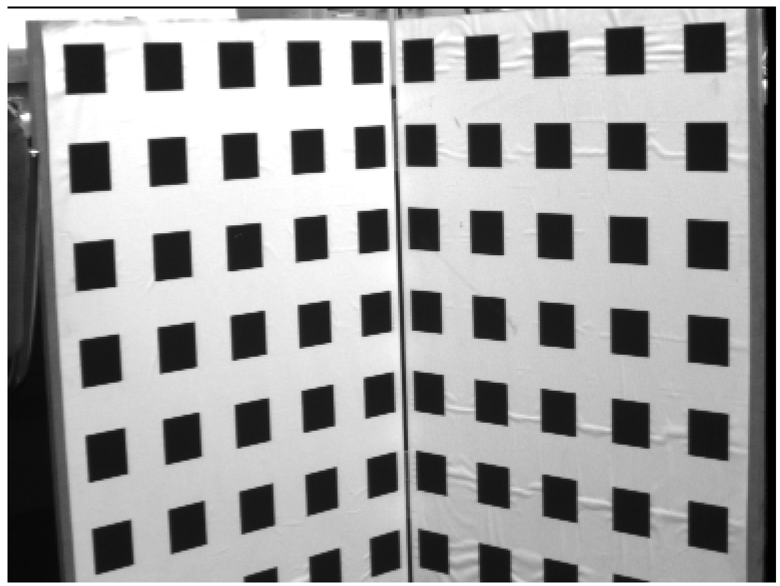
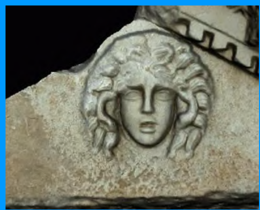
Exercises: Camera calibration





Next class:
Single View Metrology

Antonio Criminisi ETH