$$\frac{\partial S_f}{\partial t} = \frac{D_S}{L_f^2} \frac{\partial^2 S_f}{\partial x^2} + \frac{x(L_f)_t'}{L_f} \frac{\partial S_f}{\partial x} - q_{1S} \frac{O_{2f}}{K_{O_2} + O_{2f}} \frac{S_f}{K_S + S_f} X_{f1} - q_2 \frac{S_f}{K_{S,an} + S_f} X_{f2}$$