Revisiting the Fable of the Bees: A Note

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In Pigou’s terms, an externality creates a disparity between private and social costs or benefits. Thus, in the presence of an externality, markets transactions fail to internalize or account for all the costs and benefits. Without interference in the price mechanism, some transactions that would be beneficial would not be sustained. An externality occurs when the activity of one agent influences consumption or output of a second agent **and** that effect is not priced in a market. The first (or direct, competitive general equilibrium ) theorem of welfare economics fails in the presence jointness of consumption or production externalities because firms or consumers will not account of the external benefits or costs of their own actions when making choice decisions.

In Meade’s seminal taxonomy, two cases of production externalities – one-way, and reciprocal. The one-way externality was formulated as:  The reciprocal externality as follows: “…the relationship which we have examined might be a reciprocal one. While the apples may provide the food of the bees, the bees may fertilize the apples. ….” In Meade’s formulation, the output of the apple farmer A (apples) enters into the production function of the honey farmer B (bees) and the reciprocal – bees into apples: Meade demonstrates the equilibrium condition by equating elasticities in a interdependent homogenous production function. LEROY double CHECK me ON THIS TO SEE HOW MEADE SOLVES HIS.

Finding the interdependent production function unnecessarily complex, Cheung’s frames the reasoning of partial equilibrium analysis: “The reciprocal situation in which a beekeeper is able to extract honey from the same farm to which he renders pollination services poses an interesting theoretic riddle. The traditional analysis of such a condition relies on some interdependent production functions, and is, I think, unnecessarily complex. The method employed here simply treats pollination services and honey yield as components of a joint product generated by the hive.” p288-289 Cheung constructs a partial equilibrium diagram for a single input (hives) that produces a joint product of pollination (for apple production) and nectar (for honey production):



The demand functions are interrelated by the fact that apple pollination is a function of hive from B and Nectar of B is a function of hive from A. (Leroy this is not clear, please check) Demands for the non-rival pollination services and honey are horizontally summed and equated with the supply of hives to obtain the social level of hives. By the horizontal summation both pollination and nectar demand are internalized and are thus expressed in the market. Partial equilibrium, while useful to demonstrate equilibrium for a single input (hives), is unable to demonstrate the failure of the Pareto optimality as does general equilibrium.

 Leroy and garth need to reconstruct Chueng’s diagrams with math and show the equilbirum condition. Notice garth is in small letters. This analysis involves which of the two or one input are vertically summed versus horizontally summed. Leroy do we really need to do this???

Leroy, I am stumped here. There seems to be an internal contradiction in Cheung. He states that there is a single input hives. But the real inputs are pollination and necter for apples and honey, respectively. I am wondering if Chueng has to make the assumption of a single input beacuese he he is using partial? THIS IS THE KEY!!!!!!!!!! HE CANT SHOW A WORLD OF INPUTS. The optimal in GE has to contain at least 2 inputs.

In contrast to Chueng’s partial equilibrium and Meade’s production share analysis we will contruct a two-firm, two-input exchange economy general equilibrium model of an economy of the type illustrate by an Edgeworth-Bowley box and that is used to model externalities in current textbooks. The general equilibrium conditions for a production-production externality (pollination and honey) and then examines the subsets of that generalized case (see Appendix 1). In the context of the general equilibrium we can examine one way externality and reciprocal externality. To find the Pareto optimality conditions for production function with externalities we solve the construct of a Edgeworth Box between Farmers A and B, with inputs capital (K) and water (W) , for the crop (Y). The construct of the Edgeworth is used to derive the contract curve between farmers A and B. To derive a point on the contract curve we have fixed amounts of water W0 and capital K0 and we fixed the level of crop from farmer B at . The fixed level of output from farmer B is set at any level Y0 and the amounts of inputs are fixed at W0 = WA + WB and K0 = KA + KB. Two producers (farmers) A and B with production functions for output (Y) with two inputs capital (K) and water (W).

 In the absence of the externality optimality conditions the two production function are not related: $Y\_{A}=f\_{A}\left(K\_{A},W\_{A}\right) and Y\_{B}=f\_{B}\left(K\_{B},W\_{B}\right)$. The conditions for a competitive equilibrium: $MRS\_{W\_{A}K\_{A}}^{A}=MRS\_{W\_{B}K\_{B}}^{B}$. The first theorem states that an allocation of factors or goods resulting from a *competitive general equilibrium* is Pareto optimal and Pareto optimality conditions that the marginal rates of substitution for all agents must equilibrate and sum of the individual agent consumption bundles must sum to the total endowment.

**Case #1 One-way externality**

When a single input into farmer A’s production function becomes an input into farmer B’s production the result is a one-way externality. WB or WA are zero then the case defaults to a one way externality. The factor WB and WA entering into the production functions of the other farmers production function are the Meade’s “unpaid factor” -- the source of the externality. In producing YA, farmer A, generates Meade’s “unpaid factor” an externality Wa = ga(Ya) and when farmer A produces Y at some level that maximizes his profit, correspondingly the amount of input is fixed provided to farmer A is fixed i. e. $\overbar{W}\_{A}= g\_{A}\left(\overbar{Y}\_{A}\right)$. This is the case where the bees from farmer A’s bees pollinate farmer B’s apple trees. Parenthetically, when $\frac{∂Y\_{i}}{∂W\_{i}}>0, i\ne j $the externality is positive and conversely when $\frac{∂Y\_{i}}{∂W\_{i}}<0, i\ne j $the externality is negative. In other words, the definition of negative or positive externality depends not on the emitter of the externality but rather the production function of the recipient. Smoke from your cigar can be either positive or negative depending on my preference for smoking.

In producing YA, farmer A, generates the externality WA = gA(YA) and when farmer A produces YA at some level that maximizes his profit, correspondingly the amount of input is fixed provided to farmer A is fixed i. e. $\overbar{W}\_{A}= g\_{A}\left(\overbar{Y}\_{A}\right)$. The production functions are interdependent, the water input of farmer A enters into farmer B’s production function: $Y\_{A}=f\_{A}\left(K\_{A}, W\_{A}\right) and Y\_{B}=f\_{B}\left(K\_{B}, \overbar{W}\_{A}\right)$

LEROY IS THIS CORRECT OR SHOULD Yb=(Kb, Wa)or Yb=(Kb, Ya) ior somecombination thereof.



The optimality conditions show the disparity between social and private costs or the divergence from Pareto optimality. Notice how the optimality conditions or Pareto optimality conditions are derived. Production for the first farmer is being maximized subject to three constraints: (1) That the total water available is the sum of water use by farmer A plus farmer B and is equal to a fixed amount of water W0 ; and (2) That the total capital available is the sum of capital use by farmer A plus farmer B and is equal to a fixed amount of capital K0 ; and (3) That the production function of farmer B (which contains the unpaid factor WA ) is set at a fixed level . Farmer A is thus doing what is best for herself, but farmer B production function contains the externality, thus the optimality conditions will show what is best for Farmer A. With the one-way externality Farmer B's MRTS for water and capital in farmer’s B production equals Farmer A's MRTS for water and capital in farmer’s A production plus Farmer B's MRS for water in farmer’s A production as substituted farmer B's capital. The last term is the wedge or difference between social and private benefits, the Meade externality. The wedge reflects farmer B's need to find the substitute her capital against the water she receives from farmer A. For example, farmer B could install a sprinkler system to substitute for less water received from farmer A.

The first theorem of welfare economics fail in the presence of jointness of consumption or production externalities. That is an agent will not account for the external benefits or costs of their actions when making individual production or consumption decisions.

**Case #2 Two-way or reciprocal NOT cross input externality**

In producing YA, farmer A, generates the externality WA = gA(YA) and when farmer A produces YA at some level that maximizes his profit, correspondingly the amount of input is fixed provided to farmer A is fixed i. e. $\overbar{W}\_{A}= g\_{A}\left(\overbar{Y}\_{A}\right)$. And the reciprocal also enters from producing YB, farmer B, generates the externality WA = gA(YA) and when farmer A produces YA at some level that maximizes his profit, correspondingly the amount of input is fixed provided to farmer B is fixed i. e. $\overbar{W}\_{B}= g\_{B}\left(\overbar{Y}\_{B}\right)$. The production functions are reciprocally interdependent, the water input of farmer A enters into farmer B’s production function and the water input of farmer B enters into farmer A’s production function: $Y\_{A}=f\_{A}\left(K\_{A}, \overbar{W}\_{B}\right) and Y\_{B}=f\_{B}\left(K\_{B}, \overbar{W}\_{A}\right)$

Tis this how it should be??????he interrelated production functions for farmers A and B are:



This is how I had before but I like only 2 inputs better which do like leroy

LEROY, IAM NOT SURE THAT CHUENG DIDN’T HAVE IN MIND A CROSS INPUT EXTERNALITY. is the case for Chueng reciprocal externality? Relived of the partial equilibrium necessity of a single input, in a general equilibrium framework, Chuengs analysis becomes much clearer. The interrelated production functions (which Chueng thought were unnecessarily complex) are for two farmers (A and B) producing apples and honey are:



Where p is pollination services, and n is nectar. WHAT IS THE OTHER INPUT???? NOT HIVES

function: $Apples\_{A}=f\_{A}\left(K\_{A}, \overbar{hives}\_{B}\right) and Honey\_{B}=f\_{B}\left(K\_{B}, \overbar{hives}\_{A}\right)$



**Case #3 Two-way or reciprocal cross input externality**

In producing YA, farmer A, generates the externality KA = gA(YA) and when farmer A produces YA at some level that maximizes his profit, correspondingly the amount of input is fixed provided to farmer A is fixed i. e. $\overbar{K}\_{A}= g\_{A}\left(\overbar{Y}\_{A}\right)$. And the reciprocal also enters from producing YB, farmer B, generates the externality WB = gB(YB) and when farmer B produces YB at some level that maximizes his profit, correspondingly the amount of input is fixed provided to farmer A is fixed i. e. $\overbar{W}\_{B}= g\_{B}\left(\overbar{Y}\_{B}\right)$. The production functions are reciprocally interdependent, the water input of farmer A enters into farmer B’s production function and the water input of farmer B enters into farmer A’s production function: $Y\_{A}=f\_{A}\left(K\_{A}, \overbar{W}\_{B}\right) and Y\_{B}=f\_{B}\left(\overbar{K}\_{A}, W\_{B}\right)$

Leroy this is I had the case stated before In this case the water produced by farmer A enters into the production of Farmer B and the capital of farmer B enter into the production function of farmer A.Is this what mead had in mind for his formulation??????? Look at meade



Is this the formaulation that leroy solved????????????

!!!!!!!!!!Are Meade and Chueng really talking about 2 different formulations the reciprocal single input versus the reciprocal cross input. !!!!!!!!!!!!!!!!

**Case #4 Two one-way externalities** LEROY HOW DO WE STATE THIS CASE????

In producing YA, farmer A, generates the externality WA = gA(YA) and when farmer A produces YA at some level that maximizes his profit, correspondingly the amount of input is fixed provided to farmer A is fixed i. e. $\overbar{W}\_{A}= g\_{A}\left(\overbar{Y}\_{A}\right)$. In addition to the externality of input W, the externality also enters from producing YA, farmer A, generates the externality KA = gA(YA) and when farmer A produces YA at some level that maximizes his profit, correspondingly the amount of input is fixed provided to farmer B is fixed i. e. $\overbar{K}\_{B}= g\_{B}\left(\overbar{Y}\_{B}\right)$. The production functions are interdependent with two factors of production, the water input of farmer A enters into farmer B’s production function and the capital input of farmer A enters into farmer B’s production function: $Y\_{A}=f\_{A}\left(K\_{A},W\_{A} \right) and Y\_{B}=f\_{B}\left(\overbar{K}\_{A}, \overbar{W}\_{A}\right)$

LEROY, I THINK THAT THIS IS HOW I THINK THE RESULTS WILL LOOK????



Conclusions

Care must be given to postulating the each externality. Not only to the relationship of the interalted production functions as each interrelationship yields a dinctinct case. Care must also be give to scripting – sub or super spricts denote the agent producing the product or executing the production process or using the input BUT as importantly the index denotes ownership. For example Chueng doesn’t index the input hives but rather implies that the honey farmer owns the hives. By saying “that payment could be in honey or money????” For solutions to Coase problem ownership is key as a Couse soloution to externality.

Examing the fable of bees with partial equiblrium versus a general equilibrium framework provides different results. The differences in partial versus general is the definition of inputs – Cheung partial equilibrium defines a single input hives, the general equilibrium defines the two inputs that hive provide, pollination services for the orchard and honey or nectar for the beekeeper. The biological reciprocal relationship can be donstrated in general equilibrium analysis the fable of honey bees can be demonstrated to be mathematically reciprocal.

References

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Pigou, A. C. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Appendix 1. Partial equilibrium analysis of honey bees and apples.**

 As Cheung’s frames the analysis:

“The reciprocal situation in which a beekeeper is able to extract honey from the same farm to which he renders pollination services poses an interesting theoretic riddle. The traditional analysis of such a condition relies on some interdependent production functions, and is, I think, unnecessarily complex. The method employed here simply treats pollination services and honey yield as components of a joint product generated by the hive.” p288-289

LEROY MAGIC BEGINS HERE

**Appendix 2 General equilibrium framework of honey bees and apples.**

**Definitions**

**Two agents: farmer a and farmer b.**

**Two production inputs: k for capital and w for water.**

**A single output q**

**∂ partial derivative**

**λ Lagranian multiplier**

**δ total partial derivative as defined by Chang**

**d is derivative**

**Single Meade**:



Maximize:



Subject to:



The Lagrangian:

The first order conditions:





where:





Solve both equations for λ:



Expressing the solution as marginal rates of substitution:





When the use of surface irrigation water by the farm influences the production of water for the firm that uses the aquifer the production efficiency condition becomes The optimality condition for farmer 2 is:



**Cross input Chueng**:



Maximize:



Subject to:



The Lagrangian:

The first order conditions:





Where:







Thus:





Solve both equations for λ:



Expressing the solution as marginal rates of substitution:





Two Meades’

**Chueng:**



Maximize:



Subject to:



The Lagrangian:

The first order conditions:





where:







equating:



Expressing the solution as marginal rates of substitution:



 **Chueng:**



Maximize:



Subject to:



The Lagrangian:

The first order conditions:





where:







equating:



Expressing the solution as marginal rates of substitution:





**II. THE OBSERVEDPR ICING AND C ONTRACTUAL BEH AVIOR It is easy to find conclusive evidence showing that both nectar and pollination services are transacted in the marketplace: in some cities one need look no further than the yellow pages of the Telephone Directory. But the exis-tence of prices does not in itself imply an efficient allocation of resources. It is, therefore, necessary to demonstrate the effectiveness of the market in dic-tating the use even of those resources-bees, nectar, and pollen-which, ad-mittedly, are elusive in character and relatively insignificant in value. In doing so, I shall not attempt to estimate the standard sets of marginal values which an efficient market is said to equate: the burden of such a task must rest upon those who believe the government can costlessly and accurately make these estimates for the imposition of the "ideal" tax-subsidy schemes. Rather, I offer below an analysis based on the equimarginal principle. To the extent that the observed pricing and contractual behavior fails to falsify the im-plications derived from this analysis we conclude that (1) the observed behavior is explained, and (2) the observations are consistent with efficient allocation of resources.**

**A. The Analysis The reciprocal situation in which a beekeeper is able to extract honey from the same farm to which he renders pollination services poses an interesting theoretic riddle. The traditional analysis of such a condition relies on some interdependent production functions, and is, I think, unnecessarily complex.24 The method employed here simply treats pollination services and honey yield as components of a joint product generated by the hive. That is, the rental price per hive received by a beekeeper for placing his hives on a farm may be paid in terms of honey, of a money fee, or of a combination of both. The money fee or the honey yield may be either positive or negative, but their total measures the rental value of the hive. 23 See 7 U.S.C. ? 135 b, note (1970); Pub. L. No. 91-524 ? 804. My judgment is based both on the behavior of beekeepers (see next section) after the initiation of the Act and on the complexity of relevant claim forms which I have at hand. In April 1972 beekeepers associations were still lobbying for easier claiming conditions. 24 In J. E. Meade, supra note 3, at 58, this problem is set up in terms of the inter-dependent functions x1 = H1 (ll, cl, x2) and x2 =- H2 (12, c2, X1). I find Meade's analysis difficult to follow. Elsewhere, Otto A. Davis and Andrew Whinston employ the functions C1 = C1 (ql, q2) and C2 = C2 (ql, q2) in their treatment of certain "externalities." It is not clear, however, that the authors had the bee example in mind. See Otto A. Davis & Andrew Whinston, Externalities, Welfare, and the Theory of Games, 70 J. Pol. Econ. 241 (1962).**

