Faculty of Science, Engineering and Computing

RESIT COURSEWORK

Module Code: MA1030	Module Title: Introduction to Linear Algebra
Module Leader: Dr James Denholm-Price	
Student Name and ID No.:	

COURSEWORK REASSESSMENT

This is the *coursework component* of the summer reassessment (you may also have a resit exam – check your results on OSIS if you haven't already done-so).

For this reassessment you are required to complete all four of the following questions and submit solutions (with full working) to the Maths School Resit dropbox by 10AM on Friday 16th August 2013.

Deadline for Submission is

10am on 16th August 2013

to drop-in boxes in Sopwith Building.

MA1030 Introduction to Linear Algebra – Summer Resit Coursework

This coursework is due for submission by 10AM on Friday 16th August to the Student Office.

You should complete <u>all four</u> questions, showing full working throughout.

All questions carry equal marks.

1. (a) (i) Find the adjoint of the matrix $A = \begin{pmatrix} -1 & a & 2 \\ a & 4 & a \\ 2 & a & -1 \end{pmatrix}$ (where a is some scalar value).

(ii) For what value(s) of a will A have an inverse?

(iii) Hence find the solution to the system:

$$\begin{aligned} -x + 3y + 2z &= -2 \\ 3x + 4y + 3z &= 3 \\ 2x + 3y - z &= -11 \end{aligned}$$

(b) Given that for any pair of $n \times n$ matrices A and B you may assume $\det(AB) = \det(A)\det(B)$ use the rules of matrix algebra to prove that $\det(PQR) = \det(RQP)$ where P, Q and R are $n \times n$ matrices.

2. (a) (i) Find the (smaller) angle between the vectors $\mathbf{a} = \begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

(ii) Find a unit vector which is perpendicular to both the vectors **a** and **b** above.

(iii) Hence find the equation of the plane which contains the point (1, -1, 3), and which is parallel with both the vectors **a** and **b** above. (Any point in this plane may be reached by starting from (1, -1, 3) and moving in a direction given by a linear combination of the vectors **a** and **b** above ($\lambda\mathbf{a} + \mu\mathbf{b}$).)

(iv) Does the straight line with equation $\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ -8 \end{pmatrix}$ line in the plane of part (iii) above? Justify your answer.

(b) (i) Write down the vector equation of the straight line which passes through the point (-4, 1, -5) and which is perpendicular to the plane $2x - 3y + 2z = 30$.

(ii) Find the point where this straight line (from (b) part (i) above) intersects the plane $2x - 3y + 2z = 30$.

(iii) Hence calculate the shortest distance from the point (-4, 1, -5) to the plane $2x - 3y + 2z = 30$.

3. (a) Suppose that the vectors **u**, **v** and **w** are linearly independent. Show that **u** + 2**v**, **v** + 2**w** and **u** - 4**w** are linearly dependent.

(b) For which value(s) of λ do the following vectors form a linearly dependent set in $\mathbb{R}^3$?

$$\mathbf{v}_1 = \left(\lambda, -\tfrac{1}{2}, -\tfrac{1}{2}\right) \quad \mathbf{v}_2 = \left(-\tfrac{1}{2}, \lambda, -\tfrac{1}{2}\right) \quad \mathbf{v}_3 = \left(-\tfrac{1}{2}, -\tfrac{1}{2}, \lambda\right)$$

For any other value of λ, explain why $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ would form a basis in $\mathbb{R}^3$.

(c) Let *P*(t) denote a set of all real polynomials of the form

$$p(t) = a_0 + a_1 t^2 + a_2 t^4$$

where the coefficients $a_i \in \mathbb{R}$, $i = 0,1,2$. Show that

$$1 + t^2, \quad 1 - t^2, \quad 1 + t^2 + t^4$$

form a basis for *P*(t).

4. (a) Consider the matrix

$$A = \begin{pmatrix} 1 & 3 & -3 \\ -3 & 7 & -3 \\ -6 & 6 & 2 \end{pmatrix}.$$

(i) The characteristic equation for *A* is

$$(\lambda - 4)^2 (\lambda - 2) = 0 .$$

Find the eigenvalues of *A* and the corresponding eigenvectors.

(ii) Use Gaussian elimination to find the inverse of *A*.

(b) Let *B* be an $n \times n$ matrix with the eigenvalues λ_i, $i = 1, 2, \ldots, n$, and the corresponding eigenvectors $\mathbf{x}_i$, $i = 1, 2, \ldots, n$. Working from the definition of the eigenvalues and the eigenvectors of *B*, show that the matrix *A*, such that $B = P^{-1}AP$, where *P* is any non-singular matrix, has the same eigenvalues as *B*.

<u>End of coursework.</u>

(Due 10AM, 16th August 2013.)