

Sample test 1B

Problem 1. Show that all solutions of $2y' + ty = 2$ approach a limit as $t \rightarrow \infty$, and find the limiting value.

Problem 2. Find the general solutions of the differential equations

(a) $xdx + ye^{-x}dy = 0$

(b) $\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$.

Problem 3. A tank contains 100 gal of water and 50 oz of salt. Water containing a salt concentration of $\frac{1}{4}(1 + \frac{1}{2}\sin t)$ oz/gal flows into the tank at rate of 2 gal/min, and the mixture in the tank flows out at the same rate.

(a) Find the amount of salt in the tank at any time.

(b) The long-time behavior of the solution is an oscillation about a certain constant level. What is this level? What is the amplitude of the oscillation?

Problem 4. Find an integrating factor and the general solution of the equation

$$4\left(\frac{x^3}{y^2} + \frac{3}{y}\right)dx + 3\left(\frac{x}{y^2} + 4y\right)dy = 0.$$

Answers

Problem 1. See problem 32 in section 1.2.

Problem 2. See problem 11 and 32 in section 2.1.

Problem 3. See problem 5 in section 2.2.

Problem 4. See problem 30 in section 2.5.