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Center Manifolds  
for Semilinear Equations  
with Non-dense Domain  
and Applications to Hopf Bifurcation  
in Age Structured Models

Pierre Magal  
Shigui Ruan



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# Abstract

Several types of differential equations, such as delay differential equations, age-structure models in population dynamics, evolution equations with boundary conditions, can be written as semilinear Cauchy problems with an operator which is not densely defined in its domain. The goal of this paper is to develop a center manifold theory for semilinear Cauchy problems with non-dense domain. Using Liapunov-Perron method and following the techniques of Vanderbauwhede et al. in treating infinite dimensional systems, we study the existence and smoothness of center manifolds for semilinear Cauchy problems with non-dense domain. As an application, we use the center manifold theorem to establish a Hopf bifurcation theorem for age structured models.

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