

Bug 1484996 – Implementing a weighted shuffle

Suppose $w_1, w_2 \in \mathbb{R}^{>0}$. We can think of these as two ‘weights’.

Let $X_1, X_2 \sim U[0, 1]$ be independent random variables uniformly distributed in the unit interval.

Then

$$\begin{aligned} \mathrm{P}(X_1^{(1/w_1)} < X_2^{(1/w_2)}) &= \mathrm{P}(X_1 < X_2^{(w_1/w_2)}) \\ &= \int_0^1 \int_0^{X_2^{(w_1/w_2)}} 1 \, dX_1 \, dX_2 \\ &= \int_0^1 X_2^{(w_1/w_2)} \, dX_2 \\ &= \frac{w_2}{w_1 + w_2} \end{aligned}$$

$$\mathrm{P}(X_1^{(1/w_1)} > X_2^{(1/w_2)}) = \frac{w_1}{w_1 + w_2}$$

Equivalently, $X_1^{(1/w_1)} > X_2^{(1/w_2)}$ is $\frac{w_1}{w_2}$ ‘times’ as likely as $X_1^{(1/w_1)} < X_2^{(1/w_2)}$.

This motivates the following method of obtaining a weighted shuffle. Given a list of n objects with strictly-positive weights w_1, \dots, w_n , assign each the corresponding rank $x_i^{(1/w_i)}$ (where x_i is chosen uniformly at random from the unit interval), and sort the list by that rank.